Workbook answers

Chapter 1

Exercise 1.1

1 a Distance: scalar quantity, magnitude only. Displacement: vector quantity, magnitude and direction.

b Speed: scalar quantity, is the rate of change of position. Velocity: vector quantity, is the rate of change of displacement.

2 a i $v = \frac{s}{t} = \frac{200 \times 10^3}{90 \times 60} = 37\ \text{ms}^{-1}$ (2 s.f.)

ii The car may have changed its speed during its journey.

b $v = \frac{s}{t} = \frac{1.5 \times 10^3}{4.5} = 330\ \text{ms}^{-1}$ (2 s.f.)

c $v = \frac{s}{t} = \frac{6000 \times 10^3}{5 \times 24 \times 60 \times 60} = 14\ \text{ms}^{-1}$ (2 s.f.)

3 $t = \frac{s}{v} = \frac{117 \times 10^3}{97} = 1206\ \text{s} = 20$ minutes

4 a $t = \frac{s}{v} = \frac{3.78 \times 10^{16}}{3 \times 10^8} = 1.26 \times 10^8\ \text{s}$

$\left(= \frac{1.26 \times 10^8}{3.15 \times 10^7} = 4 \text{ years} \right)$

b Proxima Centauri is 4 *light-years* from the Earth.

5 a $a = \frac{v - u}{t} = \frac{1.2 - 0.6}{60} = 0.01\text{ms}^{-2}$

b $a = \frac{v - u}{t} = \frac{2 \times 10^7 - 0.0}{4.0 \times 10^{-9}} = 5 \times 10^{15}\ \text{ms}^{-2}$

c $a = \frac{v - u}{t} = \frac{30 - 90}{20 \times 60} = -0.05\ \text{ms}^{-2}$

6 Acceleration is defined as the rate of change of velocity and velocity is a vector quantity. The athlete changes direction going around the bend in the track, so his velocity changed; hence he accelerated.

7 a Using Pythagoras: displacement = $\sqrt{3^2 + 4^2} = 5$ cm

b $v = \frac{s}{t} = \frac{5 \times 10^{-2}}{100 \times 10^{-6}} = 500\ \text{ms}^{-1}$

c Direction = $\tan^{-1}\left(\frac{4}{3}\right) = 53° = 50°$ (1 s. f.) to the horizontal

d 500 ms^{-1} at 50° to the horizontal

8 a Using Pythagoras: displacement = $\sqrt{(5-1)^2 + (5 - -1)^2} = 7.2 = 7$ cm (1 s.f.)

b $v_{ave} = \frac{\text{distance travelled}}{\text{time taken}} = \frac{7}{4} = 1.75 =$ 2 cms^{-1} (1 s.f.)

c Displacement of ball = ((5 − 1), (5 − −1)) = (4, 6)

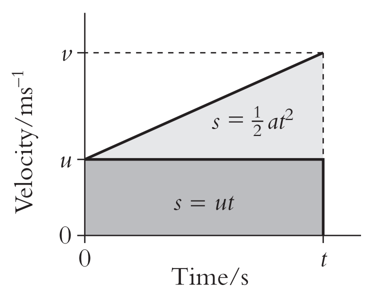
So angle to the x-axis is $\tan^{-1}\left(\frac{6}{4}\right) = 56° =$ 60° (1 s.f.)

d Velocity is 2 cms^{-1} at an angle of 60° to the x-axis.

Exercise 1.2

1 a $s = u\,t$

b



c $a = \frac{\text{change in velocity}}{\text{time}} = \frac{v - u}{t}$

d Area of triangular section = $\frac{1}{2}(v - u)\,t = \frac{1}{2}\frac{(v - u)}{t}t^2 = \frac{1}{2}at^2$

e See the triangular section on the figure in part b above.

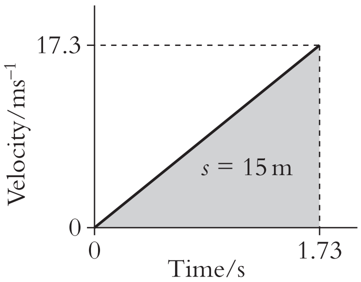
f $s = ut + \frac{1}{2}at^2$

2 a 28.8 km $\text{hour}^{-1} = \frac{28.8 \times 10^3}{60 \times 60} = 8.0\ \text{ms}^{-1}$

And $v = u + at = 8 + 2 \times 10 = 28\ \text{ms}^{-1}$

b $s = ut + \frac{1}{2}at^2 = 8 \times 10 + \frac{1}{2} \times 2 \times 10^2 =$ 180 m

3 **a**



b Using $s = \frac{1}{2}gt^2$, $t = \sqrt{\frac{2s}{g}} = \sqrt{\frac{2 \times 15}{10}} =$ 1.73 s

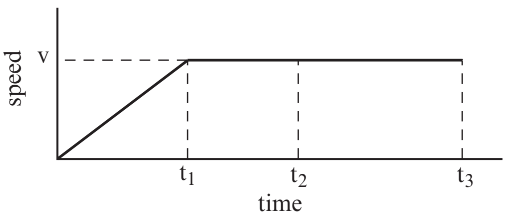
c $v = gt = 10 \times 1.73 = 17.3$ ms^{-1}

4 **a** $v = u - gt \Rightarrow t = \frac{v-u}{-g} = \frac{0-30}{-10} = 3$ s

b $s = ut - \frac{1}{2}gt^2 = 30 \times 3 - \frac{1}{2} \times 10 \times 3^2 =$ 45 m

5 Using $v^2 = u^2 + 2gs$, $s = \frac{v^2 - u^2}{2g} = \frac{36}{2 \times 9.8} =$ 1.8 m

6



Using the area under the graphs:

$v(t_3 - t_1) + \frac{1}{2}vt_1 = 200$, $v(t_2 - t_1) + \frac{1}{2}vt_1 = 100$

$\therefore vt_3 - \frac{1}{2}vt_1 = 200$ and $vt_2 - \frac{1}{2}vt_1 = 100$

$\therefore v = \frac{100}{(t_3 - t_2)} = \frac{100}{(19.19 - 9.58)} = 10.4$ ms^{-1}

Exercise 1.3

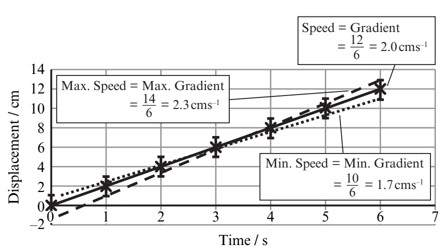
1 **a** **i** $v_{average} = \frac{\text{distance travelled}}{\text{time taken}} = \frac{12}{12} =$ 1.0 ms^{-1}

ii $v = \frac{s}{t} = \frac{12}{8} = 1.5$ ms^{-1}

b **i** Acceleration

ii Displacement

2 **a**



b See the figure in **part a**.

c Speed is the gradient of the graph. (In this question, the direction of motion is not important.)

See the figure in **part a**.

d The maximum speed is the maximum gradient—drawn from the bottom of the error bar on the point at $t = 0$ s to the top of the error bar on the point at $t = 6$ s. The minimum speed is the minimum gradient—drawn from the top of the error bar on the point at $t = 0$ s to the bottom of the error bar on the point at $t = 6$ s.

See the figure in **part a**.

e ½ the range of speeds = ½ (2.3 – 1.7) = 0.3 cms^{-1}

So speed = 2.0 ± 0.3 cms^{-1}

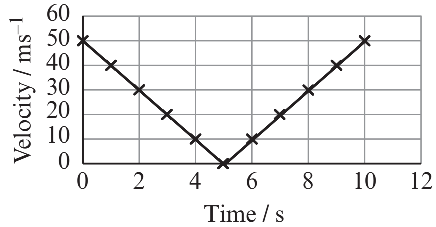
3 **a** The gradient of the graph shows the acceleration of the projectile.

This is $a = (-)\,\frac{50}{5} = (-)$ 10 ms^{-2} = g

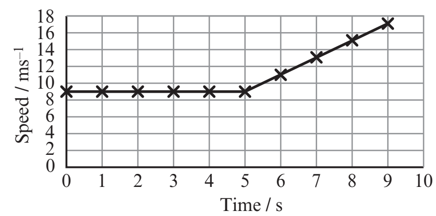
b Distance travelled = area under graph

$s = \frac{1}{2} \times 50 \times 5 = 125$ m

c



4 **a**



b Distance travelled = area under graph = $(9 \times 9) + \frac{1}{2}(17 - 9) \times 4 = 97$ m

c $s = ut_{total} + \frac{1}{2}at_{accel}{}^2 = 9 \times 9 + \frac{1}{2} \times 2 \times 4^2 =$ 97 m

(Note that because the speedboat had been travelling at a constant speed **before** it began to accelerate, the two “t’s” in the SUVAT equation are not the same.)

5 **a** The toy is moving forwards at $t = 0$ at a speed of 5 cms^{-1}. The toy slows down at a constant rate until it is momentarily stationary and then speeds up in the opposite direction at the same, constant, rate until its velocity is 7.5 cms^{-1} backwards.

b Acceleration is the gradient of the graph.

$a = \frac{-7.5 - 5}{20} = -0.625$ cms^{-2}

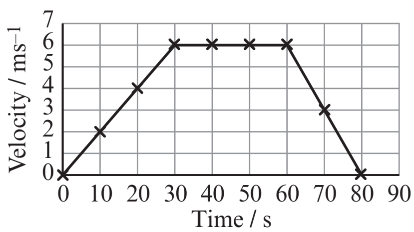
c Total displacement = area under graph =

$\frac{1}{2} \times (5 \times 8) + \frac{1}{2} \times (-7.5 \times 12) = -25$ cm

d **i** Actual distance travelled = $\frac{1}{2} \times (5 \times 8) + \frac{1}{2} \times (7.5 \times 12) = 65$ cm

ii Displacement and distance are not the same. In this case, the displacement means how far the toy was (at $t = 20$ s) from where it had been (at $t = 0$ s.)

6 **a**



b Total displacement = area under graph =

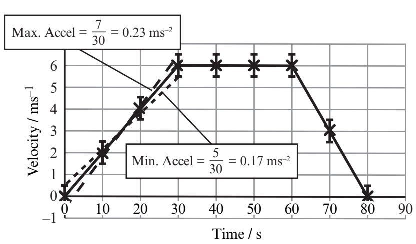
$\left(\frac{1}{2} \times 6 \times 30\right) + (6 \times 30) + \left(\frac{1}{2} \times 6 \times 20\right)$

= 330 m

c Acceleration = gradient of graph =

$\frac{6}{30} = 0.2$ ms^{-2}

d

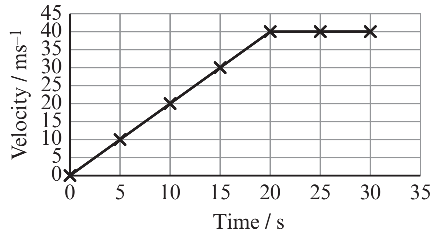


7 **a** Change of velocity = area under graph from $t = 0$ to $t = 20$ s

$= 2 \times 20$

$= 40$ ms^{-1}

b



c Using the sketch: displacement = area under graph =

$\left(\frac{1}{2} \times 40 \times 20\right) + (40 \times 10)$

= 800 m

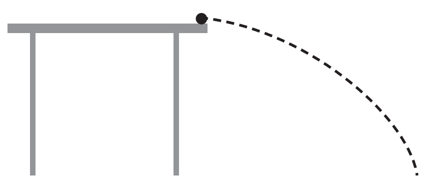
Alternative method: use the SUVAT equation $s = ut + \frac{1}{2}at^2$ and replace u with v_{final}.

Then, $s = v_{final}\, t + \frac{1}{2}at^2 = (40 \times 10) + \left(\frac{1}{2} \times 2 \times 20^2\right) = 800$ m

Note that the two "t"s are not the same.

Exercise 1.4

1 **a**



b The path is parabolic, because the horizontal component of the velocity does not change (since there are no forces acting horizontally on the marble) and the vertical component of the velocity increases (because of the force of gravity acting vertically) until the marble hits the floor.

c $t = \sqrt{\frac{2s}{g}} = \sqrt{\frac{2 \times 1}{10}} = 0.45$ s

d $s = vt = 1 \times 0.45 = 0.45$ m

2 **a** **i** $v = \sqrt{2gs} = \sqrt{2 \times 10 \times 80} =$ 40 ms^{-1}

ii $v_{total} = \sqrt{40^2 + 70^2} = 81$ ms^{-1} (2 s.f.)

In a direction of $tan^{-1}\left(\frac{40}{70}\right) = 30°$ from the horizontal

b The plane will be directly overhead when the crate hits the ground. (Both the plane and the crate have continued to travel with the same horizontal velocity and so will have travelled the same horizontal distance.)

3 **a** $t = \frac{s_{horizontal}}{v_{horizontal}} = \frac{75}{150} = 0.5$ s

b In the vertical direction: $u = 0$, and so

$s = \frac{v^2}{2g}$

$v_{vertical} = gt = 10 \times 0.5 = 5 \text{ ms}^{-1}$

So, $s = \frac{5^2}{2 \times 10} = 1.25$ m

c $v_{horizontal} = 150$, $v_{vertical} = 5$

So total vector has a magnitude of $v_{total} = \sqrt{150^2 + 5^2} = 150.1 \text{ ms}^{-1}$ (4 s.f.)

At an angle of $\theta = tan^{-1}\left(\frac{5}{150}\right) = 1.9°$ from the horizontal.

4 In the vertical direction: $v_v = 20\sin 30° = 10 \text{ ms}^{-1}$

So the time of flight, $t = \frac{v-u}{g} = \frac{10 - -10}{10} = 2$ s

In this time, Mercurio will travel a horizontal distance, $s = v_h\, t$

$v_h = 20 \cos 30° = 17.3 \text{ ms}^{-1}$ so $s = 17.3 \times 2 = 34.6$ m $= 35$ m (2 s.f.)

5 **a** $v_v = v \sin 40° = 10 \times 0.64 = 6.4 \text{ ms}^{-1}$

b $v = u - gt$

So, $t = \frac{v-u}{-g} = \frac{0 - 6.4}{-9.81} = 0.65$ s

c $s = ut - \frac{1}{2}gt^2 = 6.4 \times 0.65 - \frac{1}{2} \times 9.81 \times 0.65^2 = 2.1$ m

d In the absence of air friction, $x = 2 \times 0.65 \times 10 \cos 40° = 10$ m (2 s.f.)

6 $v_h = \frac{s_h}{t} = \frac{200}{8.0} = 25 \text{ ms}^{-1}$

$s_v = v_v t - \frac{1}{2}gt^2 \Rightarrow v_v = \frac{s_v + \frac{1}{2}gt^2}{t} = \frac{150 + \frac{1}{2} \times 10 \times 8^2}{8} = 58.75 \text{ ms}^{-1}$

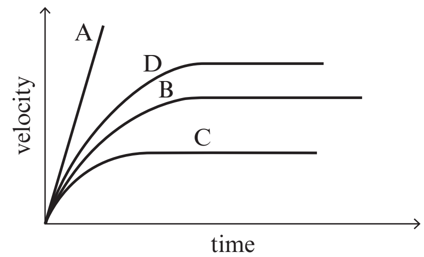
So, $v_{total} = \sqrt{25^2 + 58.75^2} = 64 \text{ ms}^{-1}$ at an angle of $tan^{-1}\left(\frac{58.75}{25}\right) = 67°$ from the horizontal.

7 **a** Since gradient $= g$, it should be 10 ms^{-2}.

b As the object's velocity increases, the effect of the fluid resistance increases. This effect reduces the acceleration of the object, thus reducing the gradient of the curve.

c When the force of resistance equals the force of gravity, the object will fall with a constant velocity (Newton's first law.). This constant velocity is called terminal velocity or terminal speed.

d **i** and **ii**



8 **a** $t = \sqrt{\frac{2s}{g}} = \sqrt{\frac{2 \times 2.5}{10}} = 0.707$ s

b $v = \frac{s}{t} = \frac{18.2}{0.707} = 25.7 \text{ ms}^{-1}$

c $t = \frac{s}{v} = \frac{11.9}{25.7} = 0.46$ s

d In 0.46 s, the ball will have fallen a vertical distance of $s = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 0.46^2 = 1.06$ m.

So the height of the ball will be $2.5 - 1.06 = 1.44$ m. This is above the height of the net (0.91 m). Therefore the tennis ball passes over the net.

e $v = \frac{230 \times 10^3}{60 \times 60} = 63.9 \text{ ms}^{-1}$

f $t = \frac{s}{v} = \frac{18.2}{63.9} = 0.285$ s

g No.

h Differences might be:

- The tennis ball is not hit horizontally from the racket.
- Air friction will act on the tennis ball.
- Spin imparted by the tennis racket may change the assumed trajectory of the tennis ball.

Effects of these might be:

- If the tennis ball is hit with an initial vertical velocity component, it will require less time to fall the 2.5 m from the top of the serve to the ground. This may allow the ball to land in the correct place on the other side of the net.
- Air friction will slow down the ball, perhaps then allowing the ball sufficient time to fall the 2.5 m.

- Spin will change the trajectory of the tennis ball so that its vertical velocity component changes at a different rate to that caused only by the effect of gravity. Such a "topspin" serve will cause the tennis ball to dip sharply as it passes over the net, reducing its horizontal velocity component and increasing its vertical velocity component, thus allowing the ball to land in the correct place.

Exam-style questions

Multiple-choice questions

1 D [1]

2 D [1]

3 A [1]

4 C [1]

5 C [1]

6 D [1]

7 C [1]

8 D [1]

Short-answer questions

9 a $\alpha = \frac{\Delta v}{\Delta t} = \frac{80 - 0}{4 \times 10^{-4}} = 2 \times 10^5\ \text{ms}^{-2}$ [1]

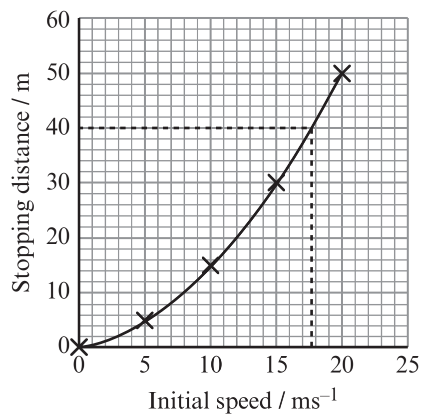
b $s = \frac{1}{2}at^2 = \frac{1}{2} \times 2 \times 10^5 \times (4 \times 10^{-4})^2 = 1.6\ \text{cm}$ [1]

c During the golf club's contact with the golf ball, the head of the club travels a distance of 1.6 cm. The average radius of a golf ball is about 2.1 cm. So the club head travels less distance than the radius of the ball. This suggests that during the club head's travel of 1.6 cm, the golf ball becomes squashed. [1]

10 a

Initial speed / ms^{-1}	Thinking distance / m	Braking distance / m	Stopping distance / m	
0	0	0	0	[½]
5	2.5	2.5	5	[½]
10	**5.0**	10	**15**	[½]
15	**7.5**	**22.5**	30	[½]
20	10	**40**	**50**	[½]

b



[2]

c $\approx 17.5 \rightarrow 18\ \text{ms}^{-1}$ [1]

11 a Highest point is when ball comes to a stop: at $t = 0.5$, $v = 0$ [1]

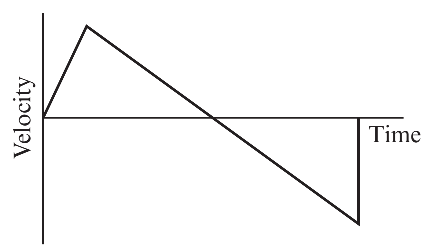
b Area under the graph between $t = 0$ and $t = 0.5$ gives displacement [1]

$= \frac{1}{2} \times 5 \times 0.5 = 1.25\ \text{m}$ [1]

c total area = area first 0.5 s + area for second 0.5 s. [1]

These two areas are equal in magnitude, but area between 0.5 s and 1.0 s is negative. So, overall area = 0. Therefore, displacement = 0. [1]

12 a



[1]

b Max. height $= \left(\frac{1}{2}at_1^2 + \frac{1}{2}gt_2^2\right) =$

$\frac{1}{2} \times 20 \times 3^2 + \left(\frac{1}{2} \times 9.81 \times \left(\frac{20 \times 3}{9.81}\right)^2\right) =$ [1]

273 m = 270 m (2 s.f.) [1]

c Total travel time = time to reach max height + time to fall to ground **[1]**

$= \left(3 + \frac{3 \times 20}{9.81}\right) + \left(\sqrt{\frac{2 \times 273}{9.81}}\right) = 16.57\ \text{s} =$ 17 s (2 s.f.) **[1]**

13 a $t = \sqrt{\frac{2s}{g}} = \sqrt{\frac{2 \times 50}{9.81}} = 3.2\ \text{s}$ **[1]**

b Horizontal distance $= ut =$ $40 \times 3.2 = 128$ m **[1]**

c Vertical component of $v = gt =$ $9.81 \times 3.2 = 31.4\ \text{ms}^{-1}$

Horizontal component of $v = 40\ \text{ms}^{-1}$ **[1]**

Magnitude of $v = \sqrt{31.4^2 + 40^2} =$ $51\ \text{ms}^{-1}$ (2 s.f.) **[1]**

Direction of $v = \tan^{-1}\left(\frac{31.4}{40}\right) = 38°$ from the horizontal. **[1]**

14 a $a = \frac{\Delta v}{\Delta t} = \frac{16 - 8}{20 - 12} = 1\ \text{ms}^{-2}$ **[1]**

b Displacement = area under graph =

$(8 \times 25) + \left(\frac{1}{2} \times 8 \times 8\right) + (8 \times 5) =$

272 m **[2]**

c $v_{ave} = \frac{\text{total displacement}}{\text{time}} = \frac{272}{25} =$ $10.9\ \text{ms}^{-1} = 11\ \text{ms}^{-1}$ (2 s.f.) **[1]**

15 a In section A, the object starts from rest and accelerates for 10 s. **[1]**

In section B, the object moves with a constant velocity for 10 s. **[1]**

In section C, the object decelerates for 10 s until it comes to a stop. **[1]**

b Velocity = gradient of graph =

$\frac{30 - 10}{20 - 10} = 2\ \text{ms}^{-1}$ **[1]**

c Average velocity $= \frac{\text{Total displacement}}{\text{time}} =$ $\frac{40}{30} = 1.3\ \text{ms}^{-1}$ (2 s.f.) **[1]**

16 a Acceleration = gradient of graph between $t = 30$ mins and $t = 80$ mins.

$= \frac{0 - 1}{(80 \times 60) - (30 \times 60)} = -3.3 \times 10^{-4}\ \text{ms}^{-2}$ **[1]**

(for minus sign) **[1]**

b Total displacement = area under graph

$= (1 \times 30 \times 60) + \left(\frac{1}{2} \times 1 \times 50 \times 60\right) -$ $\left(\frac{1}{2} \times 1 \times 50 \times 60\right) - (1 \times 50 \times 60)$

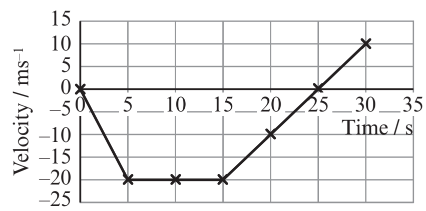
$= -1200$ m **[2]**

c Average velocity $= \frac{\text{Total displacement}}{\text{time}}$

$= \frac{-1200}{200 \times 60} = -0.1\ \text{ms}^{-1}$ **[1]**

17 a Using SUVAT: $s = \frac{1}{2}at^2 = \frac{1}{2} \times$ $-4 \times 5^2 = -50$ m **[1]**

b



[2]

c Using the graph:

Total displacement = Area under graph =

$-\left(\frac{1}{2} \times 20 \times 5\right) - (20 \times 10) -$ $\left(\frac{1}{2} \times 20 \times 10\right) + \left(\frac{1}{2} \times 10 \times 5\right) =$

-325 m **[2]**

18 a Maximum acceleration = maximum gradient

$\approx \frac{1}{5} = 0.2\ \text{ms}^{-2}$ $(0.15 < a < 0.25)$ **[2]**

b 0 m **[1]**

c The object is oscillating **[1]** about a fixed point.

Chapter 2

Exercise 2.1

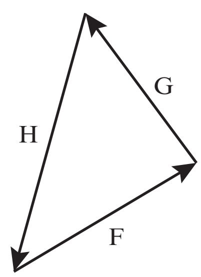
1 **a** $F_H = 6.0 \cos 40° = 4.6$ N (2 s.f.)

b $F_V = 6.0 \sin 40° = 3.9$ N (2 s.f.)

2 **a** Magnitude $= \sqrt{5^2 + 4^2} = 6.4$ N

b Angle $= \tan^{-1}\left(\frac{4}{5}\right) = 38.7°$

3 **a** For the net force to be zero, the vectors must make a closed polygon



b Adding horizontal componets:
$5 + -3 + -2 = 0$

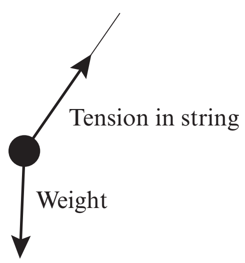
Adding vertical components:
$3 + 4 + -7 = 0$

So, net vector is (0, 0).

4 **a** Magnitude of the unspecified force = $\sqrt{6^2 + 3^2} = 6.7$ N.

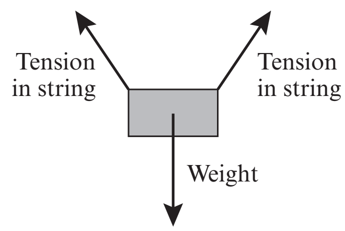
b Direction of the unspecified force = $\tan^{-1}\left(\frac{6}{3}\right) = 63.4°$ below the right-to-left horizontal axis.

5 **a**



b The vertical components of the two forces add up to zero, but there is no horizontal component of the weight of the mass, leaving the horizontal component of the tension in the string unbalanced.
Hence the mass cannot be in equilibrium.

6 **a**



b Weight, $W = 2 \times 60 \cos 35° = 98$ N (2 s.f.)

7 **a** $7.5 = T \sin 40$

$\therefore T = \frac{7.5}{\sin 40} = 11.7 = 12$ N (2 s.f.)

b $W = T \cos 40 = 8.96 = 9.0$ N (2 s.f.)

8 **a** It is a straight line showing that the extension is proportional to the stretching force.

b 2 cm

c Spring constant $= \frac{1}{\text{gradient}} = \frac{50}{8} = 6.25 =$ 6.3 Ncm^{-1} (2 s.f.)

9 **a** Along the slope: $F = W \sin \theta$

Perpendicular to the slope: $N = W \cos \theta$

$\frac{F}{N} = \frac{W \sin \vartheta}{W \cos \vartheta} \Rightarrow F = N \tan \theta$

b $\mu_s = \tan \theta$ when the book is not slipping along the slope.

c As θ increases, $W \sin \theta$ increases. When $W \sin \theta > \mu_s N$, the book will begin to slip.

10 For $\theta = 30°$ the component of weight down the slope is $W \sin 30° = 0.5\ W$.

The static frictional force is $F = \mu_s N = \mu_s W \cos 30° = 0.45 \times 0.866 \times W = 0.39\ W$, less than the component of weight down the slope, so the container slides down the slope.

11 The box is in equilibrium (moving at a constant velocity); the vertical and the horizontal forces must balance.

Vertically, $150 \sin 40° + N = mg \Rightarrow$ $N = mg - 150 \sin 40° = 25 \times 9.81 - 150 \times 0.64 = 149$ N.

Horizontally, $150 \cos 40° = \mu_d \times N \Rightarrow$ $\mu_d = \frac{150 \cos 40}{149} = 0.77$.

12 a Hooke's law: the extension of a spring, e, is proportional to the force, F, applied to stretch it, providing that the elastic limit is not exceeded.

b i $F = kx = 25 \times 0.3 = 7.5$ N

ii $F = kx = 0.3 \times 2.5 \times 10^{-3}$ $= 7.5 \times 10^{-4}$ N

c In series, a force stretching the springs makes each spring stretch by the same amount, so the overall extension will be three times as much as for one spring. So, the spring constant for one spring must be $3 \times 12 = 36$ Nm^{-1}.

d In parallel, each spring stretches only half of what it would on its own. So, the spring constant for one spring is $\frac{1}{2} \times 50 = 25$ Nm^{-1}.

13 a $F = kx = 5 \times 10^3 \times 1 \times 10^{-2} = 50$ N

b $a = \frac{F}{m} - g = \frac{50}{25 \times 10^{-3}} - 10 = 1990$ ms^{-2}

c The graph of a against t will be a straight line starting at $a = 1990$ ms^{-2} at $t = 0$ and finishing at $a = 0$ ms^{-2} at $t = 4.5 \times 10^{-3}$ s. The area under this graph will be the change in velocity of the toy.

So $Dv = \frac{1}{2} a_{t=0} t = \frac{1}{2} \times 1990 \times 4.5 \times 10^{-3} =$ 4.48 ms^{-1}

d $v^2 = u^2 + 2gs$ so $s = \frac{u^2}{2g} = \frac{4.48^2}{2 \times 10} = 1.0$ m

14 a For an object that is partly or fully submerged in a fluid, the buoyancy force acting on the object is equal to the weight of the fluid displaced by the object.

b i $P = P_o + \rho gh$

ii Upwards force on the underside of the cylinder is $F = P\,A =$ $A(P_o + \rho gh)$.

iii Downwards force from atmospheric pressure $= P_oA$.

iv Net force upwards = buoyancy force = $A(P_o + \rho gh) - P_oA = A\rho gh$.

v Weight of liquid displaced by the cylinder $= Ah\rho \times g = A\rho gh$. This verifies Archimedes' principle.

vi Its density must be the same as that of the liquid around it.

c i Volume of the block $= \frac{\text{mass}}{\text{density}} =$ $\frac{100}{1.1 \times 10^3} = 9.1 \times 10^{-2}$ m^3

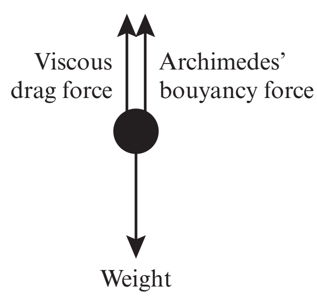
Weight of the water it displaces is $W = 9.1 \times 10^{-2} \times 1000 \times 9.81 = 893$ N.

Weight of block itself $= 100 \times 9.81 =$ 981 N

So, net force on the block $= 981 - 893$ $= 88$ N downwards.

ii Sink

15 a



b $\frac{4}{3}\pi\rho_{\text{lead}} r^3 g = 6\pi\eta r v + \frac{4}{3}\pi\rho_{\text{water}} r^3 g$

$\therefore v = \dfrac{\frac{4}{3}\pi\rho_{\text{lead}} r^3 g - \frac{4}{3}\pi\rho_{\text{water}} r^3 g}{6\pi\eta r}$

$= \dfrac{2r^2 g(\rho_{\text{lead}} - \rho_{\text{water}})}{9\eta}$

c $v = \dfrac{2 \times (1.5 \times 10^{-3})^2 \times 9.81 \times (1.14 \times 10^4 - 1.0 \times 10^3)}{9 \times 1.0 \times 10^{-3}}$

$= 51 \text{ ms}^{-1}$

d It is because of turbulence. Stokes' law does not take into account the effect of turbulence on the movement of a sphere through a viscous liquid. Such an effect is to increase the drag force acting on the sphere.

e The sphere would fall with a faster terminal velocity. This is because at a higher temperature, the viscosity of water decreases.

16 a i $F_k = \mu_k N$ and $N = mg$

$\therefore F_k = \mu_k mg = 0.02 \times 1.2 \times 10^3 \times 9.81 = 235 \text{ N}$

ii F_k is constant; it isn't affected by the speed of the car.

b $\frac{1}{2}CA\rho v^2 = 235$

$\therefore v = \sqrt{\dfrac{2 \times 235}{0.3 \times 2 \times 1.3}} = 24.5 \text{ ms}^{-1}$

c For a constant speed, driving force = $2 \times 235 = 470$ N.

d Although the dynamic frictional force remains constant (and this depends on the mass of the car), the frictional drag force increases with v^2, so at higher speeds, this force becomes very large—requiring the driving force from the engine to be very large too. Family cars

- have engines that are not designed to deliver very high driving force.
- are designed to accommodate several people inside (and in a reasonable degree of comfort)—and so have
  - large mass—making the dynamic frictional force large.
  - cross-sectional areas that are quite large, making the frictional drag force large.
- are designed without the need for high degrees of streamlining to reduce turbulence and so have fairly large values of drag coefficient, also making the frictional drag force large.

e The design of these type of cars is to reduce all frictional force and increase driving force. This is effected by

- large high-performance engines to increase the driving force from the engine.
- small cross-sectional areas to decrease the frictional drag force.
- streamlined design to decrease the drag coefficient and so decrease the frictional drag force.
- using materials with a high strength-to-mass ratio in order to reduce the dynamic friction force.

Exercise 2.2

1 a A body will continue to move with a constant velocity, or remain at rest, unless it is acted on by an unbalanced force.

b When the net force on a body is zero, the body is said to be in equilibrium.

c Yes, providing the motion is of constant velocity.

2 a Yes

b Yes

c No

d Yes

3 a The net force on a body of constant mass is equal to the product of the body's mass and its acceleration: $F = ma$

(where F is the net force, m is the mass of the object and a is its subsequent acceleration).

b One Newton is the net force required to accelerate a 1 kg mass by 1 ms^{-2} in the direction of the force.

c

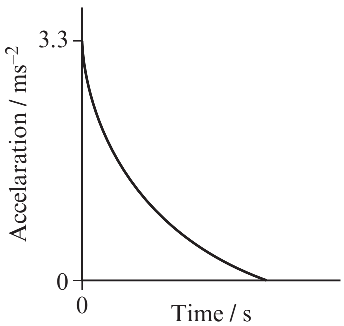
Net force / N	Mass / kg	Acceleration / ms^{-2}
120	50	2.4
900	200	4.5
1.5	6	0.25

4 $F = ma = 0.05 \times \frac{1500 - 0}{0.1} = 750$ N

5 **a** Net force on paper cone = 0.12 − 0.08 = 0.04 N

So, $a = \frac{F}{m} = \frac{0.04}{0.012} = 3.3$ ms^{-2}

b



c At the start, the forces acting on the cone are not balanced (weight > air friction force), so the cone accelerates. Increasing the speed of the cone increases the air friction force acting upwards.
The resultant force on the cone decreases, and so, using Newton's second law, the acceleration of the cone decreases. When air friction force and weight are balanced, the resultant force on the cone is zero, and the cone travels at a constant velocity.

6 **a** If body A exerts a force on body B, then body B exerts an equal and opposite force on body A.

b **i** Yes, these are a Newton's third law pair. The two forces are equal in magnitude and opposite in direction.

ii No, these are not a Newton's third law pair. The weight of the book (i.e. the gravitational force of the Earth on the book) and the gravitational force of the book on the Earth are a pair. The force of the book on the table and the normal force of the table on the book are a pair; these two forces are of an electrostatic nature caused by the deformation of the electron clouds that surround the atoms of the book and the table.

iii Yes, these are a Newton's third law pair. The two forces are caused by the oppositely charged nature of the two particles; they are equal in magnitude and opposite in direction.

iv Yes, these are a Newton's third law pair. The force of the raindrop on the ground pushes the ground (and may make a small dent), and the force of the ground on the raindrop causes the raindrop to decelerate and come to a stop.

7 **a** Using Newton's laws of motion, the sum of the forces acting on the person must equal the mass × acceleration of the person.

So, $60g + N = 0 \Rightarrow N = -60g = -60 \times 9.81 = 590$ N upwards = reading on weighing scales.

b **i** Using Newton's laws of motion, $60g + N = 60a \Rightarrow N = 60a - 60g = 60 \times 0.25 \times 9.81 - 60 \times 9.81 = -440$ N.

ii It makes the person feel lighter.

c **i** Using Newton's laws of motion, $60g + N = -60a \Rightarrow N = -60a - 60g = -60 \times 0.2 \times 9.81 - 60 \times 9.81 = -710$ N.

ii It makes the person feel heavier.

8 **a** and **b** Philosophically (and, in this case, epistemologically), this is a very difficult question to answer easily! The formal definition accepted by many of what knowledge is follows the "justified, true belief" line of thinking. In the natural sciences, we gather information by making observations. These observations lead to us making a hypothesis, which then allows us to make predictions. These predictions are then tested by performing experiments. The results of these experiments then either support or refute the prediction. Repeating this series of procedures can suggest strongly (although never prove) that our hypothesis is correct. (If the results do not support our hypothesis, then we have to make a new hypothesis.)

This then leads to us defining our hypothesis as a law. It is what we believe to be true, and our justification of it is that repeated observations of the same phenomena show the same thing. It is only when an experiment is repeated and something different is observed that we can show that our hypothesis is incorrect—and our law has to be amended or replaced with a better one. The natural sciences try to produce the best way of understanding what we observe.

c No. The best we can do is to show consistently that what we observe is in agreement with what our law claims.

d Einstein's work on relativity (see the questions in Chapter 6) showed that Newton's laws of motion have to be amended slightly under the special conditions of extremely high speed. For most cases, Newton's laws of motion continue to provide an accurate set of 'rules', which we can use to explore kinematics.

Exercise 2.3

1 a i $\omega = \frac{2\pi}{T}$

ii Since $f = \frac{1}{T}$, $\omega = 2\pi f$.

b i $\omega = \frac{2\pi}{T} = \frac{2\pi}{3.15 \times 10^7}$

$= 1.99 \times 10^{-7}$ radians s^{-1}

ii $\omega = \frac{2\pi}{60} = 0.10$ radians s^{-1}

iii $\omega = \frac{13 \times 2\pi}{3.15 \times 10^7}$

$= 2.6 \times 10^{-6}$ radians s^{-1}

c i $f = \frac{1}{30 \times 60} = 5.6 \times 10^{-4}$ Hz

ii $\omega = 2\pi f = 2 \times \pi \times 5.6 \times 10^{-4}$
$= 3.5 \times 10^{-3}$ radians s^{-1}

iii $v = r\omega = 75 \times 3.5 \times 10^{-3}$
$= 0.26\ ms^{-1}$

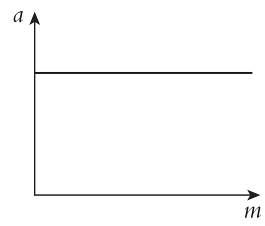
2 a A body moving in a circular path is changing its direction of motion. This means that its velocity is changing, so it must be accelerating.

b Towards the centre of the circle

c The centripetal force

d $F = \frac{mv^2}{r}$, where m is the mass of the object moving in the circular path, v is the linear speed at which the mass is moving and r is the radius of the circular path.

3 a i



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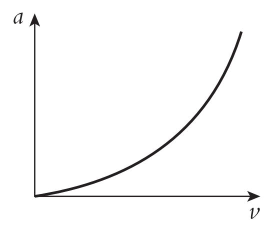

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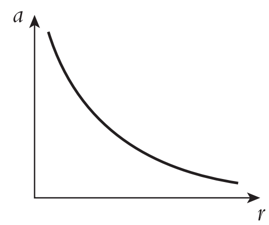
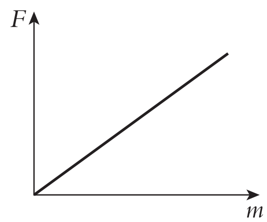


b i



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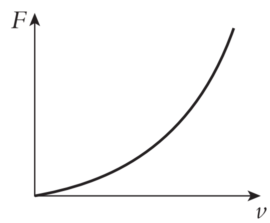

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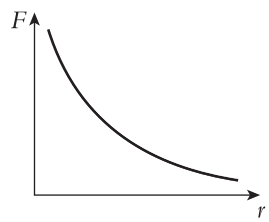


4 a The Earth's gravitational force on the Moon

b Electrical attractive force on electron

c Magnetic force due to the proton moving in a magnetic field

d Friction between the car's tyres and the road

e Tension in the string

5 $F = \frac{mv^2}{r} \Rightarrow v = \sqrt{8.23 \times 10^{-7} \times \frac{5.29 \times 10^{-11}}{9.1 \times 10^{-31}}}$
$= 6.9 \times 10^6\ \text{ms}^{-1}$

6 a i $F = \mu N$, where μ is the coefficient of static friction (as long as the car is not skidding), and $N = mg$, so $F = \mu mg$, then $F \propto m$.

ii $\frac{mv^2}{r} = \mu N = \mu mg \Rightarrow \text{v} = \sqrt{\mu rg}$, which is independent of mass, m.

iii When the road is wet or icy, the value of μ will be smaller—the friction is less—and so the value for v must be smaller. If the car exceeds this speed, there will not be sufficient friction force and the car will not be able to move in a circular path: it will skid.

b i The maximum speed on the circular part of the road depends on the square root of the coefficient of friction between the tyres and the road, the radius of the circular arc and the value of g. This value will be smaller than the motorway speed limit because the radius of the curve is a relatively small value.

ii $v = \sqrt{\mu rg} = \sqrt{0.75 \times 80 \times 9.81}$
$= 24.3\ \text{ms}^{-1}$

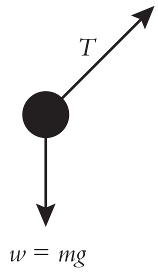
c If m is halved, then v will change by a factor of $\sqrt{\frac{1}{2}} = 0.707$ (or $1/\sqrt{2}$).

7 a i $v = r\omega = 2\pi rf = 2\pi \times 0.6 \times 2.5$
$= 9.4\ \text{ms}^{-1}$

ii $T = mr\omega^2 = 4mr\pi^2 f^2$
$= 4\pi^2 \times 0.1 \times 0.6 \times 2.5^2$
$= 14.8\ \text{N}$

b i The ball on the end of the string has mass. The tension in the string must have a component that balances the weight of the mass. This component must be in the vertical plane, so the string cannot be horizontal.

ii

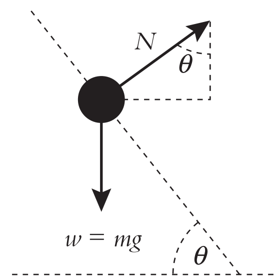


c Larger

d The horizontal component

e The vertical component is balancing the weight of the mass.

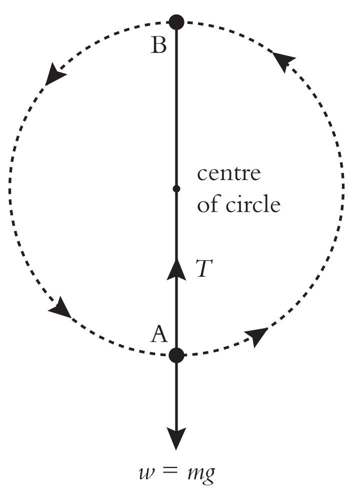
8 a



b $N \cos\theta = w = mg$ and
$N \sin\theta = \frac{mv^2}{r} \Rightarrow mg\frac{\sin\theta}{\cos\theta} = \frac{mv^2}{r}$
Therefore, $\theta = \tan^{-1}\left(\frac{v^2}{gr}\right)$

c We would expect an Olympic cyclist to cycle faster than an amateur. So, if v is larger, the equation for θ shows that θ is larger: the banking is steeper.

9 a



b The tension must balance the weight, so the centripetal force must be the sum of these two vectors: $\frac{mv^2}{r} = T - mg \Rightarrow T = \frac{mv^2}{r} + mg$.

c Now, the centripetal force must equal the sum of the tension and the weight forces:

$\frac{mv^2}{r} = T + mg \Rightarrow T = \frac{mv^2}{r} - mg$.

d The tension in the string is maximum at the bottom of the vertical circle and becomes less as the mass moves towards the top of the circle.

Exam-style questions

Multiple-choice questions

1 C [1]

2 A [1]

3 B [1]

4 C [1]

5 C [1]

6 B [1]

7 C [1]

8 D [1]

9 D [1]

10 D [1]

Short-answer questions

11 a $T_A \sin 50 = T_B \sin 40 \therefore T_B = \frac{T_A \sin 50}{\sin 40} = \frac{4000 \times 0.766}{0.643} = 4765 = 4800$ N (2 s.f.) [2]

b At constant speed, frictional drag force = 4000 cos 50 + 4765 cos 40 = 6221 = 6200 N (2 s.f.) [1]

c i The tension in the two ropes would have to be smaller. [1]

ii In warmer water the coefficient of viscosity will be smaller, leading to a smaller frictional drag force at the same speed.

12 a $F = kx \Rightarrow x = \frac{F}{k} = \frac{5}{25} = 20$ cm, [1]
so total length = 20 + 4 = 24 cm. [1]

b Four springs in series will have a spring constant = $\frac{25}{4}$,
so $\frac{F}{k} = \frac{5}{\frac{25}{4}} = 80$ cm. [1]

c Using $k = \frac{F}{x} = \frac{8}{0.02} = 400$, so 16 springs will be required. [1]

13

Motion of elevator	Reading on weighing scales / N	
Stationary	40	[1]
Moving upwards at 2.5 ms^{-1}	40	[1]
Moving downwards at 2.5 ms^{-1}	40	[1]
Accelerating upwards at 2.5 ms^{-2}	50	[1]
Accelerating downwards at 2.5 ms^{-2}	30	[1]

14 a $T \cos 20 = mg \Rightarrow T = \frac{mg}{\cos 20} = \frac{0.12 \times 9.81}{0.94} = 1.25$ N [2]

b $a = \frac{F}{m} = \frac{1.25 \sin 20}{0.12} = 3.6$ ms^{-2} (2 s.f.) [1]

c The amulet and necklace would now hang in the opposite direction (i.e. forwards) at an angle of $\tan^{-1}\left(\frac{0.12 \times 5}{0.12 \times 9.81}\right) = 27°$. [2]

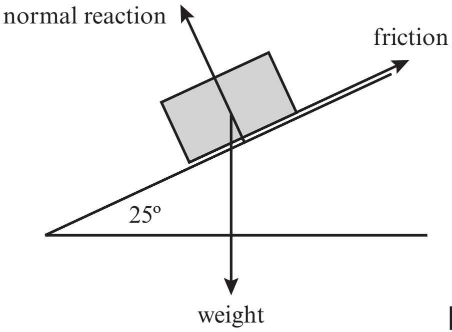
15 a $F = ma = m\frac{\Delta v}{t} = 0.125 \times \frac{40}{0.2} = 25$ N [2]

b 25 N [1]

c Newton's second law ⇒ time is larger, so the deceleration of the ball is smaller ⇒ the force required is smaller. [1]

Newton's third law ⇒ force on fielder's hands is smaller (so hurts less!) [1]

16 a



[2]

b $\mu_s = \tan 25 = 0.47$ [1]

c Yes, μ_s is independent of the mass (and area of contact) of the block, so the friction force up the slope would double (because the normal reaction force doubles), the component of the weight down the slope would double, and so these two forces would still be balanced. [1]

17 a $w = mg = \frac{4}{3}\pi r^3 \rho g$
$= \frac{4}{3} \times \pi \times (3 \times 10^{-3})^3 \times 920 \times 9.81$
$= 1.0 \times 10^{-3}$ N [1]

b $6\pi\eta r v = w \Rightarrow v = \frac{w}{6\pi\eta r}$
$= \frac{1.0 \times 10^{-3}}{6 \times \pi \times 1.8 \times 10^{-5} \times 3 \times 10^{-3}}$
$= 980$ ms^{-1} (2 s.f.) [2]

c i Higher speed [1]

ii Since the weight remains the same, the ratio of speeds is inversely proportional to the ratio of radii. Since water is more dense, the raindrop will have a smaller radius and hence a higher speed. [1]

18 a $\omega = \frac{2\pi}{24 \times 60 \times 60}$
$= 7.3 \times 10^{-5}$ radians s^{-1} [1]

b $v = r\omega = 4.23 \times 10^7 \times 7.3 \times 10^{-5}$
$= 3.1$ kms^{-1} [1]

c $a = \frac{v^2}{r} = g = \frac{(3100)^2}{4.23 \times 10^7}$
$= 0.23$ Nkg^{-1} (2.s.f.) [2]

19 a Static friction force between the road and the tyres of the car. [1]

b $v = \sqrt{\mu_s g r} = \sqrt{0.7 \times 9.81 \times 45} = 17.6$
$= 18$ ms^{-1} (2 s.f.) [2]

c 18 ms^{-1} [1]

20 a Since the vertical component of the tension must balance the weight of the skater,

$T \sin 30° = 60g \Rightarrow T = \frac{60 \times 9.81}{\sin 30}$
$= 1177$ N (1200 N to 2 s.f.). [2]

b $T \cos 30° = mr\omega^2 \Rightarrow \omega = \sqrt{\frac{T \cos 30}{m\, r}}$
$= \sqrt{\frac{1177 \times 0.866}{60 \times 2.2}} = 2.8$ radians s^{-1}
[2]

c $v = r\omega = 2.2 \times 2.8 = 6.2$ ms^{-1} (2 s.f.) [1]

Chapter 3

Exercise 3.1

1 a A Joule is defined as the amount of work done when a force of 1 N moves through a distance of 1 m (in the direction of the force).

b The principle of conservation of energy states that energy can be transferred from one form into another (or several), but it cannot be destroyed.

c i Yes. The book has gained E_P.

ii Yes, the pupil picking up the book has applied a force upwards on the book and the book has moved a distance upwards, so the pupil has done work. The pupil must have 'lost' some energy.

iii Some of the energy used by the pupil may have transformed into other forms (such as thermal energy), so the energy 'lost' by the pupil will be more than the energy gained by the book.

2 a $2\pi r = 2 \times \pi \times 1.2 = 7.5$ m

b 0 J; The centripetal force is perpendicular to the displacement of the mass at all times during its path around the circle, so the work done must be zero.

3 work done = area under graph =
$\left(\frac{1}{2} \times 60 \times 2 \times 10^{-3}\right) + (60 \times 3 \times 10^{-3}) =$
0.24 J

4 In stopping the car, the E_K of the car is transformed into thermal energy in the brakes. So we can use the idea that work done (= energy transformed) = average force × distance moved (in direction of force).

So, $\frac{1}{2}mv^2 = Fs \Rightarrow s = \dfrac{\frac{1}{2}mv^2}{F}$

$= \dfrac{\frac{1}{2} \times 1400 \times 20^2}{9 \times 10^3} = 31\text{ m}$

5 a The force could make the body

- change the direction in which it is moving: pushing your friend sideways when she is running.
- speed up (gain kinetic energy): pushing a toy car along a surface.
- change shape: crumpling up a piece of paper (deforming something requires energy).
- heat up (gain internal energy): squashing a squash ball.
- gain elastic potential energy: stretching a spring.
- gain gravitational potential energy: picking up an object from the floor.

b Whatever happens to the body, the work done on the body must be equal to the total energy gained by the body.

6 a The net work done on a system is equal to the change in kinetic energy of the system

b net work done = change on

$E_K = \frac{1}{2}m\left(v_{\text{final}}^2 - v_{\text{initial}}^2\right) = \frac{1}{2} \times 25 \times (20^2 - 10^2)$
$= 3.75$ kJ

c The size of the force and the time for which the force acted are not known. So it's not possible to know whether a large force acted for a short time or a small force acted for a long time.

7 a $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 9.1 \times 10^{-31} \times (4 \times 10^6)^2$
$= 7.3 \times 10^{-18}$ J

b Work done = change in $E_K = \frac{1}{2}m(v^2 - u^2)$
$= 3 \times E_{\text{Kinitial}} = 2.2 \times 10^{-17}$ J

8 a work done = E_K lost $= \frac{1}{2} \times 0.75 \times 1.5^2$
$= 0.84$ J

b $F = \dfrac{\text{work done}}{\text{distance}} = \dfrac{0.84}{8} = 0.1$ N

9 a i $a = \frac{F}{m} = \frac{36}{12} = 3$ ms^{-2}

ii $v = u + at = 0 + 3 \times 5 = 15$ s^{-1}

iii Distance travelled $= \frac{1}{2}at^2 =$
$\frac{1}{2} \times 3 \times 5^2 = 37.5$ m

Work done = Force × Distance =
36 × 37.5 = 1350 J

iv $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 12 \times 15^2 = 1350$ J

b The work done on the box by the force has been transferred into kinetic energy of the box.

10 a $v = \frac{140 \times 1000}{60 \times 60} = 39\ \text{ms}^{-1}$

b Work done = E_K lost

$= 0.15 \times \frac{1}{2} \times 160 \times 10^{-3} \times 39^2$

$= -18$ J (2 s.f.)

c $F = \frac{18}{15} = 1.2$ N (2 s.f.)

Exercise 3.2

1 a $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.058 \times 25^2 = 18$ J (2 s.f.)

b $E_P = mgh = 0.040 \times 10 \times 15 = 6$ J

c $E_P = \frac{1}{2}Fx = \frac{1}{2} \times 3.5 \times 0.015 = 26$ mJ (2 s.f.)

2 a $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 5 \times 2^2 = 10$ J $E_P = 0$

So, $E_{total} = 10$ J.

b $E_P = mgh = 4 \times 9.81 \times 2 = 78.5$ J (80 J to 1 s.f.) $E_K = 0$

So, $E_{total} = 80$ J.

c $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 3 \times 4^2 = 24$ J,
$E_P = mgh = 3 \times 9.81 \times 5 = 147$ J

So, $E_{total} = 24 + 147 = 171$ J. (200 J to 1 s.f.)

d $E_H = \frac{1}{2}kx^2 = \frac{1}{2}$ x $18 \times 0.08^2 = 0.058$ J (2 s.f.),
$E_P = 0$, $E_K = 0$

So, $E_{total} = 0.058$ J.

e $E_H = \frac{1}{2}Fx = \frac{1}{2} \times 5 \times 0.6 = 1.5$ J,

$E_K = \frac{1}{2}\text{mv}^2 = \frac{1}{2} \times 0.02 \times 2^2 = 0.04$ J

$E_P = mgh = 0.02 \times 9.81 \times 1.8$

$= 0.35$ J (2 s.f.)

So, $E_{total} = 1.5 + 0.04 + 0.35 = 1.89$

$= 1.9$ J. (2 s.f.)

3 a $E_P = mgh = 0.1 \times 10 \times 6 = 6$ J.

b i $t = \sqrt{\frac{2s}{g}} = \sqrt{\frac{2 \times 6}{10}} = 1.1$ s (2 s.f.)

ii $v = gt = 10 \times 1.1 = 11\ \text{ms}^{-1}$

c $mgh = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh} = \sqrt{2 \times 10 \times 6}$

$= 11\ \text{ms}^{-1}$ (2 s.f.)

4 a work done = force × distance moved in direction of force = $8.6 \times 10^4 \times 15 = 1.3 \times 10^6$ J

b E_P gained $= m\,g\,\Delta h = 3 \times 10^3 \times 9.81 \times 15 \sin 20° = 1.5 \times 10^5$ J

c Some of the work done on the block has been transformed into other forms, such as thermal energy caused by the friction between the block and the slope.
The block has also moved laterally.

5 a The gravitational force on the cup is $W = mg = 0.45 \times 9.81 = 4.4$ N (2 s.f.)

So, Work Done = F × s = 4.4 × 1.5 = 6.6 J.

b Using SUVAT, the speed of the cup is given by $v = \sqrt{2gs} = \sqrt{2 \times 9.81 \times 1.5}$

$= 5.4\ \text{ms}^{-1}$.

So, $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.45$ x $5.4^2 = 6.56$

$= 6.6$ J, as in **part a**.

c E_P lost $= mgh = 0.45 \times 9.81 \times 1.5 = 6.6$ J. This lost E_P is transferred into E_K of the cup.

d Since energy has to be transferred into other forms:

$E_P \rightarrow E_K \rightarrow$ Energy to break apart the cup + internal energy (of floor and cup pieces) + sound energy

6 a $E_{total} = E_K + E_H + E_P$

$= 0 + \frac{1}{2} \times 5 \times 10^3 \times (1 \times 10^{-2})^2 + 0$

$= 0.25$ J

b $\frac{1}{2}mv^2 + 0 + 0 = 0.25 \Rightarrow v = \sqrt{\frac{2 \times 0.25}{25 \times 10^{-3}}}$

$= 4.47\ \text{ms}^{-1}$

c $s = \frac{u^2}{2g} = \frac{4.47^2}{2 \times 10} = 1.0$ m **OR** $mgh = 0.25$

$\Rightarrow h = \frac{0.25}{25 \times 10^{-3} \times 10} = 1.0$ m

7 Using the conservation of mechanical energy,

$(E_P + E_K + E_H)_{\text{at top of bounce}} =$

$(E_P + E_K + E_H)_{\text{when trampoline bed is compressed}}$

$(60 \times 10 \times 8) + 0 + 0$

$= (60 \times 10 \times -0.7) + 0 + \left(\frac{1}{2} \times \text{k} \times 0.7^2\right)$

$\Rightarrow k = \frac{4800 + 420}{\frac{1}{2} \times 0.49} = 2.1 \times 10^4\ \text{Nm}^{-1}$

8 a i $mg\Delta h = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh}$

$= \sqrt{2 \times 10 \times 20} = 20\ \text{ms}^{-1}$

ii $t = \frac{\Delta v}{g} = \frac{20}{10} = 2$ s

b **i** $v^2 = u^2 + 2as \Rightarrow a = \frac{20^2}{2 \times 54}$
$= 3.7\ \text{ms}^{-2}$ upwards

ii $t = \sqrt{\frac{2s}{a}} = \sqrt{\frac{2 \times 54}{3.7}} = 5.4$ s. So, total time to come to a stop $= 2 + 5.4$
$= 7.4$ s.

c During stretching, the average tension in the cord is $F = ma = 70 \times (10 + 3.7)$
$= 959$ N.

So, $k = \frac{F}{x} = \frac{2 \times 959}{54} = 36\ \text{Nm}^{-1}$.

9 $420 = mgh \Rightarrow m = \frac{420}{10 \times 1.5} = 28$ kg

Assumptions: All the E_P of the falling mass is available to the spike, $g = 10\ \text{Nkg}^{-1}$, no energy is lost during the work done by the spike—and the student can pick up a mass of 28 kg!

Exercise 3.3

1 **a** work done $= E_P$ gained $= mg\Delta h$
$= 40 \times 9.81 \times 30 = 1.2 \times 10^4$ J

b output power $= \frac{\text{work done}}{\text{time taken}} = \frac{1.2 \times 10^4}{12}$
$= 1.0$ kW

2 Power $= F\,v \Rightarrow F = \frac{\text{power}}{v} = \frac{15 \times 10^3}{20} = 750$ N

3 Anton is the most powerful (3.61 W).

4 **a** power $= \frac{\text{energy transformed}}{\text{time taken}} = \frac{mg\Delta h}{t}$
$= \frac{65 \times 9.81 \times 5}{6} = 530$ W

b $E = \frac{\text{useful energy transformed}}{\text{efficiency}}$
$= \frac{65 \times 9.81 \times 5}{0.2} = 16$ kJ

5 **a** number of chocolate bars $= \frac{mgh}{\varepsilon E_{\text{chocolate bar}}}$
$= \frac{75 \times 10 \times 1085}{0.25 \times 1.1 \times 10^6} = 3$ chocolate bars

b A lot of energy will be required to walk along the 5-km track to the summit; it isn't only a vertical climb.

c $P = \frac{E}{t} = \frac{75 \times 10 \times 1085}{5 \times 60 \times 60} = 45$ W

6 **a** **i** $P = \frac{\text{useful work done}}{\text{time taken}}$
$= \frac{20 \times 70 \times 10 \times 0.25}{45} = 78$ W

ii $P = \frac{\text{useful work done}}{\text{time taken}}$
$= \frac{12 \text{ x } 70 \times 10 \times 0.35}{20} = 147$ W

b Since Anatoly's power output is almost double that of Garry's, it's most likely that Anatoly would be the one out of breath.

7 $P = Fv = mav = 0.02 \times \frac{0.34 - 0.20}{\frac{60}{72}} \times \frac{0.20 + 0.34}{2}$
$= 0.9$ mW

8 Useful work done $= mgh = 400 \times 10 \times 45$
$= 180$ kJ

Total energy required $= \frac{\text{Useful work done}}{\text{efficiency}}$
$= \frac{180\ kJ}{0.35} = 514$ kJ

Time taken $= \frac{\text{Total energy required}}{\text{power}} =$
$\frac{514\ \text{kJ}}{2.2\ \text{kW}} = 234$ s About 4 minutes.

9 **a** $mgh = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{2gh} =$
$\sqrt{2 \times 10 \times 33 \times 10^3} = 812\ \text{ms}^{-1}$

b $P = \frac{\text{work done}}{\text{time taken}} = \frac{mgh}{t}$
$= 1.36 \times 10^9 \times 10 \times 33.0 \times 10^3$
$= 4.49 \times 10^{14}$ W

c **i** $\frac{4.49 \times 10^{14}}{1.86 \times 10^9} = 2.41 \times 10^5$ times as much

ii $\frac{4.49 \times 10^{14}}{1.86 \times 10^9 \times 62500} = 3.9$ times as much

Exercise 3.4

1 **a** chemical energy $\rightarrow E_K$ + internal energy + sound energy
$E_K \rightarrow E_P$

b chemical energy $\rightarrow$ light energy + internal energy + sound energy + E_K of 'sparkles'

c **i** $E_P = mgh = 0.05 \times 10 \times 150 = 75$ J. So, the rocket must have stored a minimum of 75 J of chemical energy

ii Some of the chemical energy is required to be transferred into light energy, E_K of the 'sparkles' and sound energy (and any additional E_P the parts of the exploding rocket might have gained).

2 **a** chemical energy $\rightarrow E_K \rightarrow$ internal energy + sound energy

b Since the block is moving at a constant speed, the 'push' force from the worker must be equal to the frictional force between the block and the ground.

c It's unlikely that the force would have been applied purely in the horizontal direction. There would, therefore, have been a component of the actual force applied in the vertical direction, making the actual applied force larger than the frictional force between the block and the ground.

3 **a** $E_P \rightarrow E_K \rightarrow E_P \rightarrow E_K \rightarrow E_P$

b As the string becomes more vertically orientated, the tension in the string will increase. This will stretch the string a little, transferring some of the E_P into elastic potential energy as well as E_K.

As the angle between the string and the vertical increases, the tension in the string decreases, allowing some of the stored elastic potential energy to be transferred into E_K—and so to E_P.

4 As the comet approaches the Sun, it speeds up. Gravitational potential energy is being transferred into kinetic energy. As the comet recedes from the Sun, kinetic energy is transferred into gravitational potential energy.

5 Chemical energy (from the car's fuel) → kinetic energy (of moving parts of engine, wheels and the car itself) + internal energy (the engine—and its parts—get hot) + sound energy. Rotational kinetic energy in wheels → kinetic energy of moving gravel + internal energy (of gravel and tyres) + sound energy. Kinetic energy of car itself → E_P.

6 As the alpha particles approach the gold nuclei, they slow down. E_K is being transferred into electrical potential energy. As they move away from the gold nuclei, the opposite happens: electrical potential energy is transferred into kinetic energy.

7 **a** E_P (in springs) $\rightarrow E_K \rightarrow E_H$

b $E_P \rightarrow E_K + E_H$ as the mass moves from its highest position towards the equilibrium position. Then, $E_K + E_P \rightarrow E_H$ as the mass moves to its lowest position. On the way up, $E_H \rightarrow E_K + E_P$ as it moves towards the equilibrium position. Then, $E_K + E_H \rightarrow E_P$ as it nears its highest position again.

8 E_P of water → E_K of water → rotational E_K of turbines → electromagnetic energy in generators → electrical energy of output current. Each of these transfers will also be accompanied by energy transferred into internal energy.

9 **a** Yes. So far, scientists have not been able to show an example of the principle of conservation of energy not being obeyed. This doesn't mean that there cannot be such an example; it just means that our current understanding and our observations have not been able to falsify the conservation of energy rule.

b This is a difficult question to answer, and it may be that such a question could form the basis for substantial discussion in class or in groups. Certainly, one line of thinking would suggest that if the conservation of energy really isn't falsifiable, then all energy, whether E_P, E_K, E_H and so on, is just transferred from one to another. The implication is, therefore, that energy cannot be generated—it can only be transformed—and so the total energy in the universe must be constant. So far, it has not been possible to test this hypothesis, although as our ability to make better and better observations of the universe continues to improve, it may allow us to observe energy transformations hitherto unknown.
If such observations were made, they would only show us that the total energy of the universe was more than we had previously thought, not that the total energy of the universe is not constant.

c **i** Yes, it does seem to suggest this. The laws of thermodynamics would seem to agree with this too—and this has led to the so-called heat death of the universe hypothesis.

ii There is substantial evidence to show that since the beginning of the universe, some 13.7 billion years ago, the universe as a whole has continued to cool down. We must be careful here however, because terms such as *cool down* are associated with temperature and this is associated with the average, or typical, kinetic energy of atoms. Perhaps one argument might be something like as atoms continue to share their energy there will come a time when all atoms have the same amount of energy and no more sharing will occur. This would seem to be in agreement with the heat-death idea.

iii The two answers are not necessarily contradictory. Both processes can occur, but the net effect must be what we observe. Since our observations appear to be in agreement with the cooling of the universe over time, then, perhaps, the sharing of energy by atoms is a more dominant process (i.e. has a greater effect) than the transformation of kinetic energy into internal energy.

iv It would seem that these arguments suggest that the universe would eventually reach a constant temperature thoughout, whether this is warmer than is currently the case or cooler. More observations will continue to direct our thinking.

v It may not imply anything, but it will continue to 'allow' energy to be transferred or transformed.

Answers for Exam-style questions

Multiple-choice questions

1 B **[1]**

2 A **[1]**

3 B **[1]**

4 B **[1]**

5 A **[1]**

6 B **[1]**

7 C **[1]**

8 C **[1]**

9 C **[1]**

10 D **[1]**

Short-answer questions

11 a $E_{k\_lost} = \frac{1}{2}mv^2 = \frac{1}{2} \times 1.6 \times 2^2 = 3.2\text{ J}$ **[2]**

b $F = \frac{3.2}{1.2 \times 10^{-2}} = 270\text{ N}$ **[2]**

12 a $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 5 \times 6^2 = 90\text{ J}$ **[2]**

b $200 - 90 = 110\text{ J}$ **[1]**

c $110 = \frac{1}{2} \times 5 \times (v^2 - 6^2) \Rightarrow$

$v = \sqrt{\left(\frac{2}{5} \times 110\right) + 36} = 8.9\text{ ms}^{-1}$ **[2]**

13 a The graph shows that F is proportional to x, which is what Hooke's law states. **[1]**

b i $k = \frac{F}{x} = \frac{3}{0.1} = 30\text{ Nm}^{-1}$ **[2]**

ii $E = \frac{1}{2}F\,x = \frac{1}{2} \times 3 \times 0.1$

$= 0.15\text{ J}$ **[2]**

14 a $mgh = 0.15 \times 9.81 \times 1.2 = 1.766\text{ J}$

$\approx 1.8\text{ J}$ **[1]**

b 1.8 J **[1]**

c $v = \sqrt{\frac{2 \times 1.8}{0.15}} = 4.9\text{ ms}^{-1}$ **[1]**

So, yes, the glass is likely to break. **[1]**

15 a $E_H = \frac{1}{2}kx^2 = \frac{1}{2} \times 270 \times 0.12^2$

$= 1.94\text{ J}$ **[1]**

b $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.030 \times 8^2$

$= 0.96\text{ J}$ **[1]**

c $\varepsilon = \frac{\text{useful work done}}{\text{total energy used}} = \frac{0.96}{1.94} = 49\%$

d E_K of the bow and string (it will vibrate), internal energy in the bow (it will warm up), sound energy (a little) **[2]**

16 a Energy required for 1 book = Wh = 19.0 × 1.4 = 26.6 J

So, the number of books = $\frac{798}{26.6}$ = 30 books **[2]**

b $P = \frac{\text{useful work done}}{\text{time taken}} = \frac{798}{8 \times 60} =$ 1.7 W (2 s.f.) **[1]**

c She is not 100% efficient; some power is required for bodily functions; she will get hot. **[2]**

17 a Power is the rate of doing work;

$P = \frac{\text{work done}}{\text{time taken}}$. **[1]**

b $P = Fv = mg\frac{s}{t} = 50 \times 10 \times \frac{4.5}{12} =$ 190 W (2 s.f.) **[2]**

c $E = \frac{\text{Useful work done}}{\text{efficiency}} = \frac{mgh}{\varepsilon} =$

$\frac{50 \text{ x } 10 \text{ x } 4.5}{0.25} = 9$ kJ **[2]**

18 a The student should

- connect springs *in parallel* between two pieces of stiff board. **[1]**
- use a ruler to measure how far the springs compress when he stands on one of the boards. **[1]**
- use the equation $F = kx$, with the appropriate value of k, to find F (his weight). **[1]**

b Using $F = kx$, if $x = 5.0$ cm then

$k = \frac{65 \times 9.81}{5.0 \times 10^{-2}} = 12753 \text{ Nm}^{-1}$ **[1]**

So the student will require a minimum of $\frac{12753}{250} = 51$ springs **[1]**

19 a Remy does the most work.

Name	Work done / kg	Power / W
George	75 × 10 × 4.5 = 3375 J	3375 ÷ 8.0 = 422 W
Remy	68 × 10 × 5.5 = 3740 J	3740 ÷ 9.0 = 416 W
Andreas	82 × 10 × 4.0 = 3280 J	3280 ÷ 7.0 = 469 W

[3]

b Andreas is the most powerful. **[1]**

Chapter 4

Exercise 4.1

1 **a** In the absence of an external force, linear momentum, p, is the product of the mass of a body and its velocity; $p = mv$.

b $\text{kgms}^{-1} \equiv \text{kgms}^{-2}\ \text{s}$ And since kgms^{-2} are the base units for force (N), $\text{kgms}^{-1} \equiv \text{N s}$.

c **i** $p = mv = 50 \times 6 = 300\ \text{kgms}^{-1}$ westwards

ii $p = mv = 9.1 \times 10^{-31} \times 2 \times 10^{7} = 1.82 \times 10^{-23} = 1.8 \times 10^{-23}\ \text{kgms}^{-1}$ (2 s.f.) towards the anode

iii $p = mv = 0.11 \times 60 = 6.6\ \text{kgms}^{-1}$ towards the goal

2 **a** $p_{total} = (3.2 \times 2.5) - (5.0 \times 1.5) = 0.5\ \text{kgms}^{-1}$ northwards

b $p_{total} = \sqrt{(0.4 \times 3)^2 + (0.25 \times 4)^2} = 1.56 = 1.6\ \text{kgms}^{-1}$ (2 s.f.)

in a direction $tan^{-1}\left(\frac{(0.25 \times 4)}{(0.4 \times 3)}\right) = 40°$ above the horizontal

3 **a** $\Delta v = v_{final} - v_{initial} = 0 - 2.5 = -2.5\ \text{ms}^{-1}$

b $a = \frac{\Delta v}{t} = \frac{-2.5}{5} = -0.5\ \text{ms}^{-2}$

c $F = ma = 4.0 \times (-0.5) = -2.0\ \text{N}$

d The negative sign shows that the direction of the force was in the opposite direction to the original velocity.

4 **a** The average net force on a system is equal to the rate of change of the system's momentum. $F = \frac{\Delta p}{\Delta t}$

b $F = \frac{\Delta p}{t} = \frac{mv - mu}{t} = \frac{m(v-u)}{t} = ma$

5 **a** $\Delta p = p_{final} - p_{initial} = 0 - 4 \times 2.5 = -10\ \text{kgms}^{-1}$

b $F = \frac{\Delta p}{t} = \frac{-10}{5} = -2.0\ \text{N}$

6 $m\Delta v = Ft \Rightarrow \Delta v = \frac{Ft}{m} = \frac{180 \times 2.5}{12} = 37.5\ \text{ms}^{-1}$

7 $m\Delta v = Ft \Rightarrow t = \frac{m\Delta v}{F} = \frac{0.6 \times 1.5}{180} = 5\ \text{ms}$

8 $Ft = m\Delta v \Rightarrow m = \frac{Ft}{\Delta v} = \frac{5200 \times 5}{(30 - 10)} = 1300\ \text{kg}$

9 **a** In 1 s, the length of the column of water that will hit the wall is v.

So the volume is $V = Av$.

So the mass is $m = \rho V = \rho Av$.

b $F = \frac{\Delta p}{t} = \frac{\rho Av \times v}{1} = \rho A v^2$

c $\rho A v^2$ in the direction of the original water jet

d $F = \rho A v^2 = 1000 \times \frac{\pi \times (1.5 \times 10^{-3})^2}{4} \times (100.0)^2 = 17.7\ \text{N}$.

e $P = \frac{F}{A} = \frac{17.7}{\frac{\pi \times (1.5 \times 10^{-3})^2}{4}} = 1.0 \times 10^{7}\ \text{Pa}$

This is 100 atmospheres.

10 **a** No. of photons $s^{-1} = \frac{\text{received power}}{\text{energy of 1 photon}} = \frac{1000}{2.5 \times 10^{-19}} = 4 \times 10^{21}$ photons s^{-1}

b $F = \frac{N\Delta p}{t} = \frac{4 \times 10^{21} \times 1.3 \times 10^{-27}}{1} = 5.2 \times 10^{-6}\ \text{Nm}^{-2}$

c $F_{total} = 5.2 \times 10^{-6} \times 12 = 6.2 \times 10^{-5}\ \text{N}$

d The total force on the roof from the sunlight is $6.2 \times 10^{-5} \times 10 = 6.2 \times 10^{-4}$ N. This is a very small force compared to the weight of the solar panels themselves. No need for concern about the force from the sunlight; perhaps a need to be concerned about the weight of the solar panels!

Exercise 4.2

1 **a** *Impulse* means a change of momentum.

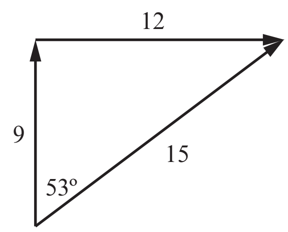
b kgms^{-1} or N s

2 **a** $J = \Delta p = p_{final} - p_{initial} = 6.5 - 4.0 = 2.5\ \text{kgms}^{-1}$ vertically upwards

b $J = \Delta p = p_{final} - p_{initial} = -2.0 - 3.0 = -5.0\ \text{kgms}^{-1}$ vertically downwards

(Note that the negative sign in the answer shows the direction is downwards.)

c



$J = \Delta p = p_{final} - p_{initial} = 15$ kgms^{-1} in a direction 53° from the vertical

3 **a** $J = Ft = 60 \times 12 \times 10^{-3} = 0.72$ N s

b $v = \frac{p}{m} = \frac{0.72}{0.2} = 3.6$ ms^{-1}

4 **a** Speed of woman $= \sqrt{2gh} = \sqrt{2 \times 10 \times 3} =$ 7.7 ms^{-1}

So $J = 0 - mv = -60 \times 7.7 = 462$ N s (upwards)

b $F = \frac{J}{t} = \frac{J}{\frac{s}{v_{ave}}} = \frac{462}{\frac{1.5 \times 10^{-2}}{7.7/2}} = 1.2 \times 10^5$ N

(Note that it isn't necessary to consider the weight of the woman in this calculation, because her weight is negligible compared to the size of the force exerted by the ground on her feet and legs.)

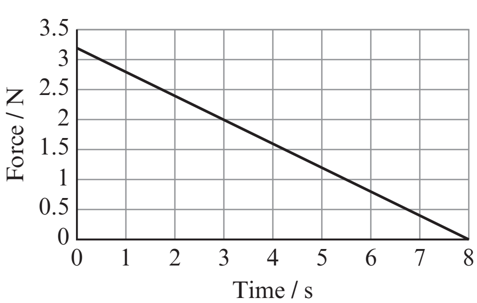
c $F = \frac{J}{t} = \frac{J}{\frac{s}{v_{ave}}} = \frac{462}{\frac{50 \times 10^{-2}}{7.7/2}} = 3.6 \times 10^3$ N

(This is 3% of **part b.**)

d The force calculated in **part b** is sufficient to break a bone in the woman's leg. When she bends her legs, the force is considerably reduced (because the time of the collision with the ground is increased)—and so is the chance of her doing herself an injury.

5 Crumple zones extend the *time* during which a crash occurs. This means that the *impulse* (the change in momentum of the car and its passengers) occurs over a longer *time*, making the *force* experienced by the passengers smaller. This reduces the chance of injury.

6 **a**

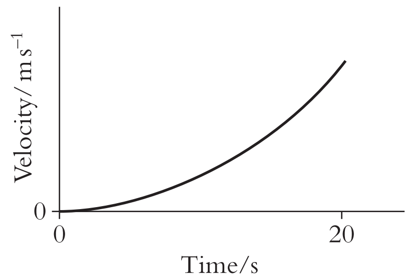


b J = area under graph $= \frac{1}{2} \times 3.2 \times 8.0$ $= 12.8 = 13$ N s (2 s.f.)

c $v = \frac{J}{m} = \frac{12.8}{0.6} = 21.3 = 21$ ms^{-1} (2 s.f.)

7 **a** Impulse = area under graph $= \frac{1}{2} \times 50 \times 20 = 500$ kgms^{-1}

b



Since the force acting on the object increases, its acceleration increases, so the gradient of the graph of v against t increases.

c $\Delta p = 500$ Therefore, $v = \frac{500}{3} = 200$ ms^{-1}. (1 s.f.)

Exercise 4.3

1 **a** The principle of conservation of linear momentum: in any interaction involving no external forces, the total momentum before the interaction is equal to the total momentum after the interaction.

b As far as we know, the principle of conservation of linear momentum is a universal law (in other words, it applies to all interactions).

2 Total momentum before = total momentum after

$\therefore 25 \times 10^{-3} \times 120 = (25 \times 10^{-3} \times 85) + (1.5 \times v)$

$\therefore v = \frac{25 \times 10^{-3} \times (120 - 85)}{1.5} = 0.58$ ms^{-1}

3 Total momentum before = total momentum after

$\therefore 4.5 \times 4.0 = (4.5 + 1.5)v$

$\therefore v = \frac{4.5 \times 4.0}{(4.5 + 1.5)} = 3.0$ ms^{-1}

4 $3.2 \times 10^3 \times 15.0 = (3.2 \times 10^3 - 800)v$

$\therefore v = \frac{3.2 \times 10^3 \times 15.0}{(3.2 \times 10^3 - 800)} = 20$ ms^{-1}

5 $(450 \times 3.0) + 0 = 0 + (m \times 5.0)$

$\therefore m = \frac{450 \times 3}{5} = 270$ g

6 $mg = \rho A v^2 \Rightarrow v = \sqrt{\frac{mg}{\rho A}} = \sqrt{\frac{5.0 \times 10^{-3} \times 9.81}{1.3 \times 1.6 \times 10^{-3}}} =$ 4.9 ms^{-1}

7 Total momentum before = total momentum after

$\therefore (3.0 \times 5.0) - (2.5 \times 4.0) = (3.0 + 2.5)v$

$\therefore v = \frac{15 - 10}{5.5} = 0.9\ \text{ms}^{-1}$ in the direction mass A had been moving.

8 Assume: average mass of Chinese person = 70 kg, g = 10 Nkg^{-1}, Earth is stationary

Velocity on landing = $\sqrt{(2 \times 10 \times 1)} = 4.5\ \text{ms}^{-1}$

Momentum of population just before landing = $1.4 \times 10^9 \times 70 \times 4.5 = 4.4 \times 10^{11}$ N s

$\therefore$ Speed of Earth (+ Chinese population) = $\frac{4.4 \times 10^{11}}{6 \times 10^{24}} = 7.3 \times 10^{-14}\ \text{ms}^{-1}$

This change of speed of the Earth is not noticeable.

Social media chat on this idea is poor physics!!

9

Conservation law	Before collision	After Collision
momentum	mu	$mv_1 + 12mv_2$
E_K	$\frac{1}{2}mu^2$	$\frac{1}{2}mv_1^2 + \frac{1}{2}12mv_2^2$

$u = v_1 + 12v_2$ *and* $u^2 = v_1^2 + 12v_2^2$

So, $v_1^2 + 24v_1v_2 + 144v_2^2 = v_1^2 + 12v_2^2$

$\Rightarrow v_1 = -\frac{132}{24}v_2 = -5.5v_2$

$\Rightarrow u = -5.5v_2 + 12v_2 = 7.5v_2$

$\Rightarrow \frac{v_1}{u} = \frac{5.5}{7.5} = 0.73$

So about 27% of its speed is lost in each collision.

Exercise 4.4

1 **a** $E_k = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$

b **i** $E_k = \frac{p^2}{2m} = \frac{12^2}{2 \times 3} = 24$ J

ii $E_k = \frac{p^2}{2m} = \frac{(5.4 \times 10^{-24})^2}{2 \times 9.11 \times 10^{-31}} =$ 1.6×10^{-17} J

c **i** $p = \sqrt{2mE_k} = \sqrt{2 \times 0.6 \times 30} =$ 6.0 kgms^{-1}

ii $p = \sqrt{2mE_k} = \sqrt{2 \times 0.058 \times 26.1} =$ 1.74 = 1.7 kgms^{-1} (2 s.f.)

2 $p_{before} = p_{after}$ Therefore, $3 \times 4 = (3 + 1) \times v \Rightarrow$ $v = \frac{3 \times 4}{3 + 1} = 3\ \text{ms}^{-1}$

3 **a** $p = mv = 0.4 \times 8 = 3.2\ \text{kgms}^{-1}$

b $p = mv = 0.4 \times -5 = -2.0\ \text{kgms}^{-1}$

c The ball has changed its momentum by $-2.0 - 3.2 = -5.2\ \text{kgms}^{-1}$.

So, the Earth must have gained momentum of 5.2 kgms^{-1}.

Since the mass of the Earth is large ($M_{Earth} = 6 \times 10^{24}$ kg), the speed at which the Earth moves will be $\frac{5.2}{6 \times 10^{24}} =$ $8.7 \times 10^{-25}\ \text{ms}^{-1}$.

This speed is too small for us to notice/measure, but that *is* what the Earth is doing!

4 **a** $J = \Delta p = m(v_{final} - v_{initial}) = 4.8 \times 10^{-26} \times (-500 - 500) = -4.8 \times 10^{-23}$ N s

(Here, the negative sign is showing that the impulse is in the opposite direction to the molecule's initial velocity.)

b 4.8×10^{-23} N s (positive because of Newton's third law)

c $P = \frac{F}{A} = \frac{J\,t\,N}{A} = \frac{4.8 \times 10^{-23} \times 1 \times 2.1 \times 10^{27}}{1} =$ 1.0×10^5 Pa

5 **a** $p = mv = 0.25 \times 450 = 112.5\ \text{kgms}^{-1}$ (= 110 kgms^{-1} to 2 s.f.)

b $p = 112.5 = (70 + 0.25)v \Rightarrow v = \frac{112.5}{70.25} =$ 1.6 ms^{-1}

c $F = \frac{\Delta p}{t} = \frac{70 \times 1.6}{0.1} = 1120$ N = (1100 N to 2 s.f.)

d The hay bale will be pushed at a speed of 1.6 ms^{-1} in the direction of the bullet.

6 **a** $v = \sqrt{\frac{2 \times E_k}{m}} = \sqrt{\frac{2 \times 8.0 \times 10^{-13}}{6.64 \times 10^{-27}}} =$ $1.6 \times 10^7\ \text{ms}^{-1}$

b $J = \Delta p = 2 \times 1.6 \times 10^7 \times 6.64 \times 10^{-27} =$ 2.1×10^{-19} N s

c $v = \frac{J}{m} = \frac{2.1 \times 10^{-19}}{3.29 \times 10^{-25}} = 6.4 \times 10^5\ \text{ms}^{-1}$

d kinetic energy = $\frac{1}{2}mv^2 = \frac{1}{2} \times 3.29 \times 10^{-25} \times (6.4 \times 10^5)^2 = 6.7 \times 10^{-14}$ J

e In an elastic collision, E_K is conserved. In the collision between the alpha-particle and the gold nucleus, the calculations have shown that the E_K after the collision will be greater than the E_K before the collision (because the gold nucleus has gained E_K.) So, if the alpha-particle really did collide elastically with the gold nucleus, it must have done so at a smaller speed. (See Chapter 19 for further insight into this phenomenon.)

7 a *Before*: $p_{total} = mu + m \times 0 = mu$

After: $p_{total} = mv_1 + mv_2$

So, applying conservation of linear momentum: $mu = mv_1 + mv_2$.

b *Before*: $E_{Ktotal} = \frac{1}{2}mu^2 + 0 = \frac{1}{2}mu^2$

After: $E_{Ktotal} = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$

So, applying conservation of E_K: $\frac{1}{2}mu^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$.

c From **parts a** and **b**: $u = v_1 + v_2$ and $u^2 = v_1^2 + v_2^2$.

Now, $u^2 = (v_1 + v_2)^2 = v_1^2 + 2v_1v_2 + v_2^2 = v_1^2 + v_2^2$

Therefore, $2v_1v_2 = 0$.

There are two possible solutions to this: $v_1 = 0$ *and* $v_2 = u$ or $v_2 = 0$ *and* $v_1 = u$ that is, the first ball stops and the second ball moves on at the speed u, or the second ball does not move and the first ball continues on at the same speed as before. This suggests that the second ball did not exist!

8 $F = \frac{\Delta(mv)}{t} = \frac{\Delta m \times v}{1} = 1250 \times 3.0 \times 10^4 = 3.8 \times 10^7$ N (2 s.f.)

9 a Total momentum before collision = $(m_{eagle}\, v_{eagle})$ eastwards + $(m_{seagull}\, v_{seagull})$ southwards

= (3.9 × 7.5) eastwards + (1.8 × 3.0) southwards

$= \sqrt{29.25^2 + 5.4^2} = 29.7$ kgms^{-1}

$\therefore v = \frac{p}{m} = \frac{29.7}{(3.9 + 1.8)} = 5.2$ ms^{-1}

b angle $= \tan^{-1}\left(\frac{5.4}{29.25}\right) = 10.5^0$ south of east

c $E_{K\ before} = \left(\frac{1}{2} \times 3.9 \times 7.5^2\right) + \left(\frac{1}{2} \times 1.8 \times 3.0^2\right) = 118$ J

$E_{K\ after} = \frac{1}{2} \times 5.7 \times 5.2^2 = 77.1$ J

$E_{K\ after} < E_{K\ before}$, so the collision was inelastic.

10 a Using conservation of momentum,

$p_{total\ before} = p_{total\ after} \Rightarrow 0 = (2.970 - 0.032) \times v + (0.032 \times 500)$

$\therefore v = \frac{-(0.032 \times 500)}{(2.970 - 0.032)} = -5.45$ ms^{-1}.

(The negative sign is showing that the rifle moves in the opposite direction to the bullet.)

b Chemical Energy$_{min} = E_{K\ rifle} + E_{K\ bullet}$

$= \left(\frac{1}{2} \times 2.938 \times 5.45^2\right) + \left(\frac{1}{2} \times 0.032 \times 500^2\right)$

= 4044 J

Exam-style questions

Multiple-choice questions

1	C	[1]
2	A	[1]
3	C	[1]
4	D	[1]
5	C	[1]
6	C	[1]
7	A	[1]
8	C	[1]
9	D	[1]
10	B	[1]

Short-answer questions

11 a $t = \frac{\Delta p}{F} = \frac{1200 \times 18}{3.0 \times 10^5} = 72$ ms **[2]**

b $s = vt = 18 \times 72 \times 10^{-3} = 1.3$ m **[1]**

c 1.3 m is sufficient for the passenger to hit the windscreen of the car. Wearing a seatbelt prevents this—and so would most probably save the passenger's life. **[1]**

12 a Before the collision, $p = mv = 6 \times 6 = 36$ kgms^{-1}

After the collision, $p = (m_1 + m_2)v = (6 + 3)v$

So, $v = \frac{36}{(6+3)} = 4$ ms^{-1} **[2]**

b $F = \frac{\Delta p}{t} = \frac{3 \times 4}{0.2} = 60$ N **[1]**

c Before the collision, $E_K = ½ \times 6 \times 6^2 = 108$ J.

After the collision, $E_K = ½ \times 9 \times 4^2 = 72$ J. **[1]**

There is less E_K after the collision, so the collision is *inelastic*. **[1]**

13 a The two forces are the same in magnitude and opposite in direction: Newton's third law. **[1]**

b Before the collision, $p = 60 \times 2\ M - 60 \times M = 60\ M$.

After the collision, $p = 3$ Mv.

Therefore, $v = \frac{60 \text{ M}}{3 \text{ M}} = 20$ kmhr^{-1} **[2]**

c Before the collision, $E_K = ½ \times 2\ M \times 60^2 + ½ \times M \times 60^2 = \frac{3}{2} \times M \times 3600 = 5400\ M$.

After the collision, $E_K = ½ \times 3\ M \times 20^2 = 600\ M$. **[1]**

This is less than the E_K before the collision, so the collision was inelastic. **[1]**

14 a Force = gradient of graph. $F = \frac{\Delta p}{t} = \frac{15}{20} = 0.75$ N **[2]**

b $\Delta E_k = \frac{p^2}{2\text{m}} = \frac{15^2}{2 \times 3} = 37.5$ J (38 J to 2 s.f.) **[2]**

15 a $4 \times 1.6 \times 10^7 = 237\ \text{v} \Rightarrow$
$v = \frac{4 \times 1.6 \times 10^7}{237} = 2.7 \times 10^5$ ms^{-1} **[2]**

b $\frac{E_{K\alpha}}{E_{KNp}} = \frac{\frac{1}{2} \times 4 \times (1.6 \times 10^7)^2}{\frac{1}{2} \times 237 \times (2.7 \times 10^5)^2} = 59.$ **[2]**

So, the alpha particle has 59 times more E_K than the Np nucleus.

16 a J = area under graph
$= 250 \times 5 \times 10^{-3} = 1.25$ N s **[2]**

b $v = \frac{J}{m} = \frac{1.25}{0.058} = 22$ ms^{-1}

c Looser strings means that the time of contact between the racket and the tennis ball will be longer. **[1]**

For the same maximum force, this will create a larger impulse and so a faster speed of the tennis ball. **[1]**

17 a J = area under graph
$= (2 \times 20) + \left(\frac{1}{2} \times 2 \times 10\right) = 50$ N s **[2]**

b $v = \frac{J}{m} = \frac{50}{2.2} = 23$ ms^{-1} (2 s.f.) **[1]**

c $\text{Power}_{\text{average}} = F_{\text{average}} \times v_{\text{average}} = 2.5 \times 11.5 = 29$ W (2 s.f.) **[2]**

18 a $F = \frac{\Delta mv}{t} = \frac{30 \times 2.5}{1} = 75$ N **[1]**

b $P = Fv = 75 \times 2.5 = 187.5 = 190$ W (2 s.f.) **[2]**

(Penalise answer of 187.5 W for incorrect no. of s.f.)

c $\Delta E_k = \frac{m\Delta v^2}{2} = \frac{30 \times 2.5^2}{2} = 93.75 = 94$ W (2 s.f.) **[1]**

19 a $F = \frac{\Delta p}{t} = \frac{\Delta mv}{t} = \frac{4.0 \times 250}{1} = 1000$ N **[1]**

b $a = \frac{F}{m} = \frac{1000}{12500} = 0.08$ ms^{-2} **[1]**

c $J = Ft = 1000 \times 20 = 2 \times 10^4$ N s **[1]**

d $\Delta v = \frac{J}{m} = \frac{2 \times 10^4}{12500} = 1.6$ ms^{-1} **[1]**

20 a $p = \sqrt{9.2^2 + 5.3^2} \times 10^{-23}$
$= 10.62 \times 10^{-23}$
$= 1.1 \times 10^{-22}$ N s (2 s.f.) **[2]**

In a direction of $\tan^{-1}\left(\frac{9.2}{5.3}\right) = 150°$ to the positron's velocity **[1]**

b $v = \frac{p}{m} = \frac{1.062 \times 10^{-22}}{3.9 \times 10^{-25}} = 270$ ms^{-1} **[1]**

c $E_K = \frac{p^2}{2m} = \frac{(1.062 \times 10^{-22})^2}{2 \times 3.9 \times 10^{-25}} = 1.4 \times 10^{-20}$ J **[1]**

Chapter 5

Exercise 5.1

1 a i The angle through which something has rotated

ii The rate at which something is rotating – or the angle through which something rotates in one second

b i $\omega = \frac{\theta}{t} = \frac{2\pi}{60} = 0.10$ radians s^{-1} (2 decimal places)

ii $\omega = \frac{\theta}{t} = \frac{2\pi}{60 \times 60} = 1.7 \times 10^{-3}$ radians s^{-1} (2 significant figures)

iii $\omega = \frac{\theta}{t} = \frac{2\pi}{60 \times 60 \times 12} = 1.5 \times 10^{-4}$ radians s^{-1} (2 significant figures)

c i $v = r\,\omega = 2.5 \times 10^{-2} \times 0.1 = 2.5$ mms^{-1}

ii $v = r\,\omega = 2.0 \times 10^{-2} \times 1.7 \times 10^{-3} = 3.4 \times 10^{-2}$ mms^{-1}

iii $v = r\,\omega = 1.5 \times 10^{-2} \times 1.5 \times 10^{-4} = 2.3$ μms^{-1}

2 a $\omega = \frac{10 \times 2\pi}{5 \times 60} = 0.21$ $rads^{-1}$

b i $v = r\,\omega = 2.5 \times 0.21 = 0.53$ ms^{-1}

ii $v = r\,\omega = 4.5 \times 0.21 = 0.95$ ms^{-1}

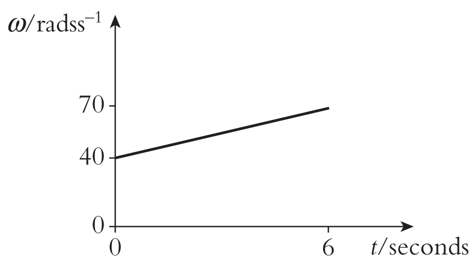
3 a Angular acceleration is the rate at which the angular velocity is changing: $\alpha = \frac{d\omega}{dt}$.

b i $\omega_i = \frac{300 \times 2\pi}{60} = 31.4$ radians s^{-1}

ii $\omega_f = \frac{120 \times 2\pi}{60} = 12.6$ radians s^{-1}

iii $\alpha = \frac{12.6 - 31.4}{5} = -3.76$ radians s^{-2}

4 a



b i $\omega_f = \omega_i + \alpha t = 40 + 5 \times 6 = 70$ radians s^{-1}

ii $\theta = \omega_i t + ½\,\alpha t^2 = 40 \times 6 + ½ \times 5 \times 6^2 = 240 + 90 = 330$ radians

So, number of rotations = $\frac{330}{2\pi} = 52.5$.

5 a $\frac{\omega_S}{\omega_E} = \frac{\left(\frac{2\pi}{24.47 \times 86400}\right)}{\left(\frac{2\pi}{86400}\right)} = \frac{1}{24.47} = 4.08 \times 10^{-2}$

b $\frac{v_S}{v_E} = \frac{r_S\omega_S}{r_E\omega_E} = \frac{6.96 \times 10^5}{6.37 \times 10^3} \times 4.08 \times 10^{-2} = 4.46$

6 a $\alpha = \frac{15 - 20}{4} = -1.25$ radians s^{-2}

b $t = \frac{\Delta\omega}{\alpha} = \frac{600}{15} = 40$ s

c $\theta = \frac{\omega_f^2 - \omega_i^2}{2\alpha} = \frac{40^2 - 15^2}{2 \times 2} =$ 344 radians (3 s. f.)

7 a $\omega_i = \frac{v}{r} = \frac{10.0}{0.400} = 25.0$ radians s^{-1}

b number of rotations = $\frac{\text{distance travelled}}{\text{circumference of wheel}} = \frac{50.0}{0.40 \times 2\pi} = 19.9$.

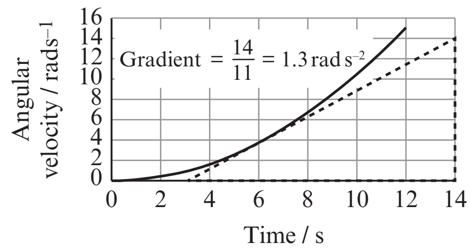
c $\alpha = \frac{\omega_f^2 - \omega_i^2}{2\theta} = \frac{0 - 25.0^2}{2 \times 2\pi \times 19.9} =$ 2.50 radians s^{-2}

d $t = \frac{\omega_f - \omega_i}{\alpha} = \frac{0 - 25.0}{2.50} = 10.0$ s

8 a α = gradient of graph = $\frac{4.5 \times 10^4}{3} = 1.5 \times 10^4$ $rads^{-2}$

b Revs = $\frac{\text{total angle}}{2\pi} = \frac{\text{Area under graph}}{2\pi} = \frac{\left(\left(\frac{1}{2} \times 4.5 \times 3\right) + (4.5 \times 2)\right) \times 10^4}{2\pi} = 2.5 \times 10^4$ revolutions

9 a i α = gradient of graph:



ii Total angle = area under graph = 15 rectangles @ 4 rad = 60 radians

∴ Average angular velocity = $\frac{60}{12}$ = 5 $rads^{-1}$

b **i** $\frac{d\omega}{dt} = \frac{4\pi}{60}t = 1.3 \text{ rads}^{-2}$

ii $\int_0^{12} \frac{2\pi}{60} t^2 dt = \left[\frac{2\pi}{180} t^3\right]_0^{12} = 60$ radians

$\therefore$ Average angular velocity $= \frac{60}{12} =$ 5 rads^{-1}

Exercise 5.2

1 **a** Torque is the product of the perpendicular force and the distance from the axis of rotation where it is applied: $\tau = Fd \sin \theta$, where F is the force applied, d is the distance from the axis of rotation and θ is the angle between the force and the line joining where the force is applied to the axis of rotation.

b Nm (**not** Joules).

c **i** $\tau = 250 \times 0.6 = 150$ N m

ii $\tau = 400 \times 3 \times \sin 30° = 600$ N m

2 **a** A couple is a pair of forces of the same magnitude acting in opposite directions; they are not collinear, so they cause a rotation of the object they are applied to.

b The two forces act in opposite directions; they combine to give zero resultant translational force. The two forces contribute an equal torque, so the resultant torque is not zero and is not in rotational equilibrium.

c A constant couple will produce a constant angular acceleration.

3 **a** The lid will require a couple that is equal to (or greater than) 15 N m. So, each force must be at least $F = \frac{\tau}{2r} = \frac{15}{2 \times 3.5 \times 10^{-2}} = 214 \text{ N} = 210$ N. (2 s.f.)

b The handles of the gadget increase the distance from the axis of rotation, so the force required now is $F = \frac{\tau}{2r} = \frac{15}{2 \times (3.5 + 15) \times 10^{-2}} = 40.5 \text{ N} =$ 41 N. (2 s.f.)

c Someone may not be able to apply a force of 214 N to open the jar with their hands. The gadget reduces the required force to 41 N.

4 **a** **i** Net force = 0

ii Net torque = 0

b **i** W is the weight of the ladder (Note that it acts from the centre of mass of the ladder.)

N_1 is the normal reaction force from the wall

N_2 is the normal reaction force from the ground

F is the static friction force between the foot of the ladder and the ground

ii $N_1 + F = 0$ and $W + N_2 = 0$

c $N_2 = 500 \text{ N} \Rightarrow F = N_1 = 0.25 \times 500 = 125$ N

$500 \times 4 \cos \theta = 125 \times 8 \sin \theta$

$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{2000}{1000}\right) = 63°$

d $\text{F} \times 4 \sin \theta + \text{N}_1 \times 4 \sin \theta = \text{N}_2 \times 4 \cos \theta$

$125 \times 4 \sin \theta + 125 \times 4 \sin \theta = 500 \times 4 \cos \theta$

$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{2000}{1000}\right) = 63°$

e $\text{F} \times 8 \sin \theta + \text{W} \times 4 \cos \theta = \text{N}_2 \times 8 \cos \theta$

$125 \times 8 \sin \theta + 500 \times 4 \cos \theta = 500 \times 8 \cos \theta$

$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{2000}{1000}\right) = 63°$

f It doesn't matter which point is taken to be the axis of rotation; all points give the same answer. Note that this suggests that if you take the axis of rotation to be the point at which more forces act than any other point, the calculation may be simpler to perform.

5 **a** $I = \sum_i m_i r_i^2$, where m is the mass of a small part of an object and r is its distance from the axis of rotation. When all contributions from all parts of an object are summed, this gives the moment of inertia.

b Mass is the property of a body that resists being accelerated. The moment of inertia, I, of a body is the property of a body that resists a body's angular acceleration.

c $\tau = I\alpha$

6 a $I = m\,r^2 = 0.20 \times 0.40^2 = 3.2 \times 10^{-2}$ kgm^2

b $I = 2 \times m\,r^2 = 2 \times 0.20 \times 0.40^2 = 6.4 \times 10^{-2}$ kgm^2

c $I = 2 \times m\,r^2 = 2 \times 0.20 \times 0.80^2 = 25.6 \times 10^{-2} = 0.256$ kgm^2

d i $I \propto$ m

ii $I \propto r^2$, so doubling r makes I four times larger.

7 a $I = \frac{1}{2}MR^2 = \frac{1}{2} \times 0.300 \times 0.2^2 = 6.0 \times 10^{-3}$ kgm^2

So, $\alpha = \frac{\tau}{I} = \frac{20 \times 0.2}{6.0 \times 10^{-3}} = 667$ radians s^{-2}

b Now the moment of inertia is $I = 6.0 \times 10^{-3} + 0.12 \times 0.12^2 = 7.7 \times 10^{-3}$ kgm^2.

So, the new angular acceleration is $\alpha = \frac{\tau}{I} = \frac{20 \times 0.2}{7.7 \times 10^{-3}} = 519$ radians s^{-2}.

8 $\omega = \alpha t \frac{\tau t}{I} = \frac{FRt}{\frac{1}{2}MR^2} = \frac{2Ft}{MR} = \frac{2 \times 4.0 \times 8.0}{6 \times 0.3} = 35.6$ radians s^{-1}

Exercise 5.3

1 a i angular momentum, $L = I\omega$, where I is the moment of inertia and ω is the angular velocity.

ii kgm^{2}s^{-1}

b i Angular impulse, ΔL, is the change in angular momentum.

ii kgm^{2}s^{-1}

c i τ_{net} is the net torque acting on a body, ΔL is the angular impulse and Δt the time during which the angular momentum changes.

ii $\tau_{net} = \frac{\Delta L}{\Delta t} = \frac{I\omega_{final} - I\omega_{initial}}{\Delta t} = \frac{I\Delta\omega}{\Delta t} = I\alpha$

iii $F = ma$

2 a $L = I\omega = mr^2\omega = 0.45 \times 1.5^2 \times 2\pi \times 2 = 12.7 = 13$ kgm^{2}s^{-1}

b $L = I\omega = \frac{2}{5}mr^2\omega = \frac{2}{5} \times 0.25 \times (3 \times 10^{-2})^2 \times 2\pi \times 50 = 0.28$ kgm^{2}s^{-1}

c $L = I\omega = \frac{1}{2}mr^2\omega = \frac{1}{2} \times 4.0 \times (4.0 \times 10^{-2})^2 \times 2\pi \times 20 = 0.40$ kgm^{2}s^{-1}

3 a $L = I\omega = \frac{2}{5}mr^2\omega = \frac{2}{5} \times 6.0 \times 10^{24} \times (6.4 \times 10^6)^2 \times \frac{2\pi}{24 \times 60 \times 60} = 7.1 \times 10^{33}$ kgm^{2}s^{-1}

b $F = \frac{\tau}{r} = \frac{\Delta L}{r\Delta t} = \frac{I\Delta\omega}{r\Delta t} = \frac{2mr^2\Delta\omega}{5r\Delta t} = \frac{2 \times 6.0 \times 10^{24} \times (6.4 \times 10^6)^2 \times 2\pi}{5 \times 6.4 \times 10^6 \times 24 \times 60 \times 60 \times 3.15 \times 10^7} = 3.5 \times 10^{19}$ N

4 a $\Delta L =$ Area under graph $= \frac{1}{2}(0.06 \times 3) \times 2 = 0.18$ kgm^{2}s^{-1}

b $\Delta L = I\Delta\omega \Rightarrow \Delta\omega = \frac{\Delta L}{I} = \frac{\Delta L}{\frac{1}{2}MR^2} = \frac{2 \times 0.18}{4.0 \times (3.0 \times 10^{-2})^2} = 1.0 \times 10^2 \Rightarrow \omega = 15 + 100 = 115$ kgm^{2}s^{-1}

c $E_{K\_rot} = \frac{1}{2}I\omega^2 = \frac{1}{2} \times \frac{1}{2} \times 4 \times (3.0 \times 10^{-2})^2 \times (115)^2 = 12$ J

5 a $L = I\omega = \frac{1}{2}MR^2\omega = \frac{1}{2} \times 2.5 \times 0.12^2 \times 2\pi \times 3 = 0.34$ kgm^{2}s^{-1}

b $\tau = \frac{\Delta L}{\Delta t} = \frac{0.34}{1.5} = 0.23$ Nm

c Conservation of angular momentum $\Rightarrow \omega = \frac{L}{I_{new}} = \frac{0.34}{\frac{1}{2} \times (2.5 + 4.5) \times 0.12^2} = 6.7$ rads^{-1}

$\therefore$ No. of revs $s^{-1} = \frac{6.7}{2\pi} = 1.1$ revs s^{-1}

6 a $E_K = \frac{1}{2}Mv^2$

b $E_{K\_rot} = \frac{1}{2}I\omega^2$

c $E_{K\_Total} = \frac{1}{2}(Mv^2 + I\omega^2)$

$= \frac{1}{2}\left(Mv^2 + \frac{2}{5}MR^2\left(\frac{v}{R}\right)^2\right) = \frac{1}{2}Mv^2\left(1 + \frac{2}{5}\right)$

$= \frac{7}{10}Mv^2$

$= \frac{7}{10} \times 163 \times 10^{-3} \times 4.0^2$

$= 1.83$ J

7 a $v = \text{at} = \frac{2st}{t^2} = \frac{2s}{t} = \frac{2 \times 1.5}{6.0} = 0.5$ ms^{-1}

b $\omega = \frac{v}{R} = \frac{0.5}{3.5 \times 10^{-2}} = 14.3$ radians s^{-1}

c $Mgh = \frac{1}{2}(Mv^2 + I\omega^2) = \frac{1}{2}\left(Mv^2 + \frac{2}{5}MR^2\left(\frac{v}{R}\right)^2\right) = \frac{7}{10}Mv^2$

$\therefore gh = \frac{7}{10}v^2$

d $h = \frac{7}{10}\frac{v^2}{g} = \frac{7 \times 0.5^2}{10 \times 9.81} = 1.8$ cm.

8 a $Mgh = \frac{1}{2}(Mv^2 + I\omega^2) =$
$\frac{1}{2}\left(Mv^2 + \frac{1}{2}MR^2\left(\frac{v}{R}\right)^2\right) =$
$\frac{1}{2}M\left(v^2 + \frac{1}{2}v^2\right) = \frac{3}{4}Mv^2$

So, $v = \sqrt{\frac{4}{3}\frac{Mgh}{M}} = \sqrt{\frac{4gh}{3}}$

Since the mass, M, and the radius, R, do not appear in this expression, the final speed of the cylinder is independent of the mass and radius of the cylinder, so both cylinders will arrive at the bottom of the sloping surface with the same linear speed.

b $v = \sqrt{\frac{4gh}{3}} = \sqrt{\frac{4 \times 9.81 \times 0.20}{3}} = 1.6\ \text{ms}^{-1}$

c This is reminiscent of the story of Galileo dropping objects of different masses from the Leaning Tower of Pisa and showing that they hit the ground at the same time—implying that they were travelling at the same speed.

d The larger mass cylinder has a larger moment of inertia. When it rolls down the slope, more of its energy is transferred into rotational kinetic energy, allowing its translational kinetic energy to be the same as the smaller cylinder's.

Exam-style questions

Multiple-choice questions

1	A	[1]
2	D	[1]
3	A	[1]
4	B	[1]
5	D	[1]
6	D	[1]
7	C	[1]
8	D	[1]
9	B	[1]
10	A	[1]

Short-answer questions

11 a i $\omega = \frac{2\pi}{24 \times 60 \times 60} =$
$7.2 \times 10^{-5}\ \text{rads}^{-1}$ **[1]**

ii $\omega = \frac{2\pi}{365 \times 24 \times 60 \times 60} =$
$2.0 \times 10^{-7}\ \text{rads}^{-1}$ **[1]**

b $\frac{v_{equator}}{v_{orbit}} = \frac{r_{Earth}\,\omega_{spin}}{r_{orbit}\,\omega_{orbit}} = \frac{6.37 \times 10^6 \times 7.2 \times 10^{-5}}{1.5 \times 10^{11} \times 2.0 \times 10^{-7}}$
$= 0.015$ **[2]**

12 a $2\pi rf = 20 \Rightarrow f = \frac{20}{2\pi r} = \frac{20}{2\pi \times 0.3} = 10.6$
$\therefore \omega = 2\pi f = 2\pi \times 10.6 = 67\ \text{rads}^{-1}$ **[2]**

b $s = \frac{v^2}{2a} = \frac{400}{5} = 80\ \text{m}$
$\therefore$ No. of revolutions $= \frac{80}{2\pi r} =$
$\frac{80}{2\pi \times 0.3} = 42$ **[1]**

c $\alpha = \frac{\Delta\omega}{\text{t}} = \frac{67}{\frac{20}{2.5}} = 8.4\ \text{rads}^{-2}$ **[1]**

13 a $\alpha = \frac{\tau}{I} = \frac{30}{850} = 35\ \text{mrads}^{-2}$ **[1]**

b $\omega = \alpha t = 35 \times 10^{-3} \times 300 = 10.5 \Rightarrow$
$\text{revs s}^{-1} = \frac{10.5}{2\pi} = 1.7\ \text{revs s}^{-1}$ **[2]**

c $v = r\omega = 0.4 \times 10.5 = 4.2\ \text{ms}^{-1}$ **[1]**

14 a $\omega_{average} = \frac{2\pi}{3} \Rightarrow \omega_{final} = \frac{2 \times 2\pi}{3} \Rightarrow$
$\alpha = \frac{\Delta\omega}{\text{t}} = \frac{2 \times 2\pi}{3 \times 3} = 1.4\ \text{rads}^{-2}$ **[1]**

b $I = \frac{\tau}{\alpha} = \frac{Fr}{\alpha} = \frac{0.5 \times 0.25}{1.4} = 0.09\ \text{kgm}^2$ **[2]**

c $m = \frac{2I}{r^2} = \frac{2 \times 0.09}{0.25^2} = 2.9\ \text{kg}$ **[1]**

15 a $GPE\ lost = mgh = 0.02 \times 9.81 \times$
$0.3 \sin 15 = 15\ \text{mJ}$ **[1]**

b $15\ mJ = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 =$
$\frac{1}{2}\left(mv^2 + \frac{2}{5}mr^2\left(\frac{v}{r}\right)^2\right) = \frac{7}{10}mv^2$ **[2]**

$\therefore v = \sqrt{\frac{10 \times 15\ mJ}{7 \times 0.02}} = 1.0\ \text{ms}^{-1}$ **[1]**

c $t = \frac{s}{v_{average}} = \frac{0.3}{0.5} = 0.6\ \text{s}$ **[1]**

16 a $L = I\omega = mr^2\omega = 1.5 \times 0.6^2 \times 30 =$
$16.2 = 16\ \text{kgm}^2\text{s}^{-1}$ (2 s.f.) **[2]**

b $KE = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 0.54 \times 900 = 243$
$= 240\ \text{J}$ (2 s.f.) **[1]**

c $\tau = \frac{\Delta L}{\Delta t} = \frac{16.2}{4.5} = 3.6\ \text{N m}$ **[1]**

17 a i $\omega = \frac{2\pi \times 300}{60} = 31.4\ \text{radians s}^{-1}$ **[1]**

ii angular momentum, $L = I\omega =$
$\frac{2}{5} \times 20.0 \times 0.40^2 \times \frac{2\pi \times 300}{60} =$
$40.2\ \text{kgm}^2\text{s}^{-1}$ **[1]**

b The principle of conservation of angular momentum states that, when the overall torque acting on a body is zero, its angular momentum remains constant. **[1]**

c The new moment of inertia is $\frac{2}{5}MR^2 + mR^2 = \frac{2}{5} \times 20.0 \times 0.40^2 + 4.0 \times 0.40^2 = 1.92$ kgm². **[1]**

So, the new angular velocity $= \frac{40.2}{1.92} = 20.9$ radians s^{-1}.

So, number of rotations per minute $= \frac{20.9}{2\pi} \times 60 = 200$. **[1]**

18 a There is no net torque involved in the skater pulling her arms inwards (whatever force is required to do this has no component that is perpendicular to the radius of the circle she is rotating around). So, angular momentum is conserved. **[1]**

b $L = I\,\omega$ = constant. So if I decreases, then ω must increase. **[1]**

c $L_{before} = I\omega = 4.5 \times 0.8 \times 2\pi = 22.6 = L_{after} \Rightarrow \omega = \frac{22.6}{0.8} = 28$ rads^{-1} **[1]**

d $E_{K\_before} = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 4.5 \times (0.8 \times 2\pi)^2 = 56.86$ J

$E_{K\_after} = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 0.8 \times (28)^2 = 313.6$ J

$\therefore \Delta E_K = 313.6 - 56.86 = 260$ J (2 s.f.) **[2]**

19 a In the absence of a net torque, angular momentum is conserved.

So, $L = I\omega = 1100 \times 1.5 = 1650$ kgm²s^{-1}

$\therefore \omega_{new} = \frac{L}{I_{new}} = \frac{1650}{1100 + (65 \times 3.5^2)} = 0.87$ rads^{-1} **[2]**

b i $KE = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 1100 \times 1.5^2 = 1200$ J (2 s.f.) **[1]**

ii $KE = \frac{1}{2}I\omega^2 = \frac{1}{2} \times (1100 + (65 \times 3.5^2)) \times 0.87^2 = 720$ J (2 s.f.) **[2]**

Chapter 6

Exercise 6.1

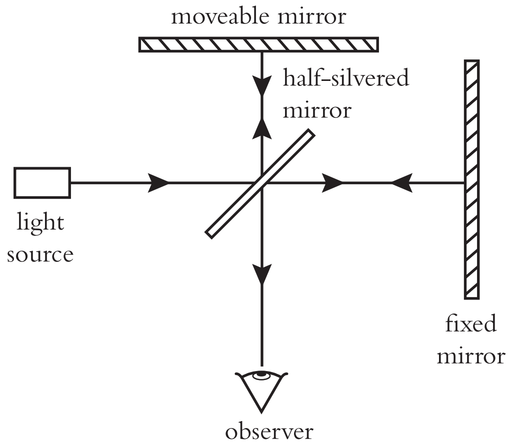
1 a i $v = v_{car} - v_{truck} = 18 - 12 = 6\ \text{ms}^{-1}$ in the same direction as the velocity of the truck.

ii $v = v_{truck} - v_{car} = 12 - 18 = -6\ \text{ms}^{-1}$ in the direction opposite to that of the car.

b i $v = v_{nitrogen} - v_{oxygen} = 500 - -438 = 938\ \text{ms}^{-1}$ in a direction upwards.

ii $v = v_{oxygen} - v_{nitrogen} = -438 - 500 = -938\ \text{ms}^{-1}$ in a direction downwards.

2 a



b If the aether existed, then a rotation of the apparatus would introduce a change in the speed of light from one direction to another. This change in speed would produce a shift in the interference pattern observed.

c The moveable mirror would introduce a change in the path length of one of the rays of light. This would result in a shift in the interference pattern.

d There was no shift in the interference pattern.

e Because there was no shift in the interference pattern, Michelson and Morley concluded that there was no aether (or that the aether had no effect on the speed of light). This was in agreement with what Maxwell and Einstein had predicted: that the speed of light was independent of any motion of the source of light or of the observer.

3 a i A frame of reference is a coordinate system and a means of measuring time that can provide a value for the position and time for a particle, anywhere and at any time.

ii An inertial frame of reference is a frame of reference in which Newton's first law of motion is obeyed.

b $6\ \text{ms}^{-1}$

c No. In the frame of reference of the moving bus, Ellie (and the two boys) are stationary. This would be the case whatever the speed of the bus is.

d Oscar sees the bus travelling forwards at $15\ \text{ms}^{-1}$ and the chocolate bar travelling forwards at $15 + 6 = 21\ \text{ms}^{-1}$.

e Yes. The bus is moving relative to Oscar in his frame of reference. So, the speed of the bus will affect how fast Oscar sees the chocolate bar moving.

f 3 m.

g $t = \frac{s}{v} = \frac{3}{6} = 0.5\ \text{s}$

h $s = v_{bus}\, t + v_{choc}\, t = (15 + 6) \times 0.5 = 21 \times 0.5 = 10.5\ \text{m}$.

i Yes. Because Oscar and Ellie measure the same time for the event to occur, their laws of physics explain their observations in the same way.

j Newton said that whatever the frame of reference is, an observer must see the same event occurring in the universe as any other observer.

k The laws of physics are the same for all inertial frames of reference.

4 a i $x' = \gamma\,(x - vt)$

ii $x = \gamma\,(x' + vt)$

iii $t' = \gamma\left(t - \frac{vx}{c^2}\right)$

iv $t = \gamma\left(t' + \frac{vx'}{c^2}\right)$

b When the clocks in both frames of reference show zero, (i.e. $t = t' = 0$) the origins of the two frames of reference coincide (i.e. $x = x' = 0$).

5 **a** $x_1' = \gamma(x_1 - vt_1)$ and $x_2' = \gamma(x_2 - vt_2)$

b $\Delta x' = x_2' - x_1' = \gamma(x_2 - vt_2) - \gamma(x_1 - vt_1) = \gamma(\Delta x - v(t_2 - t_1)) = \gamma(\Delta x - v\Delta t)$

6 **a** $t_1' = \gamma\left(t_1 - \frac{v}{c^2}x_1\right)$ and $t_2' = \gamma\left(t_2 - \frac{v}{c^2}x_2\right)$

b $\Delta t' = t_2' - t_1' = \gamma\left(t_2 - \frac{v}{c^2}x_2\right) - \gamma\left(t_1 - \frac{v}{c^2}x_1\right)$

$= \gamma\left((t_2 - t_1) - \frac{v}{c^2}(x_2 - x_1)\right) = \gamma\left(\Delta t - \frac{v}{c^2}\Delta x\right)$

7 **a** $u = \frac{v + u'}{1 + \frac{vu'}{c^2}} = \frac{0.7c + 0.3c}{1 + \frac{0.7c \times 0.3c}{c^2}} = \frac{c}{1.21} = 0.83c$

b $u = \frac{v + u'}{1 + \frac{vu'}{c^2}} = \frac{0.7c + c}{1 + \frac{0.7c \times c}{c^2}} = \frac{1.7c}{1.7} = c$ which is

what the second postulate says: the speed of light is the same for all observers in all inertial frames of reference.

8 $u = \frac{v + u'}{1 + \frac{vu'}{c^2}} = \frac{0.7c + 0.6c}{1 + \frac{0.7c \times 0.6c}{c^2}} = \frac{1.3c}{1.42} = 0.92c$

Exercise 6.2

1 **a** Invariant: a quantity is the same for all observers in all inertial frames of reference.

b **i** $\Delta s^2 = (c\Delta t)^2 - \Delta x^2 \Rightarrow \Delta s = \sqrt{(c\Delta t^2) - \Delta x^2}$

ii $c\Delta t = c(t_2 - t_1) = c\left(\gamma\left(t_2' + \frac{vx_2'}{c^2}\right) - \gamma\left(t_1' + \frac{vx_1'}{c^2}\right)\right) = \gamma\left(c\Delta t' + \frac{v\Delta x'}{c}\right)$

So, $(c\Delta t)^2 = \gamma^2\left(c\Delta t' + \frac{v\Delta x'}{c}\right)^2$.

And, $\Delta x = (x_2 - x_1) = \left(\gamma\left(x_2' + vt_2'\right) - \left(\gamma\left(x_1' + vt_1'\right)\right) = (\gamma(\Delta x' + v\Delta t')\right.$.

So, $\Delta x^2 = \Delta^2 (\Delta x' + v\Delta t')^2$.

Therefore, $\Delta s^2 = (c\Delta t)^2 - \Delta x^2 =$

$\gamma^2\left(c\Delta t' + \frac{v\Delta x'}{c}\right)^2 - \gamma^2(\Delta x' + v\Delta t')^2$.

$= \gamma^2\left(c^2\Delta t'^2 + 2c\Delta t'\frac{v\Delta x'}{c} + \frac{v^2\Delta x'^2}{c^2} - \Delta x'^2 - 2\Delta x' vt' - v^2\Delta t'^2\right)$

$= \gamma^2\left((c^2 - v^2)\Delta t'^2 - \left(1 - \frac{v^2}{c^2}\right)\Delta x'^2\right)$

$= (c\Delta t')^2 - \Delta x'^2$

$= \Delta s'^2$

So, the spacetime interval is invariant.

c **i** Rest mass is the mass of an object or particle that is stationary in its frame of reference.

ii Proper length is the length of an object measured by an observer in a frame of reference in which the object is at rest with respect to the observer.

iii Proper time is the time interval between two events occurring in a frame of reference in which the two events occur in the same position.

2 **a** **i** $t = \frac{H}{c}$

ii Yes. The container and both observers are in the same frame of reference.

b **i** Yes. The observer inside the container is in the same frame of reference as the light source, so this observer still measures the time for the light beam to reach the top of the container as $t = \frac{H}{c}$.

ii $(\text{distance travelled})^2 = (ct)^2 + (vt')^2$

iii $(ct')^2 = (ct)^2 + (vt')^2 \Rightarrow \frac{t'^2(c^2 - v^2)}{c^2} = t^2 \Rightarrow t'^2 = \frac{t^2}{1 - \frac{v^2}{c^2}}$

So, $t' = \gamma\, t$.

c It is called time dilation because the time t' is longer than the time t, since $\gamma > 1$.

3 **a** $L = x_2 - x_1 = \gamma\left(x_2' + vt'\right) - \gamma\left(x_1' + vt'\right) = \gamma\left(x_2' + vt' - x_1' - vt'\right) = \gamma\left(x_2' - x_1'\right)$

b $\left(x_2' - x_1'\right) = L'$ so $L' = \frac{L}{\gamma}$.

c This is called length contraction because L' is shorter than L, since $\gamma > 1$.

4 $u = \frac{v + u'}{1 + \frac{vu'}{c^2}} = \frac{0.5c + 0.5c}{1 + \frac{0.5c \times 0.5c}{c^2}} = \frac{c}{1.25} = 0.8c$

5 $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{1}{\sqrt{1 - 0.7225}} = 1.9$

So, the observer in the fast car measures the length of the building to be $\frac{100}{1.9} = 53$ m (2 s.f.).

6 $L' = \frac{L}{\gamma} = 12.5 \times \sqrt{1 - \frac{(0.85\,c)^2}{c^2}} = 6.6$ km (2 s.f.)

7 $t' = \gamma t = \frac{20}{\sqrt{1 - \frac{(0.9c)^2}{c^2}}} = 46$ ns

8 **a** Two identical twins age by different amounts because they move at a relative speed to each other. One twin travels to a distant place at a relativistic speed, whilst the other twin stays at home on Earth. Because each twin sees their sibling as moving relative to themselves, both twins should exhibit the same time dilation, and so each twin should consider their sibling to be younger than they are. That is why this is called a paradox.

b The twin that stays at home on the Earth.

c The twin that stays at home on Earth has remained in the same frame of reference for the whole of the other twin's journey. For the twin that has made the journey, on arrival at the distant place, the twin has changed their frame of reference because they have changed their velocity (which requires an acceleration and hence unbalanced force). This change of reference frame breaks the symmetry of the observations of the two twins and so allows both twins to agree that it is the twin who stays at home that ages the most.

9 **a** $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-0.94}} = 4.11$

So, $t'_{\frac{1}{2}} = \gamma t_{\frac{1}{2}} = 4.11 \times 3.1 \times 10^{-6} = 1.28 \times 10^{-5}$ s

b Distance travelled $= vt = 0.97 \times 3 \times 10^8 \times 1.28 \times 10^{-5} = 3.72$ km

c In the muon's frame of reference, the distance they travel is less, but the time is also less.

$t_{\frac{1}{2}} = 3.1 \times 10^{-6}$ s, and $s = \frac{s'}{\gamma} = \frac{3720}{4.11} = 905$ m, giving $v = \frac{s}{t_{\frac{1}{2}}} = \frac{905}{3.1 \times 10^{-6}} = 0.97c$.

d Number of half-lives $= \frac{15}{3.72} = 4.0$

e After 4.0 half-lives, the number of muons reaching the Earth's surface will be $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$ of the number produced. Since the number produced is very high, there will be significant numbers of muons reaching the surface.

(In fact, there will be fewer than $\frac{1}{16}$ of the number produced, because some of the muons will interact with atoms along their path through the atmosphere and never get as far as the Earth's surface.)

Exercise 6.3

1 **a** p is stationary.

b q is travelling at a constant speed.

c w is accelerating.

2 **a** Gradient of the worldline for s is $1/c$, so s is travelling at speed c.

b The gradient of the worldline for r suggests that r would be travelling faster than c. This is not possible.

3 **a** The speed of the photon is c. So, $x = ct$. Gradient $= \frac{ct}{ct} = 1$.

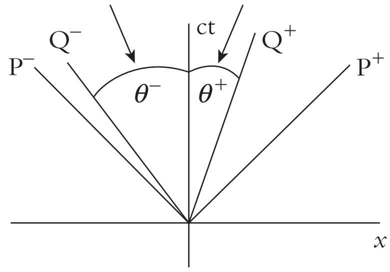
b The speed of B is v. So, $x = vt$.

Therefore, $\theta = \tan^{-1}\left(\frac{x}{ct}\right) = \tan^{-1}\left(\frac{vt}{ct}\right) = \tan^{-1}\left(\frac{v}{c}\right)$.

c Since $\tan\theta = \frac{v}{c}$, this allows us to find v in units of c.

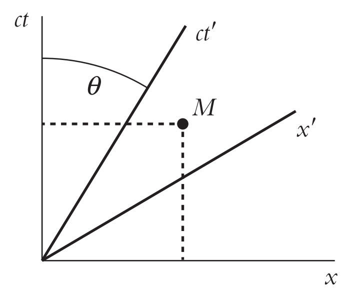
d $\theta_{max} = 45°$ because this is the angle that gives a value of v as c. Any larger value of v (i.e. any value of $\tan\theta$ greater than one) is not possible because it would mean that $v > c$.

4 **a–d** $\theta^- = \tan^{-1}(0.8) = 38.7°$ $\quad \theta^+ = \tan^{-1}(0.3) = 16.7°$

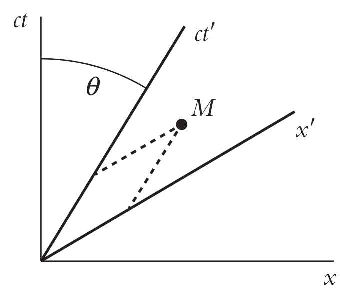


5 **a** $\theta = \tan^{-1}\left(\frac{v}{c}\right)$

b **i**



ii



c **i** $x' = \gamma x$

ii $ct' = \gamma ct$

6 a i A and B

ii A and B cannot occur simultaneously in S' because they do not lie on a line that is parallel to the x'-axis

b i B and E

ii B and E cannot occur in the same place in S' because they do not lie on a line that is parallel to the ct'-axis.

c i B and C

ii B and C cannot occur simultaneously in S because they do not lie on a line that is parallel to the x-axis.

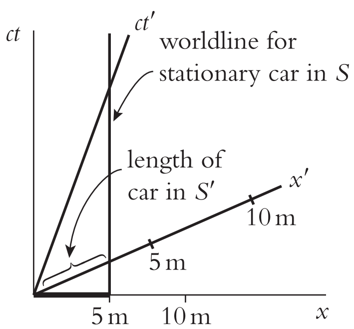
d i E and D

ii E and D cannot occur in the same place in S because they do not lie on a line that is parallel to the ct-axis.

e i Between E and B in S, and between E and D in S', it is possible to measure a proper time. This is because E and B occur in the same place in S and E and D occur in the same place in S'.

ii Between A and B in S and between B and C in S', it is possible to measure a proper length. This is because A and B occur at the same time in S and B and C occur at the same time in S'.

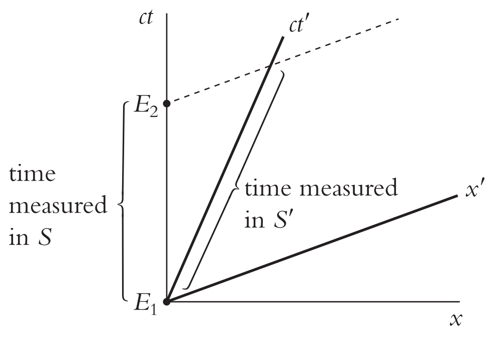
7 a



b The length of the car in S' is shorter than it is in S.

c Because of relativistic length contraction, the scale of the axes for S and S' have to be different. Relative to S, the scale for S' is different by a factor of γ.

8 a



b The time in S' is longer than it is in S.

9 a $t = \frac{s}{v} = \frac{10\text{ ly}}{0.9c} = 11.1$ years

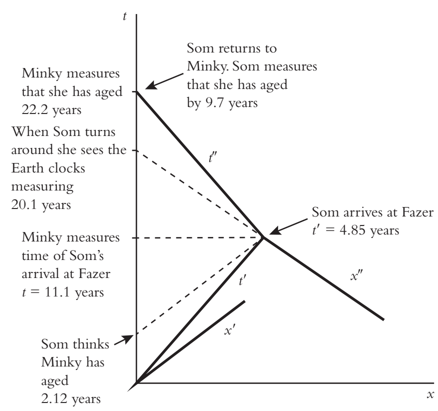
b $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-0.81}} = 2.29$

So, Som has aged $\frac{11.1}{2.29} = 4.85$ years.

c Som's clock shows 4.85 years, but she sees Minky to have aged $\frac{4.85}{\gamma} = \frac{4.85}{2.29} =$ 2.12 years.

d When Som turns around at Fazer, she changes her frame of reference because she has changed her relative motion to Minky.

e



f Som sees that the Earth clocks now show a time of 2.12 + (22.2 − 2 × 2.12) = 20.1 years.

g Minky is older by 22.2 − 9.7 = 12.5 years

Exam-style questions

Multiple-choice questions

1 A [1]

2 D [1]

3 A [1]

4 A [1]

5 B [1]

6 D [1]

7 A [1]

8 C [1]

9 A [1]

10 C [1]

Short-answer questions

11 a Since the Earth–Mars separation is a proper length,

$\Delta t = \frac{L}{v} = \frac{2.4 \times 10^{11}}{0.95 \times 3 \times 10^8} = 840$ s (2 s.f.). [1]

b $0.95c$ [1]

c $L' = \frac{L}{\gamma} = \frac{2.4 \times 10^{11}}{\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}} = \frac{2.4 \times 10^{11}}{\frac{1}{\sqrt{1-\frac{(0.95c)^2}{c^2}}}} =$

$\frac{2.4 \times 10^{11}}{3.20} = 7.5 \times 10^{10}$ m [2]

d $\Delta t = \frac{\Delta t'}{\gamma} = \frac{840}{3.20} = 260$ s [1]

12 a $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-0.75^2}} = 1.51$ [2]

b i $x' = \gamma\,(x - vt) = 1.51$
$(500 - 0.75 \times 3 \times 10^8 \times 3.0) =$
-1.0×10^9 m

ii $t' = \gamma\left(t - \frac{vx}{c^2}\right) =$

$1.51\left(3.0 - \frac{0.75 \times 3 \times 10^8 \times 500}{9 \times 10^{16}}\right) = 4.5$ s [2]

13 a It is a proper length because both the Earth and the star are stationary in the frame of reference of the observer on the Earth. [1]

b $\gamma = \frac{L}{L'} = \frac{4.244}{0.65} = 6.5292 =$
6.53 (3 s.f.) [2]

c $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow \frac{v}{c} = \sqrt{1-\frac{1}{\gamma^2}} =$

$\sqrt{1-\frac{1}{6.5292^2}} = 0.988$ [2]

14 a Taking S to be the frame of reference of the observer on the ground and S' to be the frame of reference of the observer on the train, $\Delta t' = \frac{400}{3 \times 10^8} =$
1.3×10^{-6} s. [1]

b $\Delta x = \gamma(\Delta x' + v\,\Delta t') = 1.25$
$(400 + 0.6 \times 3 \times 10^8 \times 1.3 \times 10^{-6}) =$
790 m (2 s.f.) [2]

c $\Delta t = \gamma\left(\Delta t' + \frac{v}{c^2}\Delta x'\right) = 1.25$
$\left(1.3 \times 10^{-6} + \frac{0.6 \times 3 \times 10^8 \times 400}{(3 \times 10^8)^2}\right) =$
2.6×10^{-6} s [2]

Alternatively, having found the length of the train in S, and knowing the second postulate, the time taken for the light to reach the front of the train must be $\Delta t = \frac{\Delta x}{c} = \frac{790}{3 \times 10^8} = 2.6 \times 10^{-6}$ s. [2]

15 a The distance to Proxima Centauri is
$4.0 \times 3 \times 10^8 \times 3.15 \times 10^7 =$
3.78×10^{16} m.

Therefore, $t = \frac{s}{v} = \frac{3.78 \times 10^{16}}{0.9 \times 3 \times 10^8} =$
1.4×10^8 s (= 4.44 years). [2]

b $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-0.81}} = 2.29$

So, distance to Proxima Centauri as measured by Anand $= \frac{s}{\gamma} = \frac{3.78 \times 10^{16}}{2.29} =$
1.65×10^{16} m.

Therefore, $t = \frac{s'}{v} = \frac{1.65 \times 10^{16}}{0.9 \times 3 \times 10^8} =$
6.1×10^7 s (= 1.94 years). [2]

c Anand's measurement is a proper time because he is measuring both leaving the Earth and arriving at Proxima Centauri at the same place: his rocket ship. [1]

16 a The student measures the two events, the kettle at the start and the kettle when it has boiled, at the same point in space in the student's inertial frame of reference. So, the time measured is a proper time interval. [1]

b $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{2^2}}} = \frac{1}{\sqrt{1-(0.8)^2}} = 1.67$ [2]

c $\gamma \times$ proper time $= 1.67 \times 2$ minutes $=$
3.3 minutes [2]

17 a $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-0.9995^2}} = 31.60$ **[1]**

b $\Delta t = \frac{s}{v} = \frac{4.00\text{ ly}}{0.9995c} = 4.002$ years √

c $\Delta t' = \frac{\Delta t}{\gamma} = \frac{4.002}{31.6} = 0.127$ years **[2]**

18 a i A **[1]**

ii B **[1]**

b i C **[1]**

ii A **[1]**

19 a $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-0.9025}} = 3.20$ **[1]**

b $\Delta t = \gamma \Delta t' = 3.2 \times 26 = 83$ ns **[2]**

c $s = v\Delta t = 0.95 \times 3 \times 10^8 \times 83 \times 10^{-9} = 24$ m **[2]**

Chapter 7

Exercise 7.1

1 **a** $\rho = \frac{m}{V} = \frac{22.5}{0.3 \times 0.12 \times 0.08} = 7800\ \text{kgm}^{-3}$

b $m = \rho V = 19300 \times 15730 \times 10^{-6} =$ 303.6 kg

So a pure gold bar of the given volume would have a mass that is greater than the advertised 250.000 kg. This suggests that some of the content of the gold bar has a density that is less than that of pure gold.

c $V = \frac{m}{\rho} = \frac{5}{660} = 7.6 \times 10^{-3}\ \text{m}^3$

So the volume of the potatoes is equal to the volume available in the shopping bag. However, since the potatoes are of irregular shape, they will require a larger volume because of the air gaps between them. So, yes, Arun is right to be concerned; they will not fit in his shopping bag!

2 **a** $\rho = \frac{m}{V} = \frac{3m}{4\pi r^3} = \frac{3 \times 2 \times 10^{30}}{4\pi \times (7 \times 10^8)^3} =$ $1.4 \times 10^3\ \text{kgm}^{-3}$

b The Sun is gaseous, so the density at its centre will be greater than the average and the density at its surface will be less than the average.

c $\rho_{\text{Earth}} = \frac{m}{V} = \frac{3m}{4\pi r^3} = \frac{3 \times 6 \times 10^{24}}{4\pi \times (6.4 \times 10^6)^3} =$ $5.5 \times 10^3\ \text{kgm}^{-3}$

So the density of the Earth is about 4 times the density of the Sun.

3 **a** **Language:** someone else tells us, using words we think we understand.

Sense perception: we touch something and our brain deciphers the signals from the sense receptors. Sometimes we can see – or smell – that something is hot.

Experience/memory: we may have experienced something before and have remembered.

b They are relative to us. In order for us to claim that something is hot, we need to have something else with which we can make a comparison. *Hotter* and *colder* are easier terms to use because our sensory perception really tells us that something is hotter or colder than we are. Only through considerable experience are we then able to estimate how hot or cold something is.

c In our everyday lives we do think of temperature as a measure of hotness—or coldness. And yet, it isn't easy to explain what we mean by hotness or coldness because these terms are relative to something else—usually us. If we accept that hotness is some kind of measure of how much energy something has, then it is reasonable to replace the word hotness with temperature—and we are beginning to sound much more like a physicist.

d The sensation of something feeling hot is the brain's way of deducing, from the stimuli it recieves, that there is a net flow of thermal energy from the hot(ter) object to us.

e The sensation of something feeling cold is the brain's way of deducing that there is a net flow of thermal energy from us to the cold(er) object.

f We use our experience. Because it is likely that we have touched many things that are hotter—or colder—than we are and may have learned what their temperature is, we remember and recall. For example, if we touch an ice cube, we will know that its temperature has to be 0 °C or less. Our sense of touch is not very good at putting an exact figure to the temperature of something; it is good at deducing whether something is hotter or colder than we are.

g No. It is well known that a burn from steam is worse than a burn from an equal amount of boiling water. They feel different not because they are at different temperatures, but because they contain different amounts of energy that can be transferred to us when we touch them. The steam contains more energy, so it feels hotter.

h Yes, our sense of touch is not absolute. What we feel depends on the temperature of what we feel with. Generally it isn't necessary for us to be able to give an accurate value to the temperature of what we touch, only that what we touch is hotter, or colder, than we are. So, no, we don't need to be concerned with our sense of touch.

i If we touch something with our fingers that is hotter than we are, our brain tells us that it is hot. A physicist would say that our sense of touch is telling us that there is a net flow of thermal energy *from* the hotter object *to* our fingers. Touching something colder results in a net flow of thermal energy *from* our fingers *to* the colder object.

j Yes, the two models are very similar. A difference in temperature forces the flow of thermal energy. A difference in electrical potential forces a net flow of electrical energy—that is, a current.

k Two objects at the same temperature exchange thermal energy at the same rate, so there is no net flow of thermal energy between them.

4 a Internal energy is the sum of the total random kinetic energy and the total inter-atomic, or inter-molecular, potential energy of the atoms, or molecules, of the sample.

b By changing the total random kinetic energy of the atoms, or molecules, (i.e. by adding thermal energy—heating) or by doing work on the sample (which may change the kinetic energy and/or the potential energy of the atoms or molecules).

5 a Sample C. The total inter-molecular potential energy is greater in the water vapour than in the liquid or solid phase, because the bonds holding the molecules together have been broken. The total random kinetic energy of the molecules is the same in all three samples.

b Sample A. Because the inter-molecular bonds effectively act as a negative amount of potential energy, and the total kinetic energy of the molecules is the same as for sample B and C, the total internal energy is least in sample A.

6 a No. There will be a range of energies.

b No.

c Kinetic energy and potential energy

d The kinetic energy of the atoms, or molecules, will increase. The potential energy of the atoms, or molecules, will stay the same.

e $\overline{KE} = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{2}mv_i^2$

f $c = \sqrt{\frac{1}{N}\sum_{i=1}^{N}v_i^2}$

g $\overline{KE} = \frac{1}{N}\sum_{i=1}^{N}\frac{1}{2}mv_i^2$ and $c^2 = \frac{1}{N}\sum_{i=1}^{N}v_i^2$

so $\overline{KE} = \frac{1}{2}mc^2$

7 a 0 K means that the average random kinetic energy of the atoms, or molecules, is zero. (Note this implies that they are not moving.)

b No. You cannot have negative kinetic energy.

c −273.15 °C (usually, −273 is sufficient)

d Yes, the two scales have the same incremental values.

e $T(°\mathrm{C}) = T(\mathrm{K}) + 273$

Exercise 7.2

1 Specific heat capacity: the energy required to heat up 1 kg of a substance by 1 K.
Heat capacity: the energy required to heat up a sample/body by 1 K.

2 **a** heat capacity $= \dfrac{\text{energy change}}{\text{change in temperature}} = \dfrac{300}{0.5} = 600\ \text{JK}^{-1}$

b $\Delta T = \dfrac{\text{energy}}{\text{heat capacity}} = \dfrac{3600}{450} = 8\ \text{K}$

c Object A, since heat capacity is the energy required to warm up a body by 1 K, and the object with the smallest heat capacity will be the one to heat up the most.

d $E = m\,c\,\Delta T = 5 \times 10^3 \times 420 \times (1540 - 15) = 3.2 \times 10^9\ \text{J}$

e $\Delta T = \dfrac{E}{m \times c} = \dfrac{2.5 \times 10^3}{50 \times 10^{-3} \times 1600} = 31.25$

Therefore, $T_{\text{final}} = 31.25 + 15 = 46\ °\text{C}$.

3 **a** **i** changing from liquid to vapor/gas with no change in temperature

ii changing from vapor/gas to liquid with no change in temperature

iii changing from liquid to solid with no change in temperature

iv changing from solid to liquid with no change in temperature

b

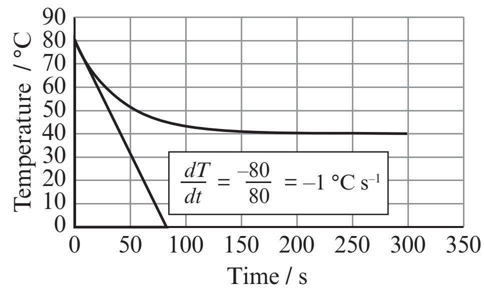
Change of phase	E_K	E_P
melting	stays the same	increases
vaporising	stays the same	increases
soldifying	stays the same	decreases
condensing	stays the same	decreases

4 $E = m \times L = 1.5 \times 2.3 \times 10^6 = 3.5 \times 10^6\ \text{J}$ (2 s.f.)

5 **a** The graph shows a decreasing gradient. So, for a constant specific heat capacity (which we assume), the rate at which energy is being lost is decreasing with time.

b 40 °C

c



d $\dfrac{dE}{dt} = mc\dfrac{dT}{dt} = 2 \times 680 \times (-1) = 1360\ \text{Js}^{-1} = 1400\ \text{Js}^{-1}$ (2 s.f.)

6 **a** Energy lost by aluminium $= m\,c\,\Delta T = m\,c\,(T_{\text{initial}} - T_{\text{final}}) = 0.4 \times 900 \times (800 - T_{\text{final}})$

b Energy gained by water $= m\,c\,\Delta T = m\,c\,(T_{\text{final}} - T_{\text{initial}}) = 2.5 \times 4200 \times (T_{\text{final}} - 20)$

c $0.4 \times 900 \times (800 - T_{\text{final}}) = 2.5 \times 4200 \times (T_{\text{final}} - 20)$

$\Rightarrow ((2.5 \times 4200) + (0.4 \times 900)) \times T_{\text{final}} = (0.4 \times 900 \times 800) + (2.5 \times 4200 \times 20)$

Therefore, $T_{\text{final}} = \dfrac{(0.4 \times 900 \times 800)\ + (2.5 \times 4200 \times 20)}{(2.5 \times 4200) + (0.4 \times 900)} = \dfrac{2.88 \times 10^5\ + 2.1 \times 10^4}{10\,500\ + 360} = 46\ °\text{C}$ (2 s.f.)

d No energy has been lost to the surroundings.

7 $E = (m\,s\,DT) + (m\,L) = (1.5 \times 4200 \times (100 - 15)) + (1.5 \times 2.3 \times 10^6)$

$= 5.355 \times 10^5 + 3.45 \times 10^6 = 3.99 \times 10^6\ \text{J}$

So, $t = \dfrac{E}{P} = \dfrac{3.99 \times 10^6}{1.5 \times 10^3} = 2.7 \times 10^3\ \text{s}$

(= 45 minutes).

8 Energy gained by ice $= m\,c_{\text{ice}}\,\Delta T + m\,L + m\,c_{\text{water}}\,\Delta T = m\,c_{\text{ice}}\,\Delta T + m\,L + m\,c\,(T_{\text{final}}) =$

$(0.05 \times 2100 \times 18) + (0.05 \times 3.3 \times 10^5) + (0.05 \times 4200 \times T_{\text{final}})$

Energy lost by water $= m\,c\,\Delta T = 0.25 \times 4200 \times (T_{\text{final}} - 12)$

$(0.05 \times 2100 \times 18) + (0.05 \times 3.3 \times 10^5) + (0.05 \times 4200 \times T_{\text{final}}) = 0.25 \times 4200 \times (T_{\text{final}} - 12)$

$((0.05 \times 4200) - (0.25 \times 4200))\ T_{\text{final}} = (0.25 \times 4200 \times 12) - ((0.05 \times 2100 \times 18) + (0.05 \times 3.3 \times 10^5))$

So, $T_{\text{final}} = \frac{(0.25 \times 4200 \times 12) - ((0.05 \times 2100 \times 18) + (0.05 \times 3.3 \times 10^5))}{((0.05 \times 4200) - (0.25 \times 4200))}$

$= 7\ °C$ (1 s.f.).

Exercise 7.3

1 a All the atoms will be vibrating about a fixed position with a range of random kinetic energies in random directions.

b They will start to vibrate more violently (because they will have gained energy).

c These further atoms will also start to vibrate more violently. In turn, they will then collide with more atoms and so on along the object.

d There is a very large number of atoms between the left-hand edge of the object and the right-hand edge. Collisions from one atom to the next take a small time to occur, but since there are many atoms, there will be many collisions. This will take some time to happen.

e Conduction is a slow process (which explains why it takes a long time for the handle of a spoon in a cup of hot tea to become hot).

f The free electrons will collide with the vibrating atoms, gain energy and move about with an increased speed. Since they are free to move anywhere, they will move in all directions—including to the right—at a fast speed. When these fast-moving electrons collide with atoms, they will transfer a little of their energy to the atoms.

g The fast-moving electrons transfer energy more quickly than the vibrating atoms transfer energy, because they are free to move anywhere. This allows the atoms at the right-hand edge of the metal object to gain energy relatively quickly compared to atoms in a non-metal. We say, therefore, that metals are better conductors than non-metals.

h i Conduction, generally, is most effective in solid materials. This is because the atoms are close together and so their vibrations can pass on energy to the atoms next to them fairly easily. Also the free electron density will contribute to the transfer of energy through the conductor.

ii Conduction, generally, is least effective in gases. This is because the atoms and molecules of gas are widely spaced apart making the collisions of one atom to the next less frequent. Also, there may be no free electrons to contribute to the transfer of energy through the material.

iii There are two factors acting here: the material itself, wool, is a poor conductor (it has a very low thermal conductivity, about 0.04 $\text{J s}^{-1}\ °\text{C}^{-1}\text{m}^{-1}$; the thermal conductivity of human skin is about 5 times larger than this), and the wool traps air in the spaces between the strands. Air has a very low thermal conductivity, about half that of wool.

2 a The fabric seat will feel neither hot nor cold. The material of the fabric is a poor conductor—or a good insulator—so the transfer of thermal energy between your fingers and the seat does not happen easily. Hence, what you 'feel' is that the fabric seat is neither hot nor cold.

b The metal legs of the chair are good conductors, so when you touch them they will 'feel' cold. This is because your fingers are at a higher temperature than the metal legs and so there is a net thermal energy transfer, by the process of conduction, from your fingers to the metal chair legs. Because the metal legs are good conductors, this thermal energy transfer occurs easily, making you feel that the legs of the chair are cold.

3 a Some factors might include

- the mass of atoms.
- the spacing of atoms.
- the density of free electrons in the material.
- the strength of the atomic bonds between atoms.

b **i** $\frac{\Delta Q}{\Delta t} \propto \frac{1}{l}$

ii $\frac{\Delta Q}{\Delta t} \propto A$

iii $\frac{\Delta Q}{\Delta t} \propto k$

iv $\frac{\Delta Q}{\Delta t} \propto \Delta T$

c $\frac{\Delta Q}{\Delta t} = kA\frac{\Delta T}{l}$

4 **a** $\frac{\Delta Q}{\Delta t} = kA\frac{\Delta T}{l} = 200 \times 8 \times 10^{-4} \times 2 =$ 0.32 W

b $\frac{\Delta Q}{\Delta t} = kA\frac{\Delta T}{l} \Rightarrow A = \frac{l}{k\Delta T}\frac{\Delta Q}{\Delta t} = \frac{0.65}{420 \times 40} \times 52 = 2.0 \times 10^{-3}$ m^2 (or 20 cm^2)

c $\frac{\Delta Q}{\Delta t} = kA\frac{\Delta T}{l} \Rightarrow \frac{\Delta T}{l} = \frac{1}{kA}\frac{\Delta Q}{\Delta t} = \frac{1}{40 \times 25 \times 10^{-4}} \times 3.5 = 35$ °Cm^{-1}

d $\frac{\Delta Q}{\Delta t} = kA\frac{\Delta T}{l} \Rightarrow k = \frac{l}{A\Delta T}\frac{\Delta Q}{\Delta t} = \frac{4 \times 10^{-3}}{2 \times 1.6 \times (30.9 - 30)} \times 600 =$ 0.83 Js^{-1} °C^{-1}m^{-1}

5 **a** The warm air from the convection heater is less dense and so rises, displacing the cooler air above it and forming a convection current. Since the molecules of air move at a fast speed, the transfer of thermal energy to the room if effected quickly, thus warming up the room in a fairly short time.

b The chill factor is created from the motion of cold air (i.e. wind). Warm bodies will transfer thermal energy to the air around them (by conduction and radiation), and this air is then blown away quickly by the wind, keeping a large temperature difference between the warm body and the air around it. This temperature difference continues to drive the transfer of heat from the warm body to the air. Because the air around the warm body is continually being blown away—by this forced convection—the air 'feels' colder than it actually is.

6 **a** The human body's working temperature is lower than the temperature of its surroundings. As the runner produces more energy, it cannot be transferred away by the usual methods of conduction, convection or radiation.

b Energy can only be lost by the evaporation of water from the surface of the skin: sweating. This evaporation causes cooling. So, the runners have to keep well hydrated throughout the race.

7 **a** radiated power = $\sigma\epsilon AT^4$

b power = $\sigma\epsilon AT^4 = 5.67 \times 10^{-8} \times 1 \times 4\pi \times (6.96 \times 10^8)^2 \times 5770^4 =$ 3.83×10^{26} W

c $\frac{\text{solar power}}{m^2} = \frac{3.83 \times 10^{26}}{4\pi \times (1.496 \times 10^{11})^2} =$ 1360 Wm^{-2}

8 **a** $\frac{\Delta Q}{\Delta t} = \sigma\epsilon A(T^4_{\text{radiator}} - T^4_{\text{room}}) =$ $5.67 \times 10^{-8} \times 0.55 \times 1 \times 0.5 \times ((273 + 60)^4 - (273 + 20)^4) =$

77 W (2 s.f.)

b Radiators are painted white because they look nice! (Actually, in Victorian Britain, cast iron radiators were painted black—and so were more efficient at radiating energy into a room. Such radiators were often hidden behind screens, because they were considered an eyesore!)

9 **a** 800 nm is in the infrared part of the electromagnetic spectrum.

b $T = \frac{2.9 \times 10^{-3}}{800 \times 10^{-9}} = 3600$ K (2 s.f.)

c From Wien's displacement law, if T increases, λ_{max} decreases.

Exam-style questions

Multiple-choice questions

1 B

2 D

3 C

4 B

5 C

6 C

7 D

8 D

9 C

10 A

Short-answer questions

11 a E_p lost = mg Δh = 0.6 × 10 × 0.8 = 4.8 J **[1]**

b $E_p \rightarrow E_K \rightarrow$ Internal energy **[1]**

c $\Delta T = \frac{\text{internal energy gained}}{\text{heat capacity}} = \frac{50 \times 4.8}{96} = 2.5$ K **[2]**

So, $T = 20 + 2.5 = 22.5$ °C. **[1]**

12 a The molecules of water have a range of values of energy **[1]**
Those molecules with the largest energies are able to break free of the water surface and become water vapour. **[1]**

b The temperature of the water is a measure of the average kinetic energy of the water molecules. The molecules that evaporate are those with the largest energies **[1]**
so when these are 'lost', the average kinetic energy decreases. The temperature decreases and the water cools. **[1]**

13 a Energy from the heater passes through the aluminium by conduction **[1]** (the passing of energy from one atom to the next). This takes a long time because there are so many atoms between the heater and the thermometer. So, the thermometer does not record an increase for a couple of minutes. **[1]**

b In one minute, the energy supplied to the aluminium is 60 × 50 = 3000 J.

During this time the temperature of the aluminium rises by 3.3 °C. So, using the energy equation:

$shc = \frac{\text{energy supplied}}{\text{mass} \times \Delta T} = \frac{3000}{1 \times 3.3} =$
909 ≈ 900 Jkg^{-1} °C^{-1} **[2]**

c Once the temperature of the aluminium block is greater than the ambient temperature, the aluminium block will start to share its energy with the surroundings. This causes the *rate* of temperature increase to slow down. A constant rate of 3.3 °Cmin^{-1} would produce a temperature of 103 °C, but the rate is not constant; it is decreasing **[2]**

14 a mass × *slhf* = power × time ⇒
$t = \frac{m \times slhf}{\text{power}} = \frac{0.5 \times 3.3 \times 10^5}{100} =$
1650 s (27.5 minutes) **[2]**

b mass × *shc* × Δ*T* = power × *t* ⇒
$t = \frac{m \times shc \times \Delta T}{\text{power}} = \frac{0.5 \times 4200 \times 100}{100} =$
2100 s (35 minutes) **[2]**

c mass × *slhv* = power × *t* ⇒
$t = \frac{m \times slhv}{\text{power}} = \frac{0.5 \times 2.26 \times 10^6}{100} =$
11 300 s (3.1 hours) **[1]**

15 a 60 °C **[1]**

b $\frac{\Delta Q}{\Delta t} = mc\frac{\Delta T}{\Delta t} \Rightarrow c = \frac{1}{\left(m\frac{\Delta T}{\Delta t}\right)}\frac{\Delta Q}{\Delta t} =$

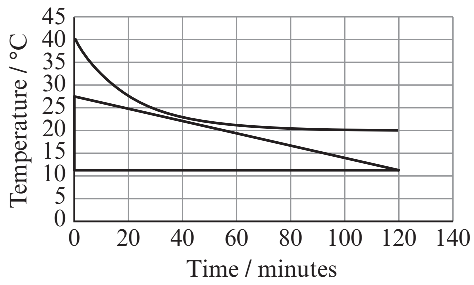
$\frac{1}{0.6 \times \left(\frac{60-20}{10 \times 60}\right)} \times 100 =$

2500 Jkg^{-1} °C^{-1} **[2]**

c $Q = mL \Rightarrow L = \frac{Q}{m} = \frac{100 \times 30 \times 60}{0.6} =$
3.0 × 10^5 Jkg^{-1} **[2]**

16 a The gradient shows the rate at which the temperature is changing. This rate depends on the difference between the temperature of the water and the temperature of the room **[1]**. As the water cools, the difference between the temperature of the water and the temperature of the room decreases, therefore the rate at which the water cools also decreases, hence the decreasing gradient of the graph. **[1]**

b



$\frac{dE}{dt} = 0.2 \times 4200 \times \frac{-(27.5 - 11.5)}{120} =$
-110 Jmin^{-1} (2 s.f.) **[2]**

c The black-coloured cup would lose energy by radiation at a faster rate. So, the graph would have a steeper gradient. That is it would take less time for the water to cool to the same temperature of the room. **[1]**

17 a $\frac{\Delta Q}{\Delta t} = kA\frac{\Delta T}{l} = 75 \times \frac{\pi \times 0.12^2}{4} \times$
$\frac{100}{0.5} = 169.7 = 170$ W **[2]**

b $\frac{\Delta Q}{\Delta t} = 170 \times \frac{390}{75} \times 0.5 = 442 =$
440 W **[1]**

c The rate of thermal energy transfer through each of the bars must be the same. So for the copper bar:
$\frac{\Delta Q}{\Delta t} = 390 \times A \times \frac{100 - T}{1}$.
for the iron bar: $\frac{\Delta Q}{\Delta t} = 75 \times A \times \frac{T - 0}{0.5}$.

So, $390 \times A \times (100 - T) = 150 \times A \times T$

$\therefore T = \frac{39\,000}{540} = 72$ °C. **[2]**

18 a $Q = m\,c\,\Delta T = 1 \times 4200 \times (100 - 20) = 336$ kJ **[1]**

$\therefore t = \frac{336\ kJ}{1000\ W} = 336$ s $= 5.6$ minutes **[1]**

b No energy lost/all energy from heater goes into heating water OR constant energy transfer from heater **[1]**

c Powder radiated $= \sigma\epsilon A T^4 =$
$5.67 \times 10^{-8} \times 1 \times 10^{-3} \times 373^4 = 1.1$ W **[2]**

19 a $\frac{\text{power radiated}}{A} = \sigma\epsilon T^4 =$
$5.67 \times 10^{-8} \times 1 \times 2000^4 =$
$90.7 \times 10^4 = 0.91$ MWm^{-2} **[2]**

b $\epsilon = \frac{0.23}{0.91} = 0.25$ **[1]**

c $P = 0.23 \times 4\pi 0.04^2 \times 3^4 =$
0.37 MW **[2]**

20 a Using Wien's displacement law:
$\lambda_{max} = \frac{2.9 \times 10^{-3}}{2500} = 1.16 = 1.2$ µm
(2 s.f.) **[2]**

b 1.2 µm is in the infrared part of the electromagnetic spectrum. **[1]**

c $T = \frac{2.9 \times 10^{-3}}{50 \times 10^{-9}} = 5.8 \times 10^4$ K **[1]**

Chapter 8

Exercise 8.1

1 a Radiation

b Temperature, surface area, emissivity, Stefan–Boltzmann constant

c $P = \sigma\varepsilon AT^4$, where P is the radiated power, σ is the Stefan–Boltzmann constant, ε is the emissivity of the surface and T is the absolute temperature.

d i A black body is a body that absorbs all radiation incident on it.

ii Real bodies reflect some of the radiation incident on them.

iii Emissivity, ε, is defined as the ratio of the power radiated by a body to that radiated by a black body of the same size and at the same temperature:

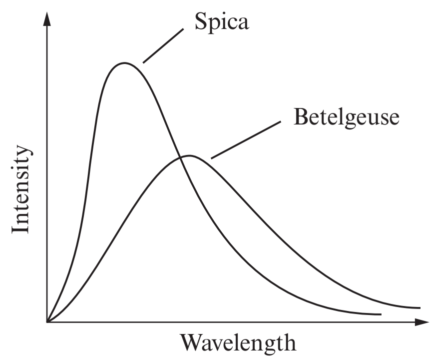
$$\varepsilon = \frac{\text{power radiated by a body}}{\text{power radiated by a black body of the same size and at the same temperature}}$$

2 a $P_{\text{radiated}} = \sigma\varepsilon AT^4 = 5.7 \times 10^{-8} \times 0.75 \times 4\pi \times 0.12^2 \times (150 + 273)^4 =$ 248 W = 250 W (2 s.f.)

b $P_{\text{absorbed}} = -\sigma\varepsilon AT^4 = -5.7 \times 10^{-8} \times 0.75 \times 4\pi \times 0.12^2 \times (20 + 273)^4 = -57$ W (2 s.f.)

c Net exchange of energy per second = 248 − 57 = 191 W

3 a



b i The hotter star will have its peak intensity at a smaller wavelength

ii Wien's displacement law.
$\lambda_{\text{peak}} = \frac{2.9 \times 10^{-3}}{T}$, where λ_{peak} is the wavelength at which maximum intensity occurs and T is the absolute temperature.

iii It doesn't! There is no emissivity term in Wien's law.

c $T = \frac{2.9 \times 10^{-3}}{650 \times 10^{-9}} = 4462 = 4500$ K (2 s.f.)

d i $\lambda_{\text{peak}} = \frac{2.9 \times 10^{-3}}{4300} = 670$ nm (2 s.f.)

ii With the peak in the spectrum at this wavelength (almost right at the far red end of the visible spectrum), there will be a large amount of energy at wavelengths that are too long to be visible. These infrared wavelengths will add to the visible part of the emission to make the luminosity (the total emitted power) larger than one would expect by considering the visible wavelengths only.

e $T = \frac{2.9 \times 10^{-3}}{1.063 \times 10^{-3}} = 2.7$ K

4 a Using Wien's displacement law:
$T = \frac{2.9 \times 10^{-3}}{400 \times 10^{-9}} = 7250 = 7300$ K (2 s.f.)

b $d = \sqrt{\frac{L}{4\pi b}} = \sqrt{\frac{7.2 \times 10^{27}}{4\pi \times 2.8 \times 10^{-10}}} =$
1.43×10^{18} m $= \frac{1.43 \times 10^{18}}{9.46 \times 10^{15}} = 151$ ly

5 a i Albedo is the ratio of the power reflected from a surface to the total power incident on the surface:

$$\alpha = \frac{\text{power reflected by surface}}{\text{total power incident on surface}}.$$

ii Albedo has no units; it is a factor only.

b The processes of conduction and convection both require the presence of atoms and molecules to facilitate the transfer of thermal energy. Around planets and stars there are no atoms and molecules (so very few that it is reasonable to assume there are none), so conduction and convection cannot occur.

c i A high emissivity means a low albedo—and vice versa.

ii $\alpha + \varepsilon = 1$

Exercise 8.2

1 a The Earth has an atmosphere; the Moon does not. The Earth's atmosphere is responsible for the higher surface temperature.

b The atmosphere of the Earth has the bigest effect on the albedo of the Earth.

c One reason is that the cloud cover in the atmosphere varies both over short and long periods of time, from hours to days to seasons during the year.

Another reason is that the terrain of the Earth's surface varies considerably, from polar ice caps to deep oceans to dense jungle and forestry to urban areas.

d i Polar regions have very high albedos.

ii Since a large proportion of the incident radiation is reflected—and therefore cannot be absorbed—it follows that the surface temperature of such areas will remain fairly constant.

e i The surface of the oceans, particularly where the ocean depth is substantial, have low albedos.

ii Although these areas absorb solar radiation readily, their specific heat capacity (i.e. that of water) is a high value. This suggests that only small changes in surface temperature would occur.

f More reflected solar radiation would result in a lower surface temperature.

2 a $S = \frac{P}{4\pi d^2}$

b Incident power $= \pi R^2 S$

c Incident power $\text{m}^{-2} = \frac{\pi R^2 S}{4\pi R^2} = \frac{S}{4}$

d Mean power absorbed $\text{m}^{-2} = \frac{S(1-\alpha)}{4}$

e i Power absorbed $\text{m}^{-2} = \frac{S(1-\alpha)}{4} = \frac{1400 \times (1-0.3)}{4} = 245\ \text{Wm}^{-2}$

So, to be in thermal equilibrium, the Earth would have to radiate 245 Wm^{-2}.

ii $T = \sqrt[4]{\frac{P}{\sigma}} = \sqrt[4]{\frac{245}{5.7 \times 10^{-8}}} = 256\ \text{K}$

iii At $T = 288$ K, the Earth is radiating $5.7 \times 10^{-8} \times 288^4 = 392\ \text{Wm}^{-2}$.

So, it will have to absorb $392 - 245 = 147\ \text{Wm}^{-2}$ extra.

iv The extra power is being radiated back to the surface by the atmosphere.

v This is the greenhouse effect.

3 a i $\lambda_{max} = \frac{2.9 \times 10^{-3}}{5700} = 510$ nm

ii This is in the visible part of the E.M. spectrum.

b i Ultraviolet is in the approximate range of 100–400 nm.

ii Visible light is in the range of 400–700 nm.

iii Infrared is in the approximate range of 700 nm–1 mm.

c Yes.

d **Ultraviolet:** because it tans our skin (and excesses of ultraviolet light cause skin cancer)

Visible: because we can see light from the Sun

Infrared: because it warms our bodies when we go outside in the sunshine

e i Ozone is a molecule of oxygen consisting of three oxygen atoms, O_3.

ii The energy of a short-wavelength ultraviolet photon is sufficient for it to break the atomic bond between two of the oxygen atoms. So, if the photon is absorbed, two of the oxygen atoms will break apart, dissociating the molecule.

iii Since the ultraviolet photons are being absorbed by the O_3, less ultraviolet radiation reaches the Earth's surface.

iv Ultraviolet radiation—particularly the shorter wavelengths—is harmful to life on Earth. So, the absorption of ultraviolet radiation by O_3 prevents life on the Earth from harm.

v Less ozone means less absorption. This means that more ultraviolet radiation would reach the surface and create harmful effects to life on the Earth.

vi Over the Antarctic

vii Countries in the southern hemisphere close to the Antarctic, such as Australia, New Zealand, South Africa, Argentina and Chile

viii Scientists have shown that ozone can be destroyed by chlorofluorocarbons (CFCs) and hydrochlorofluorocarbons (HCFCs), which had been used in refrigeration systems and as propellants in aerosol cans. The Montreal Protocol of 1989 persuaded world leaders to reduce the production and use of CFCs and HCFCs.

4 a At a temperature of 15 °C (288 K), the peak in the intensity of radiated energy occurs at a wavelength of $\lambda_{\text{peak}} = \frac{2.9 \times 10^{-3}}{288} = 1 \times 10^{-5}$ m (10 μm).

This is in the infrared region of the electromagnetic spectrum.

b i A molecule will be able to vibrate in several different ways. For example, one atom in the molecule may oscillate in a radial direction, whilst another may oscillate perpendicularly to the bond between it and another atom. These different oscillation modes will require different quantised amounts of energy, so for any given molecule there will be a set of discrete energy states available in which its atoms can oscillate.

ii Wavelengths larger than visible light will be in the infrared region of the electromagnetic spectrum.

iii Atmospheric molecules will be able to absorb those infrared wavelengths that enable them to change their oscillation modes. With a large variety of molecules, there will be a large number of wavelengths that can be absorbed. This means that much of the range of infrared wavelengths will be absorbed by the atmospheric molecules, meaning that the atmosphere cannot be transparent to infrared radiation.

c Power absorbed from the Sun m^{-2} = 245 W m^{-2}

Power radiated by the Earth's surface m^{-2} into space = $0.62 \times \sigma \times T^4$

So, $T = \sqrt[4]{\frac{245}{0.62 \times 5.7 \times 10^{-8}}} = 288.6 \approx 288$ K, as observed.

5 a In the troposphere

b Evaporation from oceans, seas, lakes, rivers

c Farming (e.g. rice fields) and irrigation of grassed areas, evaporation from man-made reservoirs, canals and lakes, petroleum and fossil fuel burning

d Condensation and the formation of precipitation (rain, mostly, as well as sleet and snow)

e An increase in water vapour concentrations would likely increase the albedo of the Earth. This would result in a *decrease* in the surface temperature.

6 a Microbial action in soils under natural vegetation, oceans and seas, oxidisation of ammonia in the atmosphere

b Agriculture and cultivation of soils, biomass burning, manure from animal farming, fossil fuel combustion, cars and other transport, man-made fertiliser production, synthetic material production and sewage treatment plants

c Nitrogen fixation by plants and photo-dissociation in the atmosphere

d Increased amounts of nitrous oxide would increase the greenhouse effect, making the surface temperature *increase*.

e Because most man-made nitrous oxide comes from food production: farming and agriculture. The demand for more and more food has made it difficult to find other ways of producing food that don't also produce nitrous oxide.

7 a Permafrost, ice cores, and glaciers; wetlands, forest fires, plants, seepage from coal and natural gas deposits, ocean floors, microbial actions in freshwater ecosystems, termites

b Rice farming, animal farming, decaying organic matter in landfills, waste-water treatment plants, transportation and use of fossil fuels

c Global warming will cause permafrost in the arctic regions, such as Siberia, to melt. This will release large amounts of methane into the atmosphere causing a kind of positive feedback effect, warming the Earth's surface even more.

d Hydroxyl and chlorine radicals present in the troposphere react with methane and remove it from the atmosphere.

e An increase in methane in the atmosphere will enhance the greenhouse effect.

8 a Decomposition of organic material, volcanic activity, wildfires, respiration by animals, outgassing from oceans, seas lakes and rivers, weathering of carbonated rocks

b Burning of fossil fuels, deforestation, cement production

c The biggest change has been the reduction in the burning of certain fossil fuels, for example high-tar coal. Governments worldwide have met frequently to propose measures for the further reduction in man-made carbon dioxide, such as the Paris Agreement of December 2015.

d Photosynthesis by plants, trees and vegetation, direct absorption by oceans, seas, lakes and rivers, natural production of soil and peat

e An increase in atmospheric carbon dioxide will enhance the greenhouse effect further, causing further global warming.

9 a Increased amounts of man-made greenhouse gases, particularly carbon dioxide, have created extra heating of the Earth's surface by the greenhouse effect. This has added to the natural greenhouse effect to cause global warming since the industrial revolution of the eighteenth century.

b Carbon dioxide, methane, nitrous oxide

c Further enhancement of the greenhouse effect will cause significantly more global warming.

d The Paris Agreement of 2015 has made governments take action to reduce the amount of man-made greenhouse gases in an attempt to reduce the effect of the enhanced greenhouse effect and so reduce global warming. Some industrial companies continue to propose chemical methods for the removal of greenhouse gases from the atmosphere.

Exam-style questions

Multiple-choice questions

1 B [1]

2 B [1]

3 C [1]

4 D [1]

5 B [1]

6 B [1]

7 D [1]

8 B [1]

9 A [1]

10 B [1]

Short-answer questions

11 a i $\lambda_{max} = \frac{2.9 \times 10^{-3}}{6000} = 483$ nm

≈ 480 nm [2]

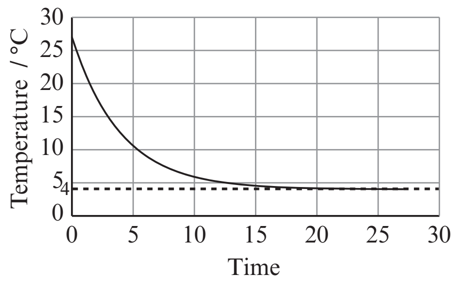
ii Visible region [1]

b $\varepsilon = \frac{40 \times 10^6}{\sigma T^4} = \frac{40 \times 10^6}{5.7 \times 10^{-8} \times 6000^4} = 0.54$ [2]

12 a $P_{net} = \sigma A\left(T_{room}{}^4 - T_{joint}{}^4\right)$
$= 5.7 \times 10^{-8} \times 0.01 \times (277^4 - 300^4)$
$= -1.26\ \text{W}$ **[2]**

b Since the meat is losing energy to the room (which we may assume remains at a constant temperature of 4 °C), its temperature is decreasing. This is decreasing the power being radiated. **[1]**

c



Any curve showing a decreasing gradient **[1]**

Levelling off at 4 °C **[1]**

13 a $\frac{b_{Venus}}{b_{Earth}} = \frac{\left(\frac{L}{4\pi d_{Venus}{}^2}\right)}{\left(\frac{L}{4\pi d_{Earth}{}^2}\right)} = \frac{d_{Earth}{}^2}{d_{Venus}{}^2} = \frac{1}{0.7^2} = 2.0$ **[2]**

b The atmosphere of Venus has a more pronounced greenhouse effect than that of the Earth. **[1]**

c Any from carbon dioxide, sulfur dioxide (and sulfuric acid droplets), carbon monoxide **[2]**

14 a $s = vt = 3 \times 10^8 \times 500 = 1.5 \times 10^{11}\ \text{m}$ **[1]**

b $S = \frac{L}{4\pi d^2} = \frac{3.83 \times 10^{26}}{4\pi \times (1.5 \times 10^{11})^2}$
$= 1354 \approx 1.4\ \text{kWm}^{-2}$ **[2]**

c S is spread over an area of πR^2. But the surface area of the Earth is $4\pi R^2$. So the average solar power m^{-2} is $\frac{\pi R^2 S}{4\pi R^2} = \frac{S}{4}$. **[1]**

d Some of the solar radiation is reflected by the atmosphere (about 30%).

So $0.7 \times \frac{S}{4} = 0.7 \times \frac{1400}{4} = 245\ \text{Wm}^{-2}$ **[1]**

15 a Gases in the Earth's atmosphere (such as carbon dioxide, methane, water vapour and nitrous oxide) absorb some of the radiation emitted by the Earth and re-radiate it back to the surface. **[1]**
This means that the surface temperature is warmer than it would be in the absence of the greenhouse gases. **[1]**

b Any from carbon dioxide, methane, nitrous oxide **[2]**

c An increase in the water vapour content of the atmosphere would increase the Earth's albedo. **[1]**
This means more solar radiation would be reflected, leaving less to be absorbed by the Earth, hence a cooler temperature. **[1]**

16 a $\lambda_{max} = \frac{2.9 \times 10^{-3}}{5700} = 509\ \text{nm} \approx 510\ \text{nm}$ **[2]**

b i Ozone in the stratosphere absorbs solar ultraviolet radiation. **[1]**

ii The energy of visible light photons is too great to be absorbed by molecules in order for them to change their vibrational states and it is too small to be absorbed by ozone. So visible light passes through the atmosphere without significant absorption. **[1]**

iii The energy of infrared photons is the right amount for molecules to increase their vibrational states and so infrared photons are readily absorbed by molecules in the atmosphere. **[1]**

17 a i In the troposphere **[1]**

ii In the stratosphere **[1]**

b Ozone can be dissociated (i.e. broken down into O_2 and O) when it absorbs ultraviolet radiation. **[1]**

Since ultraviolet radiation is potentially harmful to life on the Earth, the ozone removes a large amount of it before it can reach the Earth's surface, thus protecting the Earth and its inhabitants. **[1]**

c CFCs (chlorofluorocarbons) OR HCFCs (hydrochlorofluorocarbons) **[1]**

18 a $L = \sigma A T^4 = 5.7 \times 10^{-8} \times 4\pi \times (7 \times 10^6)^2 \times (2 \times 10^4)^4 = 5.6 \times 10^{24}$ W **[2]**

b $\lambda_{max} = \frac{2.9 \times 10^{-3}}{20000} = 1.45 \times 10^{-7}$ m = 150 nm (2 s.f.) **[1]**

c $b = \frac{L}{4\pi d^2} = \frac{5.6 \times 10^{24}}{4\pi \times (1.5 \times 10^{11})^2} = 19.8 = 20$ Wm^{-2} (2 s.f.) **[2]**

19 a The ratio of reflected radiation to the total incident radiation **[1]**

b Variations in cloud cover; different terrain **[2]**

c A nuclear war would produce large amounts of particulate and gaseous pollutants. These pollutants would enter the atmosphere and increase the albedo of the Earth. **[1]**

An increase in albedo would decrease the amount of solar radiation absorbed by the Earth's surface, upsetting the energy balance and decreasing the surface temperature. **[1]**

20 a $L = 4\pi d^2 S = 4\pi \times (1.5 \times 10^{11})^2 \times 1.36 \times 10^3 = 3.8 \times 10^{26}$ W **[2]**

b $T = \sqrt[4]{\frac{L}{4\pi R^2 \sigma}}$

$= \sqrt[4]{\frac{3.8 \times 10^{26}}{4\pi \times (6.96 \times 10^8)^2 \times 5.7 \times 10^{-8}}}$

$= 5752 = 5700$ K (2 s.f.) **[2]**

c There may be. The peak intensity radiated by the Sun is at a wavelength of $\lambda_{max} = \frac{2.9 \times 10^{-3}}{5700} =$ 509 nm ≈ 510 nm, which is in the middle of the visible region of the electromagnetic spectrum. **[1]** (Calculation or just stated wavelength is sufficient.)

Chapter 9

Exercise 9.1

1 **a** A mole is defined as the amount of a substance that has the same number of particles as there are atoms in 12 grammes of $^{12}_{6}C$.

b *Molar mass* means the mass, in grammes, of 1 mole of a substance.

c **i** $N = \frac{20}{56} \times 6.02 \times 10^{23} = 2.15 \times 10^{23}$ atoms

ii $N = \frac{20}{235} \times 6.02 \times 10^{23} = 5.1 \times 10^{22}$ atoms

iii Number of molecules $= \frac{20}{18} \times 6.02 \times 10^{23} = 6.7 \times 10^{23}$

So, number of atoms $= 6.7 \times 10^{23} \times 3 = 2.0 \times 10^{24}$ atoms (2 s.f.).

2 **a** One atom of ^{12}C has a mass of $\frac{12 \times 10^{-3}}{6.02 \times 10^{23}} = 2 \times 10^{-26}$ kg.

b $u = \frac{2 \times 10^{-26}}{12} = 1.66 \times 10^{-27}$ kg

3 **a** mass = molar mass × number of moles = 65.4 g × 8 = 523.2 g = 523 g (3 s.f.)

b number of atoms = number of moles × $N_A = 8 \times 6.02 \times 10^{23} = 4.82 \times 10^{24}$ atoms

c mass of 1 atom $= \frac{\text{mass of sample}}{\text{number of atoms}} = \frac{523 \times 10^{-3}}{4.82 \times 10^{24}} = 1.09 \times 10^{-25}$ kg

OR

mass of 1 atom $= \frac{\text{molar mass}}{N_A} = \frac{65.4 \times 10^{-3}}{6.02 \times 10^{23}} = 1.09 \times 10^{-25}$ kg

4 **a** An atom of hydrogen comprises 1 proton and 1 electron.

molar mass = mass of 1 atom (≈ mass of proton) × N_A =

$1.673 \times 10^{-27} \times 6.02 \times 10^{23} =$

1.0×10^{-3} kg = 1.0 g

b The mass of the electron is about $\frac{1}{1840}$ of the mass of the proton, so, to 2 s.f., adding the mass of the electron will not make a significant difference.

5 **a** Number of similar worlds $= \frac{N_A}{\text{Earth's population}} = \frac{6.02 \times 10^{23}}{7.9 \times 10^{9}} = 7.6 \times 10^{13}$

b No. Current estimates suggest that our galaxy could have up to 6×10^{9} planets similar to the Earth, capable of sustaining life. So, not enough!

c Yes. Current estimates suggest there may be of the order of 10^{11} galaxies other than our own. If all of these galaxies could contain a similar number of possible planets, then there would be $10^{11} \times 6 \times 10^{9} = 6 \times 10^{20}$ possible other worlds. This would be enough for 7.9 million moles of people!

6 **a** $V = 4\pi R^2 d = 4\pi \times (6.4 \times 10^{6})^2 \times 3000 = 1.2 \times 10^{18}$ m^3

b $M = \rho V = 1000 \times 1.2 \times 10^{18} = 1.2 \times 10^{21}$ kg. This is about $\frac{1}{5000}$ of the Earth's mass.

c Molar mass of water = 18 g = 1.8×10^{-2} kg

So, number of moles $= \frac{1.2 \times 10^{21}}{1.8 \times 10^{-2}} = 6.7 \times 10^{22}$ moles.

The known universe is estimated to contain about 1×10^{21} stars. So the number of moles of water in the Earth's oceans is about 67 times the number of stars in the known universe.

d $N = nN_A = 6.7 \times 10^{22} \times 6 \times 10^{23} = 4 \times 10^{46}$ molecules

7 **a** **i** Mass of $N_2 = 0.77 \times 1.3 = 1.001$ kg = 1.0 kg (2 s.f.)

ii Number of moles of $N_2 = \frac{1.001}{0.028} = 35.75$ moles = 36 moles (2 s.f.)

iii Mass of atom of $N_2 = \frac{1.001}{35.75 \times 2 \times 6.02 \times 10^{23}} = 2.2 \times 10^{-26}$ kg

b **i** Mass of $O_2 = 0.23 \times 1.3 = 0.299$ kg = 0.30 kg (2 s.f.)

ii Number of moles of $O_2 = \frac{0.299}{0.032} =$ 9.34 moles = 9.3 moles (2 s.f.)

iii Mass of atom of O_2 = $\frac{0.299}{9.34 \times 2 \times 6.02 \times 10^{23}} = 2.66 \times 10^{-26}$ kg = 2.7×10^{-26} kg

8 **a** **i** Number of moles of gold = $\frac{19300}{0.197} =$ 9.8×10^4 moles

ii Volume of 1 atom of gold = $\frac{1}{9.8 \times 10^4 \times 6.02 \times 10^{23}} = 1.7 \times 10^{-29}$ m^3

b **i** Number of moles of copper = $\frac{8960}{0.0635} =$ 1.4×10^5 moles

ii Volume of 1 atom of copper = $\frac{1}{1.4 \times 10^5 \times 6.02 \times 10^{23}} = 1.2 \times 10^{-29}$ m^3

c The ratio of their sizes is $\sqrt[3]{\frac{1.7}{1.2}} = 1.12$. So, despite their masses being different by a factor of over 3, their 'sizes' are only about 12% different.

d Since most of the mass of an atom is located in the nucleus and the nucleus takes up a very small part of the space of an atom, a larger mass in a larger nucleus doesn't have to mean a larger atom.

9 **a** Mass of gin = $940 \times 44 \times 10^{-6} = 41$ g

b Number of moles = $\frac{41}{430} = 0.095$ moles (Gin is a very complex molecule!!)

c Number of atoms = $53 \times 0.095 \times 6.02 \times 10^{23} = 3.0 \times 10^{24}$ atoms.

Exercise 9.2

1 **a** An ideal gas

- obeys each of the three empirical gas laws under all conditions: Boyle's law, Charles' law and the pressure law;
- consists of particles whose volume is negligible compared to the volume of space the gas occupies;
- consists of particles that collide elastically;
- has a density sufficiently low that the vast majority of collisions are with the walls of the container, not with each other; and
- has no potential energy—this means that an ideal gas cannot be composed of molecules.

b For a real gas to approximate an ideal gas, it should

- have a low pressure/density,
- not liquefy or solidify and
- have a fairly high temperature.

This usually implies that

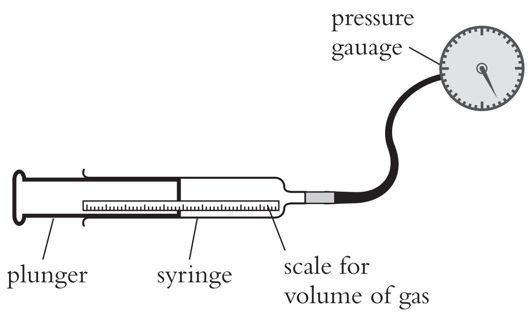
- molecules are small compared with the volume they occupy,
- there are no forces between molecules and
- molecules collide elastically.

c Gases exert pressure because

- large numbers of particles, moving quickly, collide with the walls of their container;
- each collision exerts a force on the container wall; and
- the sum of all the forces from all the collisions divided by the area of the container walls gives the pressure.

2 **a** Boyle's law: $P \propto \frac{1}{V}$ when the temperature is constant.

b *Example*: a syringe of gas



Equipment: Syringe with a volume scale, pressure gauge fitted to end of syringe

Measurements: Volume of air in syringe, using scale on side of syringe, pressure of air inside syringe, using pressure gauge

Method: Change the volume of the air inside the syringe by slowly moving the plunger inwards and outwards. This is the independent variable. Measure the resulting pressure. This is the dependent variable. For each value of volume, record the volume and pressure of the air. Then, plot a graph of *pressure* against *volume*. This should show that $P \propto \frac{1}{V}$.

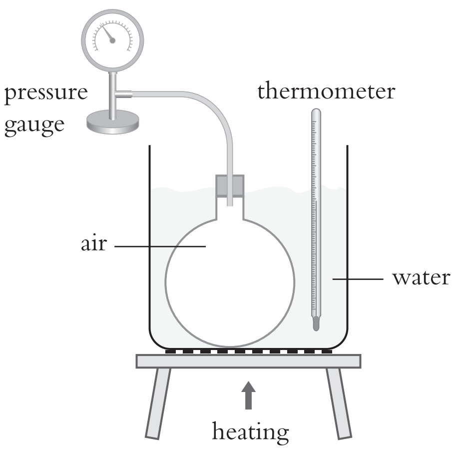
c $P_1 \times V_1 = P_2 \times V_2$

Therefore, $P_2 = \frac{P_1 V_1}{V_2} = \frac{100 \times 20}{4} = 500$ kPa.

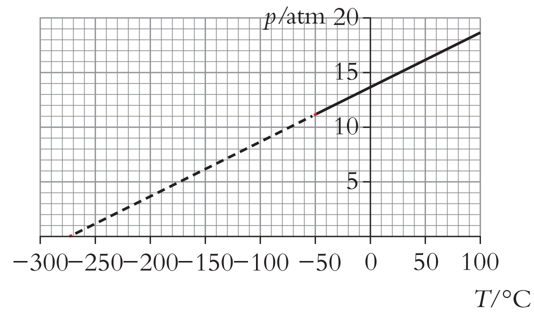
3 Assuming that the temperature remains constant, as the air bubble rises through the water, the pressure around the bubble decreases. Boyle's law states that $P_1 \times V_1 = P_2 \times V_2$ so, if the pressure reduces, the volume of the air bubble must increase.

4 **a** Pressure law: if the volume of a sample of an ideal gas is constant, then the pressure of the gas is proportional to its absolute temperature: $P \propto T$.

b



A fixed volume container of gas is attached to a pressure gauge. The temperature of the gas can be varied by heating the beaker of water in which it is immersed. The temperature of the water (and hence the temperature of the gas) is measured with a thermometer. The pressure of the gas is measured with a pressure gauge. Values of temperature and pressure are recorded in a table. A graph of *pressure* (on the *y*-axis) against *temperature* (on the *x*-axis) should produce a straight line. Extrapolating the graph backwards to where the pressure is zero leads to a value of absolute zero.



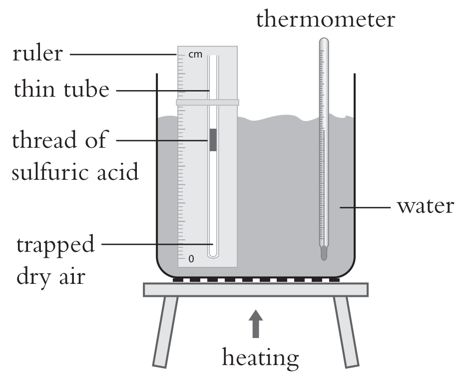
Note: Immersing the container of gas in a pot of liquid nitrogen provides a useful extra measurement for the graph at a temperature of −196 °C. This extra point 'anchors' the straight-line graph and makes the value extrapolated for absolute zero more accurate.

5 At 0 K, atoms do not have any kinetic energy; they do not move around. This means they cannot exert forces on the walls that give rise to pressure.

6 $\frac{P_1}{T_1} = \frac{P_2}{T_2}$ Therefore, $P_2 = \frac{P_1 \times T_2}{T_1} = \frac{1.01 \times 10^5 \times (273 - 60)}{(273 + 10)} = 8 \times 10^4$ Pa (1 s.f.).

7 **a** Charles's law: if the pressure of a sample of ideal gas is constant then the volume of the gas is proportional to its absolute temperature; $V \propto T$.

b



A small amount of liquid in a thin, uniform capillary tube traps a volume of air beneath it. If the top end of the capillary tube is open, atmospheric pressure keeps the pressure of the trapped volume of air constant.

The capillary tube is attached to a ruler so that the length of the trapped volume of air can be measured; in a uniform tube, this volume is proportional to the length. The capillary tube and ruler are then immersed in a beaker of water, which can be heated. The temperature of the water (and hence the air in the capillary tube) is measured with a thermometer. Values of the temperature of the trapped air (the independent variable) and volume of the trapped air (the dependent variable) are recorded in a table. A graph of *volume* against *temperature in °C* should produce a straight line. If another graph of *volume* against *temperature in Kelvin* is drawn, a straight line passing through the origin occurs, showing that $V \propto T$.

8 Extrapolate the graph of volume against temperature in °C backwards (i.e. to negative values of T in °C). Where the straight line meets the Temperature axis, at a value of 0 for the Volume, will be a value for absolute zero. Note: if liquid nitrogen is available, measuring the volume of the trapped air at a temperature of −196 °C will 'anchor' the graph and produce a more accurate value for absolute zero.

9 a Since $V \propto T$, the absolute temperature quadruples.

b Absolute temperature is a measure of the average kinetic energy $\left(E_K = \frac{1}{2} m c^2\right)$ of the molecules. If the temperature is quadrupled, then the average speed of the molecules must have doubled.

10 For constant pressure, $\frac{V_1}{T_1} = \frac{V_2}{T_2}$.

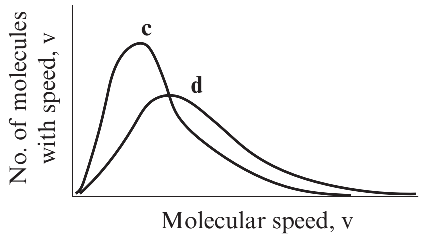
So, $T_2 = \frac{V_2 \times T_1}{V_1} = \frac{300 \times (273 + 20)}{150} = 586 \text{ K} =$ $(586 - 273)\ °\text{C} = 313\ °\text{C}$ (300 °C to 1 s.f.).

Exercise 9.3

1 a No.

b No.

c and d



e Maxwell–Boltzmann distributions

2 a In a gas consisting of many molecules, the molecules will have a range of velocities. Simply finding the average of these will almost certainly produce an answer of zero—because velocity is a vector quantity. So, by squaring all the velocities, then finding their mean and then square-rooting the mean, we will get a value for the 'average' speed, which we call the root-mean-square speed.

b For 1 mole of gas, $\frac{1}{2}N_A m\overline{v^2} = \frac{3}{2}N_A kT$

$= \frac{3}{2}RT$,

but $N_A m = M$ the molar mass.

So, $\overline{v^2} = \frac{3RT}{M} \Rightarrow v_{rms} = \sqrt{\frac{3RT}{M}}$.

c $v_{rms} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3 \times 8.31 \times (273 + 25)}{32 \times 10^{-3}}}$

$= 482 \text{ ms}^{-1}$

3 **a** 5

b $\frac{5+4+8+7+5+6+6+5+3+4+5+9+8+6+7+6+5+9+10+5+6+7+7+8+5}{25}$
$= 6.24$

c $\sqrt{\frac{1}{25} \times (9 + 32 + 175 + 180 + 196 + 192 + 162 + 100)}$
$= 6.46$

d rms > mean > mode

e **i** y

ii z

iii x

4 **a** $p = -mv$

b $p = mv$

c $\Delta p = 2mv$

d $\Delta t = \frac{2l}{v}$

e $F = \frac{\Delta p}{\Delta t} = \frac{2mv}{\left(\frac{2l}{v}\right)} = \frac{mv^2}{l}$

f $\frac{N}{3}$

g $F = \frac{N}{3}\left(\sum_{i=1}^{\frac{N}{3}}\right)\frac{mv_i^2}{l} = \frac{1}{3}\left(\frac{Nmc^2}{l}\right)$

h $A = l^2$

i $P = \frac{F}{A} = \frac{\frac{1}{3}\left(\frac{Nmc^2}{l}\right)}{l^2} = \frac{1}{3}\left(\frac{Nmc^2}{l^3}\right)$

j $V = l^3$

k $P = \frac{1}{3}\left(\frac{Nmc^2}{l^3}\right) = \frac{1}{3}\left(\frac{Nmc^2}{V}\right) \Rightarrow PV = \frac{1}{3}Nmc^2$

5 **a** $M = Nm$

b $\rho = \frac{M}{V} = \frac{Nm}{V} \Rightarrow P = \frac{1}{3}\rho c^2$

c $c = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 1.01 \times 10^5}{1.3}} = 480\ \text{ms}^{-1}$

6 **a** For an ideal gas, the internal energy consists only of kinetic energy. If each atom/molecule has an average E_K of $\frac{1}{2}mc^2 = \frac{3}{2}kT$, then for N atoms/molecules, the total internal energy must be $U = \frac{3}{2}NkT$.

b The implication is that a given number of atoms/molecules of any ideal gas, whatever kind of atoms or molecules they consist of, will have the same internal energy at any given absolute temperature.

c An atom of hydrogen has less mass than an atom of oxygen. If they are to have the same kinetic energy, then the hydrogen atom will have to be travelling faster than the oxygen atom so that $\frac{1}{2}m_{\text{hydrogen}}v^2_{\text{hydrogen}} = \frac{1}{2}m_{\text{oxygen}}v^2_{\text{oxygen}}$.

7 $U = \frac{3}{2}NkT = \frac{3}{2} \times 6.02 \times 10^{23} \times 1.38 \times 10^{-23} \times (273 + 100) = 4.6\ \text{kJ}$.

8 **a** $PV = \frac{1}{3}Nmc^2 = \frac{2}{3}N\left(\frac{mc^2}{2}\right) = \frac{2}{3}N\left(\frac{3}{2}kT\right) = NkT$

b Since $n = \frac{N}{N_A}$, we can write $PV = nN_AkT$.

c $PV = nN_AkT = nRT$

$R = N_Ak = 6.02 \times 10^{23} \times 1.38 \times 10^{-23} = 8.31\ \text{JK}^{-1}\ \text{mole}^{-1}$

d $T = \frac{PV}{nR} = \frac{1.013 \times 10^5 \times 2.24 \times 10^{-2}}{1 \times 8.31} = 273\ \text{K}$
or 0 °C

9 **a** $U = \frac{3}{2}NkT$ and $PV = NkT \therefore U = \frac{3}{2}PV$

b $\frac{3}{2}NkT = \frac{3}{2} \times 6.02 \times 10^{23} \times 1.38 \times 10^{-23} \times 273 = 3.4 \times 10^3\ \text{J}$

$\frac{3}{2}PV = \frac{3}{2} \times 1.013 \times 10^5 \times 22.4 \times 10^{-3} = 3.4 \times 10^3\ \text{J}$

c **i** a little

ii a lot

iii not at all

iv a lot

v a lot

Exam-style questions

Multiple-choice questions

1 C [1]

2 A [1]

3 B [1]

4 C [1]

5 C [1]

6 C [1]

7 B [1]

8 B [1]

9 B [1]

Short-answer questions

10 a Any [2] from the following:

- Obeys each of the three empirical gas laws under all conditions: Boyle's law, Charles' law and the pressure law
- Consists of particles whose volume is negligible compared to the volume of space the gas occupies
- Consists of particles that collide elastically
- Has a density sufficiently low that the vast majority of collisions are with the walls of the container, not with each other
- Has no potential energy

b $\frac{V_i}{T_i} = \frac{V_f}{T_f} \Rightarrow V_f = T_f \frac{V_i}{T_i} = 303 \times \frac{650}{293} =$ 672 = 670 cm^3 (2 s.f.) [2]

c The gas has increased its volume. This means that some of the energy it had gained has been used as work to expand the container. Q – work done on container = ΔU [1]

11 a $n = \frac{PV}{RT} = \frac{6.0 \times 10^6 \times 3 \times 10^{-3}}{8.31 \times 300} =$ 7.2 moles [1]

b $N = nN_A = 7.2 \times 6.02 \times 10^{23} =$ 4.3×10^{24} gas atoms [1]

c $V = \frac{\text{volume of container}}{N} = \frac{3 \times 10^{-3}}{4.3 \times 10^{23}} =$ 7×10^{-27} m^3 [1]

d *Volume of a gas atom* ~ 10^{-30} m^3. This is, to the nearest order of magnitude, 10^{-4} of the volume it occupies. So the assumption is valid. [2]

12 a $n = \frac{PV}{RT} = \frac{1.5 \times 10^6 \times 0.1}{8.31 \times (273 + 27)} = 60$ moles [2]

b $M = n \times$ molar mass $= 60 \times 0.029 =$ 1.74 = 1.7 kg (2 s.f.) [1]

c 20% of 60 moles = 12 moles. Therefore, there are $12 \times 6.02 \times 10^{23} \times 2$ atoms = 1.4×10^{25} oxygen atoms present. [2]

13 a $P = \frac{1}{3}\rho c^2 \Rightarrow c = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 100 \times 10^3}{1.2}} =$ 500 ms^{-1}

b $c \propto \sqrt{T}$ So if c is halved, then T must change by a factor of $\frac{1}{\sqrt{2}}$.

So, new $T = \frac{1}{\sqrt{2}} \times 313 = 221 =$ 220 K (2 s.f.). [2]

14 a $m = \frac{1}{2} \times \frac{\text{molar mass of bromine molecules}}{N_A}$ $= \frac{1}{2} \times \frac{0.16}{6.02 \times 10^{23}} = 1.33 \times 10^{-25}$ kg. [1]

b $KE = \frac{3}{2}RT = \frac{3}{2} \times 8.31 \times 400 =$ 5 kJ (1 s.f.) [2]

c $c = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 400}{1.33 \times 2 \times 10^{-25}}} =$ 250 ms^{-1} (2 s.f.) [2]

15 a $T = \frac{PV}{nR} = \frac{1 \times 10^4 \times 0.2}{0.5 \times 8.31} = 480$ K (2 s.f.) [2]

b The gas is at a low density and a moderately high temperature. [1]

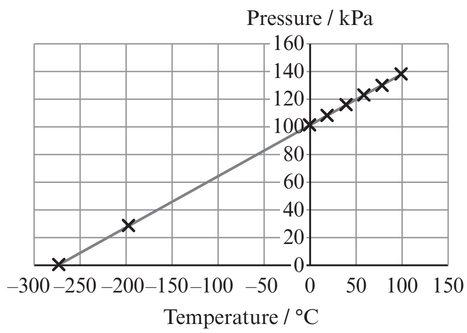
c $P \propto T$ for a fixed volume, so new $T = 480 \times 2 = 960$ K [1]

16 a $N = nN_A = 2 \times 6.02 \times 10^{23} =$ 1.2×10^{24} atoms (2 s.f.) [1]

b $k = \frac{PV}{NT} = \frac{2 \times 10^5 \times 2.5 \times 10^{-2}}{1.2 \times 10^{24} \times 302} =$ 1.38×10^{-23} JK^{-1} [2]

c $R = kN_A = 1.38 \times 10^{-23} \times 6.02 \times 10^{23} =$ 8.31 JK^{-1} mole^{-1} [1]

17 a



(**[1]** for correctly plotted points; **[1]** for straight line)

b Using liquid nitrogen is the only way to achieve this temperature. Commercially available liquid nitrogen will boil as soon as it is released from its flask; boiling temperature of nitrogen is −196 °C. **[1]**

c The student would have predicted the value of absolute zero to be between −270 °C and −275 °C. See extrapolation on graph in **part a**. **[1]**

18 a Using Boyle's law: $P_1V_1 = P_2V_2 \Rightarrow P_2 = \frac{P_1V_1}{V_2} = \frac{1.0 \times 10^5}{\frac{1}{2}} = 2.0 \times 10^5$ Pa **[2]**

b The gas is compressed slowly, so there is time for the increase in internal energy of the gas—because of the work done on the gas—to be shared with the surroundings. So, the temperature of the gas does not increase (a necessary condition for Boyle's law). **[1]**

c The pressure would be greater **[1]**. If the compression occurs quickly, the work done on the gas will increase the temperature of the gas. The atoms of gas will move faster and collide with the syringe walls more frequently and with greater force. This makes the pressure inside the syringe greater than that calculated in **part a**. **[1]**

Chapter 10

Exercise 10.1

1 a i Internal energy is the sum of all the potential and the kinetic energies of all the particles in the gas.

ii $U = \frac{3}{2}nRT = \frac{3}{2} \times 3.0 \times 8.31 \times 300 =$ 11.2 kJ

iii Since the gas is ideal, there is no potential energy. So, in this case, $U \propto T$. If T is halved, then U is halved.

b i An open system is one that allows mass to enter or leave.

ii A closed system is a system that prevents mass from entering or leaving.

iii An isolated system is a system that prevents any form of energy or mass from entering or leaving.

2 a i N is the number of particles in the system; k is Boltzmann's constant; T is the absolute temperature of the system.

ii $U = \frac{3}{2}NkT$

$N = nN_A$ and $k = \frac{R}{N_A}$

Substituting gives

$$U = \frac{3}{2}nN_A\frac{R}{N_A}T$$
$$= \frac{3}{2}nRT.$$

iii Since $PV = nRT$, $U = \frac{3}{2}PV$.

b From the equations for U, U can be changed by changing the

- number of particles, N, in the system;
- temperature, T, of the system;
- pressure, P, of the system; and
- volume, V, of the system.

3 a Thermal energy is being given **to** the gas.

b Thermal energy is being removed **from the** gas.

c The internal energy of the gas has **increased**.

d The internal energy of the gas has **decreased**.

e Work has been done **by** the gas **on** its surroundings—that is the gas has expanded.

f Work has been done **on** the gas by external forces—that is the gas has been compressed.

4 a $U = \frac{3}{2}NkT$, but $N = nN_A$ (where N_A is Avogadro's constant) and $kN_A = R$.

So, $U = \frac{3}{2}NkT = \frac{3}{2}nN_AkT = \frac{3}{2}nRT = \frac{3}{2}PV$.

If P is constant, $\Delta U = \frac{3}{2}P\Delta V$.

b $\Delta U = \frac{3}{2}P\Delta V = \frac{3}{2} \times 2.0 \times 10^6 \times (0.15 - 0.25) = -3.0 \times 10^5$ J

c Work done $= P\Delta V = 2.0 \times 10^6 \times (0.15 - 0.25) = -2.0 \times 10^5$ J

5 a Isobaric process

b i $T_X = \frac{PV}{nR} = \frac{3 \times 10^6 \times 2 \times 10^{-2}}{4.0 \times 8.31} = 1800$ K (2 s.f.)

ii $T_Y = \frac{PV}{nR} = \frac{3 \times 10^6 \times 5 \times 10^{-2}}{4.0 \times 8.31} = 4500$ K (2 s.f.)

c i $\Delta U = \frac{3}{2}P\Delta V = \frac{3}{2} \times 3.0 \times 10^6 \times (5 - 2) \times 10^{-2} = 1.35 \times 10^5$ J

ii Increased (This should be obvious because the temperature has increased.)

d $W = P\Delta V = 3 \times 10^6 \times (5 - 2) \times 10^{-2} = 9 \times 10^4$ J

e As the gas expands at a constant pressure, it absorbs heat from the surroundings.

6 a i A is an isobaric process: the pressure is remaining constant.

ii B is an isovolumetric process: the volume is remaining constant.

b i Work done = area under graph = $2 \times 10^5 \times 15 \times 10^{-3} = 3.0$ kJ ii At Z, the gas must be at a lower temperature. Points X and Z must lie on different isotherms, and because Z will be on an isotherm that is lower than X, it must be at a lower temperature.

iii ΔU must be negative because its temperature has fallen. But, work has been done on the gas, so W is also negative. Therefore, $Q = \Delta U + W$ must be negative. This means that heat has been lost to the surroundings from the gas.

c i No. Work done = $P\Delta V$, and here, $\Delta V = 0$. So, process B does not involve any work being done on or by the gas.

ii At point Z, the gas had lost internal energy compared to point X. At Y, it has the same internal energy as at point X. So, during process B, the gas must have gained heat from the surroundings.

7 a i Work done = area under graph = $5 \times 10^5 \times 15 \times 10^{-3} = 7.5$ kJ

ii Z is at a higher temperature than Y.

iii $\Delta U > 0$ and work done is on the surroundings by the gas; therefore, $W > 0$, so $Q > 0$. This means that heat must be absorbed by the gas from the surroundings.

b i No. There is no change in volume, so there is no work done on or by the gas.

ii Since X must be at a lower temperature than Z and no work is done, $\Delta U < 0$, so $Q < 0$. This means that heat is being lost by the gas to its surroundings.

8 a i For an isothermal process, PV = constant. From the graph,

$8 \times 1 = 4 \times 2 = 2.66 \times 3 = 2 \times 4$ So, the process is isothermal.

ii The gas has been compressed, so work has been done on the gas by the surroundings.

iii Conservation of energy implies that heat must have been transferred from the gas to the surroundings.

b i An adiabatic process is one in which no exchange of heat occurs. That is $Q = 0$.

ii Quickly. If the process occurs quickly there isn't sufficient time for heat to be exchanged between the surroundings and the gas.

iii The gas has been compressed, so work has been done on the gas by the surroundings.

iv The area under the curve (i.e. $\int PdV$) is the work done (on the gas by the surroundings).

v Conservation of energy implies that the work done on the gas is equal in magnitude to the increase in internal energy of the gas.

Since $Q = 0$ and the gas is being compressed ($W < 0$), the internal energy, U, of the gas must be increasing ($\Delta U > 0$): the gas is heating up. Work done on the gas is being transferred into the internal energy of the gas.

9 a Work done = area under graph from W to X = $5 \times 10^5 \times 15 \times 10^{-3} = 7.5$ kJ

b No.

c Work done = area under the graph from Y to Z = $2 \times 10^5 \times -15 \times 10^{-3} = -3.0$ kJ

d Net work done = 7.5 − 3.0 = 4.5 kJ

e The area of the rectangle enclosed by the four thermodynamic processes.

f There must be an input of energy from an external source. Such a source might be, for example, the fuel in a car engine.

10 a

Process	Q	U	W
A	added	increases	by the gas
B	removed	decreases	none
C	removed	decreases	on the gas
D	added	increases	none

b The area enclosed by the thermodynamic cycle represents the net work done by the gas.

Exercise 10.2

1 a i Adding Q will increase the average kinetic energy of the particles of gas.

ii The internal energy of the gas will increase.

iii No. The gas can only do work on the surroundings if the volume of the container increases. Since the volume remains constant, no mechanical work is done.

b i The particles of gas hit the container walls harder (and more often) so, if the pressure remains constant, the area of the container walls must increase. So, the volume of the container increases.

ii Pressure has units of Pa (or Nm^{-2}), volume has units of m^3, so the product of P and V must have units of $\text{N m}^{-2}\,\text{m}^3 = \text{N m} = \text{J}$ (Joules).

iii $W = P\Delta V$

c i $Q = \Delta U + W$

ii The first law of thermodynamics

2 a i There is no change in internal energy. U depends on T. Since T is constant, U must be constant too.

ii $Q = \Delta U + W = 0 + 5.0\text{ kJ} = 5.0\text{ kJ}$

iii Q is gained by the gas.

b i $Q = 0$

ii $\Delta U = Q - W = 0 - (-2.0\text{ kJ}) = 2.0\text{ kJ}$

iii The temperature of the gas increases.

3 a $W = P\Delta V = 1.01 \times 10^5 \times (18 - 12) = 6.06 \times 10^5\text{ J}$

b $\Delta U = Q - W = Q - P\Delta V = 4.5\text{ MJ} - 6.06 \times 10^5 = 3.9\text{ MJ}$ (2 s.f.)

4 a Initially, $U = \frac{3}{2}PV = \frac{3}{2} \times 4.0 \times 10^5 \times 6.0 \times 10^{-2} = 3.6 \times 10^4\text{ J}$.

After change, $U = \frac{3}{2}PV = \frac{3}{2} \times 3.0 \times 10^5 \times 8.0 \times 10^{-2} = 3.6 \times 10^4\text{ J}$

$\therefore \Delta U = 0$.

b i The gas has expanded, so $W > 0$.

ii $Q > 0$

c When $\Delta U = 0$, $Q = W$.

5 a Because the process is isothermal, $\Delta U = 0$.

b $Q = 0 + W$, so $Q = W$. Therefore, the gas gains heat of 2.5 kJ from its surroundings.

6 a $Q = \Delta U + W$. For an isothermal process, $\Delta U = 0$. So, $Q = W = -4.5\text{ kJ}$ (the gas loses heat to its surroundings).

b For an adiabatic process, $Q = 0$. So, $0 = \Delta U + W$. Since work is being done on the gas, the value of W is negative, so the value of ΔU must be positive.
This means that the gas will be gaining internal energy; its temperature must increase.

7 a i $T = \frac{PV}{nR} = \frac{5 \times 10^5 \times 1}{2.5 \times 8.31} = 2.4 \times 10^4\text{ K}$

ii $U = \frac{3}{2}nRT = \frac{3}{2} \times 2.5 \times 8.31 \times 2.4 \times 10^4 = 7.5 \times 10^5\text{ J}$

OR

$U = \frac{3}{2}PV = \frac{3}{2} \times 5 \times 10^5 \times 1 = 7.5 \times 10^5\text{ J}$

b i $T = \frac{PV}{nR} = \frac{5 \times 10^5 \times 0.25}{2.5 \times 8.31} = 6.0 \times 10^3\text{ K}$

ii $U = \frac{3}{2}nRT = \frac{3}{2} \times 2.5 \times 8.31 \times 6.0 \times 10^3 = 1.9 \times 10^5\text{ J}$ (2 s.f.)

OR

$U = \frac{3}{2}PV = \frac{3}{2} \times 5 \times 10^5 \times 0.25 = 1.9 \times 10^5\text{ J}$ (2 s.f.)

OR

$\Delta U \propto \Delta V$, so if V is quartered, U is quartered $\Rightarrow U_{\text{new}} = \frac{U_{\text{old}}}{4} = \frac{7.5 \times 10^5}{4} = 1.9 \times 10^5\text{ J}$ (2 s.f.).

iii $W = P\Delta V = 5 \times 10^5 \times (0.25 - 1.0) = -3.75 \times 10^5\text{ J}$ (The minus sign is showing that the work is done **on** the gas.)

iv $Q = \Delta U + W = (1.9 - 7.5) \times 10^5 + (-3.75 \times 10^5) = -9.4 \times 10^5\text{ J}$

c **i** Q is lost from the gas (indicated by the minus sign in the previous answer.)

ii The answers suggest that whenever a gas is compressed with no change in pressure, its temperature falls (i.e T becomes smaller) and heat is lost from the gas (i.e. $Q < 0$).

iii Since this is the opposite to compressing the gas, T should increase (and hence U will increase), and Q will become greater than 0 (i.e. heat is gained from the surroundings allowing the gas to increase its temperature as well as to do work on the surroundings).

8 **a** **i** $T = \frac{PV}{nR} = \frac{1.0 \times 10^5 \times 0.15}{4.0 \times 8.31} = 451.3 =$ 450 K (2 s.f.)

ii $U = \frac{3}{2} nRT = \frac{3}{2} \times 4.0 \times 8.31 \times 451.3 =$ 2.25×10^4 J

OR

$U = \frac{3}{2} PV = \frac{3}{2} \times 1.0 \times 10^5 \times 0.15 =$ 2.25×10^4 J

b **i** $T = \frac{PV}{nR} = \frac{5.0 \times 10^5 \times 0.15}{4.0 \times 8.31} = 2256.5 =$ 2.3×10^3 K (2 s.f.)

OR

Since T ∝ P at constant volume, if P is changed by a factor of 5, so is T.

$\therefore T_{new} = 5 \times T_{old} = 5 \times 451.3 =$ $2256.5 = 2.3 \times 10^3$ K (2 s.f.)

ii $U = \frac{3}{2} nRT = \frac{3}{2} \times 4.0 \times 8.31 \times 2256.5 =$ 1.125×10^5 J $= 1.1 \times 10^5$ (2 s.f.)

OR

$U = \frac{3}{2} PV = \frac{3}{2} \times 5 \times 10^5 \times 0.15 =$ 1.1×10^5 J (2 s.f.)

OR

$\Delta U \propto \Delta P$, so if P changes by a factor of 5, U is also changed by a factor of 5

$\Rightarrow U_{new} = 2.25 \times 10^4 \times 5 = 1.1 \times 10^5$ J (2 s.f.).

iii Since there is no change in the volume of the gas, $W = 0$.

iv $Q = \Delta U + W = (1.125 - 0.225) \times 10^5 + 0 = 9.0 \times 10^4$ J

c **i** Q is gained, since the value of $Q > 0$.

ii The answers suggest that whenever a gas undergoes an increase in pressure with no change to its volume, its temperature increases and it gains heat from the surroundings ($Q > 0$).

iii Since this is opposite to **part b vi**, the gas will decrease in temperature—and hence also decrease its internal energy—and will lose heat to the surroundings.

9 **a** **i** Since the gas expands, $W > 0$.

ii Since $Q = 0$, $\Delta U < 0$.

iii The temperature of the gas decreases.

b **i** Since the gas is compressed, $W < 0$.

ii Since $Q = 0$, $\Delta U > 0$.

iii The temperature of the gas increases.

10 **a** $Q = mL = 1.0 \times 2.26 \times 10^6 = 2.26 \times 10^6 =$ 2.26 MJ

b $W = P\Delta V = 1.01 \times 10^5 \times (1.67 - 1 \times 10^{-3}) =$ 1.69×10^5 J

c $\Delta U = Q - W = 2.26 \times 10^6 - 1.69 \times 10^5 =$ $2.09 \times 10^6 = 2.09$ MJ

Exercise 10.3

1 **a** **i** Each counter has a chance of $\frac{1}{6}$ to be exchanged, so $N = \frac{1}{6} \times 60 = 10$.

ii $N = \frac{1}{6} \times 6 = 1$

iii Amaya now has 51, and Andrew now has 15.

iv Amaya's body has cooled a little, and Andrew's body has warmed a little.

b **i** $N = \frac{1}{6} \times 51 = 8$

ii $N = \frac{1}{6} \times 15 = 2$

iii Amaya now has 45, and Andrew now has 21.

iv Amaya's body has cooled a little more, and Andrew's body has warmed a little more.

c **i** One would expect Amaya and Andrew to have the same number of counters.

ii Amaya's temperature has decreased and Andrew's temperature has increased until their two temperatures are the same.

d **i** Yes, statistically each player would exchange $N = \frac{1}{6} \times 33 = 5$ counters on each turn.

ii Their two bodies are in thermal equilibrium because they are exchanging energy at the same rate.

e **i** The probability of energy being transferred from the hotter body to the colder body is much greater than the probability of energy being transferred from the colder body to the hotter body.

ii The probability of this happening is too small for it to occur. Since the game progresses with 'turns' a 'one-off' highly unlikely event (e.g. the colder body produces one or more sixes and the hotter produces no sixes) will soon be overcome by the next turn or turns, resulting in the net transfer of energy from the hotter body to the colder body—and not the other way around. This is why hotter bodies cool and colder bodies warm up, because the chance of that happening is far greater than the reverse.

2 **a** 2

b **i** 6

ii S has increased by a factor of 2.6.

c **i** 24

ii S has increased by a factor of 1.8 (2 s.f.).

c **i** 120

ii S has increased by a factor of 1.5 (2 s.f.).

d **i** Ω increases at an increasing rate.

ii S increases at a decreasing rate.

e **i** S has decreased by a relatively small amount

ii S has increased by a relatively large amount

iii The net change to S is positive. That is S has increased.

iv S always increases.

3 **a** Entropy decreases. Reason: water molecules in frozen ice are more regularly arranged and so have fewer microstates.

OR

Water loses heat as it freezes. Since the change in entropy is porportional to heat input, entropy must decrease.

b Entropy increases. Reason: increased number of microstates; there are more ways in which the gas molecules can be arranged within the volume they occupy.

OR

An isothermal expansion must be accompanied by a net input of heat. Since the change in entropy is proportional to heat input, entropy must increase.

c Entropy increases. Reason: the pieces of the glass are now less ordered; they have a greater number of microstates.

OR

The E_p the glass had on the table has been converted into heat (and energy to break the glass.) Since the change in entropy is porportional to heat input, entropy must increase.

4 **a** 50 °C

b **i** $Q = mc\Delta T = 2.0 \times 4200 \times (80 - 50) = 2.52 \times 10^5$ J

ii 2.52×10^5 J

c **i** $T_{ave-cold} = 35$ °C

ii $T_{ave-hot} = 65$ °C

d **i** $\Delta S_{hot} = \frac{-Q}{T_{ave-hot}} = \frac{-2.52 \times 10^5}{(273 + 65)} = -746\ \text{JK}^{-1}$

ii $\Delta S_{cold} = \frac{Q}{T_{ave-hot}} = \frac{2.52 \times 10^5}{(273 + 35)} = 818\ \text{JK}^{-1}$

iii $\Delta S_{total} = \Delta S_{hot} + \Delta S_{cold} = -746 + 818 = 72\ \text{JK}^{-1}$, which is >0.

5 **a** gravitational potential → kinetic → thermal

b $\Delta S = \frac{Q}{T} = \frac{40 \times 10 \times 3}{20 + 273} = 4.1\ \text{JK}^{-1}$

6 **a** **i** The cooling tea produces a negative entropy change. The warming surroundings produced a positive entropy change. The temperature of the tea is warmer than that of the surroundings, so the magnitude of the entropy change for the tea is smaller than that of the surroundings. So, the net entropy change is positive.

ii The atoms of sodium and chlorine are losing energy (atomic bonds are being created) so the entropy change for the crystal is negative. The energy lost by these atoms is gained by the surroundings, making the entropy change of the surroundings positive. This entropy change of the surroundings is greater than that of the crystal (because the number of microstates in the surroundings is much greater than the number of microstates in the crystal), making the net entropy change positive.

iii During sweating, the most energetic atoms of sweat break free of the atoms around them (evaporating). The average energy of the atoms left behind is lower—hence the cooling. This will create a negative entropy change. However, the addition of those high energy atoms into the surroundings creates a greater positive entropy change. This allows the net entropy change to be positive, following the second law of thermodynamics.

b $\Delta S_{\text{net}} = \Delta S_{\text{hot}} + \Delta S_{\text{cold}} = \frac{-Q}{T_{\text{hot}}} + \frac{Q}{T_{\text{cold}}} = \frac{-2kJ}{700} + \frac{2kJ}{300} = 3.8\ \text{JK}^{-1}$

7 **a** **i** $T_A = \frac{PV}{nR} = \frac{4 \times 10^5 \times 2 \times 10^{-2}}{4 \times 8.31} =$ 241 K (3 s.f.)

$T_B = \frac{PV}{nR} = \frac{2 \times 10^5 \times 4 \times 10^{-2}}{4 \times 8.31} = 241$ K (3 s.f.)

So, states A and B are at the same temperature.

ii The internal energy of the gas is the same in states A and B.

b **i** Work done = area under the curve from A to B

$= 1.33 \times (2 \times 10^{-2} \times 2 \times 10^5) = 5.3$ kJ

ii 5.3 kJ

iii $\Delta S = \frac{\Delta Q}{T} = \frac{5.3\ kJ}{241} = 22\ \text{JK}^{-1}$

c The loss of entropy from the surroundings must be less than the gain in entropy of the gas. This is possible if the temperature of the surroundings is greater than the temperature of the gas.

8 **a** $T = \frac{PV}{nR} = \frac{6 \times 10^5 \times 4 \times 10^{-2}}{1 \times 8.31} = 2.9 \times 10^3$ K

b Since $T \propto PV$, for Z, $6 \times 10^5 \times 2 \times 10^{-2} = 12 \times 10^3$ and, for Y, $3 \times 10^5 \times 4 \times 10^{-2} = 12 \times 10^3$

So, Y and Z are at the same temperature.

c **i** Its temperature decreases.

ii Its internal energy decreases.

iii Its entropy decreases. This suggests that the entropy of the surroundings must increase implying that thermal energy is transferred from the gas to the surroundings. This is consistent with an isovolumetric cooling of the sample of gas.

d **i** Its temperature decreases.

ii Its internal energy decreases.

iii Its entropy decreases. The gas has been compressed isobarically, so the surroundings have done work on the gas. But, since the gas has undergone a decrease in its internal energy (its temperature has decreased), it must have transferred some thermal energy to the surroundings so that the first law is obeyed. For the second law to be obeyed (and it must be!), the gain in entropy of the surroundings must be greater than the loss of entropy of the gas. The transfer of thermal energy from the gas to the surroundings must be greater than it would be for an isovolumetric cooling of the gas. This also means a greater change in entropy of the surroundings than the loss of entropy of the gas.

9 **a** $Q = mc\Delta T = 0.25 \times 4200 \times (30 - 85) = -5.78 \times 10^4$ J

b $T_{ave} = \frac{30 + 85}{2} = 57.5$ °C

c $\Delta S_{coffee} = \frac{-5.78 \times 10^4}{57.5 + 273} = -175\ \text{JK}^{-1}$

d $\Delta S_{classroom} = \frac{+5.78 \times 10^4}{20 + 273} = 197\ \text{JK}^{-1}$

e $\Delta S_{total} = \Delta S_{coffee} + \Delta S_{classroom} = -175 + 197 = 22\ \text{JK}^{-1}$

10 **a** The ice cube melts quickly because the metal plate is a good conductor of thermal energy, allowing heat to be exchanged with the metal plate at a high rate.

b The loss of energy from the surroundings causes a negative entropy change. But the gain of energy by the ice cube causes a larger positive entropy change (because the value of Q is the same for both processes, but T is lower for the ice cube). So, the net entropy change is greater than zero.

c $\Delta S = \frac{Q}{T} = \frac{10 \times 10^{-3} \times 334 \times 10^3}{273} = 12.2\ \text{JK}^{-1}$

Exercise 10.4

The questions in this section will help you apply what you have learnt so far about thermodynamic processes, the first law and the second law to systems we use every day.

1 **a** A heat engine is a device that transfers thermal energy into useful mechanical work.

b Thermal energy, Q_H, is removed from a hot reservoir (assumed to remain at a constant temperature, T_H). Some of this thermal energy is transferred into useful mechanical work, W, and the remainder of the thermal energy, Q_C, (i.e. $Q_H - W$) is transferred to the heat sink (also assumed to remain at a constant temperature).

c Efficiency, η, is defined as $\frac{\text{useful work done}}{\text{total energy supplied}}$.

So, $\eta = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H} = 1 - \frac{Q_C}{Q_H}$.

2 **a** The car engine takes chemical energy, in the form of fuel, and, by combustion, tranforms it into thermal energy. This thermal energy is then transformed into useful mechanical work—kinetic energy.

b Energy from the fuel is 'wasted' in the forms of

- hot exhaust gases,
- mechanical vibration of the engine parts,
- unwanted thermal energy in the hot engine and
- other energy-requiring aspects of the car: lights, air conditioning, and so on.

c Hot exhaust gases, unwanted thermal energy of the hot engine

d **i** $\eta = \frac{W}{Q_{hot}}$

ii $Q_{hot} = W + Q_{cold} \Rightarrow \eta = \frac{W}{Q_{hot}} = \frac{Q_{hot} - Q_{cold}}{Q_{hot}} = 1 - \frac{Q_{cold}}{Q_{hot}}$

iii In practice, there is also wasted energy in other forms of energy, not just thermal. This makes the efficiency of the engine smaller than the equation in **part d i**.

e Yes. From the first law of thermodynamics, $Q = \Delta U + W$, and ΔU contributes to the lost thermal and other forms of energy. Since the energy has been transformed into forms that have a greater number of microstates than the fuel had had, the entropy of the engine and its surroundings must have increased, thus showing this to be an example of the second law of thermodynamics in practice.

f Yes, this is reasonable—and it is suggesting that the increased entropy is acting as a kind of thermal energy that is unavailable for doing useful work.

g This is the Kelvin form of the second law of thermodynamics.

3 a $\eta = 1 - \frac{T_{cold}}{T_{hot}}$

So, at higher values of T_{cold} the efficiency will be less.

b i $\eta = 1 - \frac{T_{cold}}{T_{hot}} = 1 - \frac{273 + 8}{273 + 700} = 71\%$

ii $\eta = 1 - \frac{T_{cold}}{T_{hot}} = 1 - \frac{273 + 23}{273 + 700} = 70\%$

c The change in electrical energy production is $\frac{6}{46} = 13\%$.

The change in theoretical efficiency is only 1%.

The result was, perhaps, surprising, since the change in theoretical efficiency would predict only a 1% difference in energy output. The performance of the generators suggests that they do not operate as a Carnot heat engine and that energy losses are substantial.

4 a Otto cycle

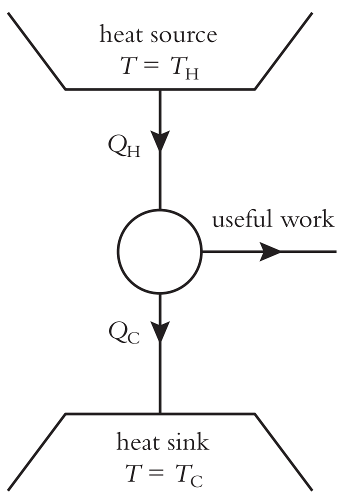
b The area enclosed by the four processes.

c i D

ii B

d D

e



f Some of the heat taken from the heat source is given to the heat sink. So, not all the heat from the heat source is available to produce useful work.

5 a Carnot cycle

b A and C are isothermic processes. B and D are adiabatic processes.

c A

d i A

ii C

e A

f See diagram from answer **4e**.

Useful work done = $W = Q_H - Q_C$

Total energy used = Q_H

So efficiency = $\eta = \frac{Q_H - Q_C}{Q_H} = 1 - \frac{Q_C}{Q_H}$.

g i $\Delta S_H = -\frac{Q_H}{T_H}$

ii $\Delta S_C = \frac{Q_C}{T_C} = \frac{Q_H - W}{T_C}$

iii $\Delta S_{net} = \frac{Q_H - W}{T_C} - \frac{Q_H}{T_H}$

iv $\frac{Q_H - W}{T_C} - \frac{Q_H}{T_H} \geq 0 \Rightarrow \frac{Q_H - W}{Q_H} \geq \frac{T_C}{T_H} \Rightarrow$

$1 - \frac{W}{Q_H} \geq \frac{T_C}{T_H} \Rightarrow \frac{W}{Q_H} - 1 \leq -\frac{T_C}{T_H}$

$\therefore \frac{W}{Q_H} = \eta \leq 1 - \frac{T_C}{T_H}$

h i $\eta = 1 - \frac{T_C}{T_H} = 1 - \frac{273 + 27}{273 + 327} =$ 0.5 or 50%

ii In a real power station, there are other sources of energy loss, such as lost heat in the processes of conduction, convection and radiation or mechanical energy losses due to friction.

iii T_H could be increased, or T_C could be decreased.

iv The value of T_H is limited by engineering and thermodynamic issues. T_C is usually determined by the temperature of the ambient conditions (this is why it is usual to build nuclear power stations near large masses of cold water, such as the sea).

6 a $\eta = 1 - \frac{T_C}{T_H} = 1 - \frac{273 + 250}{273 + 550} = 0.364 = 36\%$ (2 s.f.)

b Frictional losses and other thermal energy losses to the surroundings will mean that some of the useful mechanical work done is transferred into non-useful energy.

7 a

Process	Q	U	W
A	removed	none	on the gas
B	none	increases	on the gas
C	added	none	by the gas
D	none	decreases	by the gas

b Heat is being transferred to the hot gas during the isothermal compression (A) and is gained from the cold gas during the isothermal expansion (C). To make this possible, work is done on the cold gas (B) to heat up the gas so that it can become hot enough to transfer thermal energy.

c This is how a refrigerator works.

8 a Heat, Q_C, is extracted from the heat sink by the mechanical work done. Heat, Q_H, is then deposited into the heat source. In this case, conservation of energy gives $Q_H = Q_C + W$.

b The Clausius form of the second law of thermodynamics states that it is not possible to exchange heat from a colder body to a hotter body without the use of mechanical work. The heat pump shows that mechanical work is done on the gas in order to allow it to transfer thermal energy from a cold body to a hot body.

9 a $\frac{Q_H - Q_C}{Q_H} = \frac{T_H - T_C}{T_H} \therefore \frac{Q_H - 6.0\text{ kJ}}{Q_H} = \frac{(273 + 25) - (273 + 4)}{(273 + 25)} = 0.07$

$\therefore Q_H(1 - 0.07) = 6.0\text{ kJ} \Rightarrow Q_H = \frac{6.0\text{ kJ}}{0.93} = 6.45\text{ kJ}$

b $W = Q_H - Q_C = 6.45 - 6.0 = 0.45\text{ kJ}$

Exam-style questions

Multiple-choice questions

1 A [1]

2 C [1]

3 B [1]

4 C [1]

5 A [1]

6 B [1]

7 D [1]

8 B [1]

9 B [1]

10 A [1]

Short-answer questions

11 a $T \propto PV$ (or equivalent) [1]

At X: $PV = 4 \times 10^5 \times 10 \times 10^{-3} = 4 \times 10^3\text{ J}$

At Y: $PV = 1 \times 10^5 \times 40 \times 10^{-3} = 4 \times 10^3\text{ J}$

So, the process is isothermal. [1]

b Work done = area under graph. [1]

$\approx (30 \times 10^{-3} \times 1 \times 10^5) + \left(\frac{1}{2} \times 3 \times 10^5 \times 10 \times 10^{-3}\right) + \left(\frac{1}{2} \times 1 \times 10^5 \times 20 \times 10^{-3}\right) = 5.5\text{ kJ}$ [1]

c It has done work on the surroundings, but its temperature has remained the same, so it must have received heat from the surroundings. [1]

12 a Q is the amount of heat exchanged by a system with its surroundings. A positive Q means that heat is supplied to the gas from the surroundings. [1]

ΔU is the change in the internal energy of the gas. ΔU will be positive if the internal energy of the gas increases—that is its temperature increases. [1]

W is the work done by the gas on its surroundings. W is positive if the gas expands, doing work on the surroundings. [1]

b i Since the surroundings are hotter, heat will be absorbed by the system, making Q positive. The system then does work on its surroundings, so W is also positive.

$\therefore 50 = \Delta U + 30 \Rightarrow \Delta U = 50 - 30 = 20\text{ J}$ [1]

ii The surroundings undergo an entropy decrease given by $\Delta S = \frac{50}{T_{hot}}$. **[1]**
But the gas undergoes an entropy increase given by $\Delta S = \frac{20+30}{T_{cold}}$. **[1]**
$\frac{20+30}{T_{cold}} - \frac{50}{T_{hot}} > 0$, so the net entropy increases. **[1]**

13 a $U = \frac{3}{2}PV = \frac{3}{2} \times 8 \times 10^4 \times 3 \times 10^{-3} =$ 360 J **[2]**

b $T = \frac{PV}{nR} = \frac{8 \times 10^4 \times 3 \times 10^{-3}}{0.04 \times 8.31} = 722$ K $=$ 720 K (2 s.f.) **[1]**

c Work done by gas, $W = P\Delta V =$ $8 \times 10^4 \times (9-3) \times 10^{-3} = 480$ J
$\Delta U = \frac{3}{2}P\Delta V = \frac{3}{2} \times 8 \times 10^4 \times (9-3) \times 10^{-3} = 720$ J **[1]**
$\therefore Q = 480 + 720 = 1200$ J **[1]**

14 a An isothermal process takes place at constant temperature. This implies that there is no change in internal energy—that is $\Delta U = 0$. **[1]**
An adiabatic process is one in which there is no exchange of heat with the surroundings – that is $Q = 0$. **[1]**

b i $W = P\Delta V = 1.5 \times 10^5 \times (4-60) \times 10^{-3} = -8.4 \times 10^{-3}$ J (Note that the minus sign is showing that the work has been done on the gas.) **[1]**

ii Since $PV \propto$ T and V has decreased, T must have decreased. $\therefore$ U has decreased. **[1]**

iii $Q = \Delta U + W$ Since both ΔU and W are negative, Q must be negative.
Therefore, the gas has lost thermal energy to the surroundings. **[1]**

15 a Isobaric expansion **[1]**

b work done = area of rectangle = $6 \times 10^{-2} \times 3 \times 10^5 = 1.8 \times 10^4$ J **[2]**

c B **[1]**

d $\Delta U = \frac{3}{2}\Delta(PV) = \frac{3}{2} \times (8.0 - 2.0) \times 10^3$ $= 9 \times 10^3$ J (Note that the change is positive, so the internal energy of the gas has increased.) **[1]**

16 a $Q = mc\Delta T = 0.3 \times 4200 \times (10-80) = 8.82 \times 10^4$ J **[1]**

b $\Delta S = \frac{Q}{T_{ave}} = \frac{-8.82 \times 10^4}{\frac{1}{2} \times (283 + 353)} =$ -277 JK^{-1} **[2]**

c The water loses entropy (-277 JK^{-1}),
but the heat sink gains entropy $\left(\frac{8.82 \times 10^4}{283} = 312 \text{ JK}^{-1}\right)$, **[1]**
so the net change in entropy is $312 - 277 = 35$ JK^{-1}, which is >0. (So, the second law of thermodynamics is obeyed.) **[1]**

17 a The gas is heated $\Rightarrow Q > 0$; the container is rigid $\Rightarrow W = 0$. **[1]**
$\therefore \Delta U > 0$ **[1]**

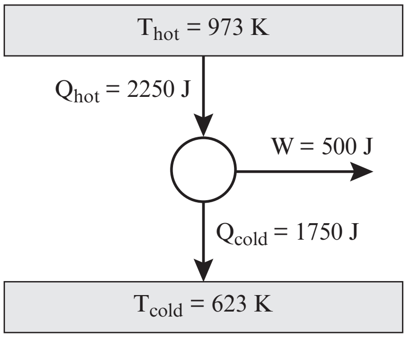
b Gas will cool $\Rightarrow \Delta U < 0$. **[1]**
Gas will be compressed $\Rightarrow W < 0$. **[1]**
$\therefore Q < 0$ **[1]**

18 a Compressing the gas rapidly does not give the gas enough time to exchange heat with the surroundings. So, $Q = 0$. **[1]**
But work has been done on the gas $\therefore W < 0$.
So $\Delta U > 0$ and hence the temperature of the gas increases. **[1]**

b The gas is compressed, so $W < 0$ **[1]**

Since the compression occurs slowly, there is sufficient time for the gas to exchange heat with the surroundings. The work done on the gas causes the gas to transfer thermal energy to its surroundings, so $Q < 0$. If $Q = W$, then there is no change in the internal energy. Therefore, the temperature if the gas remains constant. **[1]**

19 a



[2]

b $\eta = \frac{500}{2250} = 22\%$ (2 s.f.) **[1]**

c $\Delta S_{hot} = -\frac{2250}{973} = -2.31\ \text{JK}^{-1}$

$\Delta S_{cold} = \frac{2250 - 500}{623} = 2.81$

$\therefore \Delta S_{total} = 2.81 - 2.31 = 0.5\ \text{JK}^{-1}$ **[2]**

Chapter 11

Exercise 11.1

1 a The motion of the free electrons is random There is no net flow of charge in any direction, so there is no current.

b If a current were flowing, there would have to be a net flow of charge in one direction. This could be shown by adding extra motion vector arrows, all in the same direction, to all of the free electrons.

c A better conductor might have more free electrons. So, the diagram would have to have more small circles representing free electrons. (There are other factors that would be involved, but, for the purpose of this question, the number of free electrons is the main requirement.)

2 a v

b nAv

c $Q = nAve$ in 1 s

d $I = \frac{\Delta Q}{\Delta t} = nAve$

e i $v = \frac{I}{nAe}$

$= \frac{0.1}{(1 \times 10^{29}) \times (1 \times 10^{-6}) \times (1.6 \times 10^{-19})}$

$= 6.25 \times 10^{-6}\ \text{ms}^{-1}$

ii The free electrons *drift* very slowly because there are so many of them.

3 a Number of moles per kg $= \frac{1000}{63.5} =$ 15.7 moles kg^{-1}

b Number of atoms per kg $= 15.7 \times 6.02 \times 10^{23} = 9.5 \times 10^{24}$ atoms kg^{-1}

c Number of atoms per m^3 $= 9.5 \times 10^{24} \times 8900 = 8.4 \times 10^{28}$ atoms m^{-3}

d $n = 2 \times 8.4 \times 10^{28} = 1.6 \times 10^{29}$ free electrons m^{-3}

4 a An Amp is that constant current flowing in two long, straight, parallel conductors of negligible cross-sectional area, 1 m apart in a vacuum, such that the force between the two conductors is 2×10^{-7} N per metre length of conductor.

b If the values in the definition are inserted into the equation, the resulting force per unit length of conductor is 2×10^{-7} N:

$\frac{F}{l} = \mu_o \frac{I_1 I_2}{2\pi r} = 4\pi \times 10^{-7} \times \frac{1 \times 1}{2\pi \times 1}$

$= 2 \times 10^{-7}\ \text{Nm}^{-1}$.

c i $\frac{F}{l} = \mu_o \frac{I_1 I_2}{2\pi r} = 4\pi \times 10^{-7} \times \frac{1 \times 1}{2\pi \times 0.25}$

$= 8 \times 10^{-7}\ \text{Nm}^{-1}$

ii The two conductors will be forced together. If the currents are antiparallel, then the two conductors will be forced apart.

5 a i E_P

ii E_P transfers to E_K and then to thermal energy (and some sound) when the apple hits the ground.

iii No. The apple/tree/Earth is not a heat engine; thermal energy cannot be transformed into E_K, which then transforms into E_P.

iv The apple falls to the ground because it moves from where it had greater potential energy to where it has less potential energy.

b i eletrical potential energy → kinetic energy → thermal energy

ii The free electrons move from where they had greater EPE to where they have less EPE. Yes, the two answers are essentially the same.

c i Yes, there is a potential difference between where the apple is on the tree and where the apple is on the ground.

ii Yes, the gravitational potential difference causes the apple to move from a place with greater gravitational potential to a place with less gravitational potential.

iii Yes, the electrons move because of an electrical potential difference; they move from a place where the electrical potential is high to a place where the electrical potential is less.

iv potential difference = energy transferred per unit charge: $V = \frac{W}{Q}$

v Volt

vi We create a potential difference by introducing a cell or other source of emf.

6 a $V = \frac{W}{Q} = \frac{4}{0.25} = 16\text{ V}$

b $V = \frac{W}{Q} = \frac{2}{(1.25 \times 10^{18}) \times (1.6 \times 10^{-19})} = 10\text{ V}$

c $V = \frac{W}{Q} = \frac{24}{0.4 \times 60} = 1\text{ V}$

7 a work done $= q \times \Delta V = 1.6 \times 10^{-19} \times 1 = 1.6 \times 10^{-19}\text{ J}$

b i $6.4 \times 10^{-19}\text{ J} = \frac{6.4 \times 10^{-19}}{1.6 \times 10^{-19}} = 4\text{ eV}$

ii $3.2 \times 10^{-13}\text{ J} = \frac{3.2 \times 10^{-13}}{1.6 \times 10^{-19}} = 2 \times 10^{6}\text{ eV} = 2\text{ MeV}$

iii $2 \times 10^{-15}\text{ J} = \frac{2 \times 10^{-15}}{1.6 \times 10^{-19}} = 1.25 \times 10^{4}\text{ eV}$ (10 keV 1 s.f.)

c i $3\text{ eV} = 3 \times 1.6 \times 10^{-19} = 4.8 \times 10^{-19}\text{ J}$

ii $200\text{ keV} = 200 \times 10^{3} \times 1.6 \times 10^{-19}\text{ J} = 3.2 \times 10^{-14}\text{ J}$

iii $7.4\text{ MeV} = 7.4 \times 10^{6} \times 1.6 \times 10^{-19} = 11.8 \times 10^{-13} = 1.2 \times 10^{-12}\text{ J}$

8 a i The free electrons bounce off, losing some of their kinetic energy and slowing down temporarily.

ii $E_K \rightarrow$ thermal energy

iii Yes. The free electrons move and collide, slow down and move again repeatedly. So, they never achieve a fast speed.

iv The atoms of the conductor gain energy from the electrons and vibrate more.

v This causes the conductor to heat up. Note that it also makes it more difficult for the free electrons to drift along, because the increased vibrations of the atoms take up more space and deprive the free electrons of the space needed for them to move along.

b i A greater current means a greater movement of charge per second. This can be achieved by more electrons drifting along and electrons moving faster.

Both of these will result in the atoms of the conductor gaining more energy from the collisions they have with the free electrons, thus increasing the heating effect of the current.

9 a Some factors might be the following:

Factor	Reason
Length of conductor	A longer conductor will have more atoms with which the free electrons can collide.
Cross-sectional area of conductor	A thinner conductor will cause the free electrons to move faster. Faster electrons will transfer more energy to the atoms when they collide.
Free electron density	Free electron density will determine how many free electrons move.
Strength of atomic bonds	Weaker bonds will allow the atoms to vibrate with a greater amplitude. This will cause the vibrating atoms to take up more space within the conductor.
Atomic spacing	Smaller spaces between atoms will mean less space in which the free electrons can move.
Mass of atoms	Mass of atoms will contribute to how big they are (and so how much space they take up inside the conductor) and the speed at which they vibrate.

b i Long = more atoms to gain energy from the free electrons. And long = bigger resistance, so greater heating effect.

ii Very thin = faster-moving free electrons (because of conservation of charge.) Faster electrons collide with the atoms harder and more often, creating a greater heating effect.

iii The main reason that the filament is made of tungsten is that tungsten has a very high melting temperature, which allows the filament to get white-hot without the problem of it melting.

10 a Resistance: the ratio of the voltage across a component to the current flowing through it, $R = \frac{V}{I}$, measured in ohms, Ω.

b Volts are J/C, so base units are kgm^{2}s^{-2}/A s = kgm^{2}s^{-3}A^{-1}.

The base unit for current is A.

Therefore, base unit for resistance = kgm^{2}s^{-3}A^{-2}.

c i $R = \frac{V}{I} = \frac{0.5}{250 \times 10^{-3}} = 2\ \Omega$

ii $R = \frac{V}{I} = \frac{5}{50 \times 10^{-6}} = 1 \times 10^5\ (100\ \text{k}\Omega)$

iii $R = \frac{V}{I} = \frac{120}{30 \times 10^{-3}} = 4 \times 10^3\ (4\ \text{k}\Omega)$

11 a Resistivity: the resistance of a sample of material that has a cross-sectional area of 1 m^2 and a length of 1 m.

b $R = \rho\frac{l}{A} = 4 \times 10^{-8} \times \frac{1 \times 10^{-2}}{2 \times 10^{-6}} = 2 \times 10^{-4}\ \Omega$

c Length is 100 times shorter, and the cross-sectional area is 100 × 100 times smaller, so resistance will be 100 times larger.

$R = 100 \times 1.7 \times 10^{-8} = 1.7 \times 10^{-6}\ \Omega$

d i 10 km

ii Rod is 10^4 times longer and has cross-sectional area 10^4 times smaller.

So, new resistance is $R = 10^4 \times 10^4 \times 1.7 \times 10^{-8} = 1.7\ \Omega$.

e $\rho = R\frac{A}{l} = (2.2 \times 10^3) \times \frac{\pi \times (1.2 \times 10^{-3})^2}{2 \times 10^{-2}}$
$= 0.5\ \Omega\ \text{m}$

f $A = r\frac{l}{R} = (4 \times 10^{-4}) \times \frac{3 \times 10^{-2}}{100}$
$= 1.2 \times 10^{-7}\ \text{m}^2$

12 a B is $\frac{1}{2}$ as long and $\frac{1}{4}$ of the cross-sectional area, so its resistance will be $\frac{1}{2} \times 4 \times 100 = 2 \times 100 = 200\ \Omega$.

b $\frac{1}{2}$ of the radius means $\frac{1}{4}$ of the cross-sectional area. But this also means length is four times longer.

So, $R = 4 \times 4 \times 100 = 1.6\ \text{k}\Omega$.

13 a At any point on the graph, the resistance is given, strictly, by the value of the voltage divided by the value of the current.

Note: this is **not** the same as the gradient of the graph!

Mathematically, $R = \frac{V}{I}$ **not** $R = \frac{dV}{dI}$.

b i $R = \frac{V}{I} = \frac{5}{0.20} = 25\ \Omega$

ii $R = \frac{V}{I} = \frac{12}{0.48} = 25\ \Omega$

c At all points along the graph, the value of $\frac{V}{I}$ is the same. Another way of saying this is that at all points on the graph $V \propto I$.

d Ohm's law states that the voltage across a component is proportional to the current flowing through it at a constant temperature. This leads to the familiar equation: $V = IR$.

14 a i Potential difference, V

ii Temperature difference, ΔT

b i $R = \rho \frac{l}{A}$

ii $R = \frac{1}{k}\frac{l}{A}$

c i $I = \frac{dq}{dt}$ $\left(\frac{q}{t}\text{ would be acceptable too}\right)$

ii $I = \frac{dQ}{dt}$ $\left(\frac{Q}{t}\text{ would be acceptable too}\right)$

d $V = IR = \frac{dq}{dt} \times \rho \frac{l}{A}$

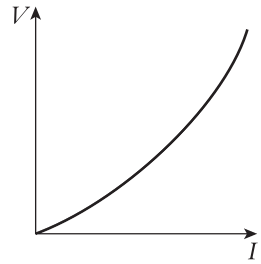
e $\Delta T = \frac{dQ}{dt} \times \frac{1}{k}\frac{l}{A}$

f

Term	Electrical case	Thermal case
What makes current flow?	V	ΔT
Expression for current	$\frac{dq}{dt}$	$\frac{dQ}{dt}$
Expression for resistance	$R = \rho \frac{l}{A}$	$R = \frac{1}{k}\frac{l}{A}$
Overall equation	$V = \rho \frac{l dq}{A dt}$	$\Delta T = \frac{l dQ}{k A dt}$

15 a The light bulb will not stay at a constant temperature if the voltage across the bulb is varied. As the voltage across a light bulb is increased, the temperature of the light bulb will increase (as more current flows, more energy is transferred to thermal energy), meaning Ohm's law cannot be obeyed.

b



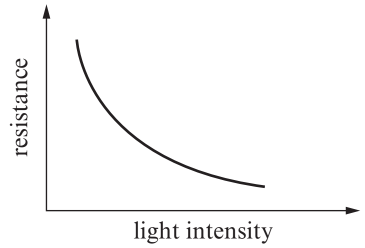
c i As current increases, the resistance of the light bulb increases.

ii As the current increases, electrons collide more frequently (and harder) with atoms of the filament. Each collision transfers energy in the form of thermal energy to the atoms, making them vibrate more violently. This makes it more difficult for the electrons to pass along the filament of the light bulb. We measure this difficulty as the increase in resistance of the light bulb.

Since R is defined as $\frac{V}{I}$, an increase in V does not produce a corresponding similar increase in I, so the ratio of V to I increases, thus increasing the resistance.

16 a When the temperature increases, the atoms of the thermistor vibrate more violently. This more violent vibration is sufficient to cause some of the bound electrons to be shaken free, increasing the free electron density. With more electrons to be able to move, there will be a larger current for the same voltage, hence a decrease in resistance.

17 a



b Individual photons of light, incident on some of the bound electrons, will give the bound electrons enough energy to break free of the atom. This will increase the free electron density and so decrease the resistance of the LDR. An increased light intensity means an increased number of photons per second. More photons mean more bound electrons can be removed and so the decrease in resistance. As atoms lose bound electrons, however, some free electrons will start to be captured by the relatively positively charged atoms, so further increases in light intensity will not remove as many electrons, and so the decrease in resistance continues but by less and less.

Exercise 11.2

1 a V

b I

c $P = IV$

d i $P = IV$ and $V = IR$ so $P = I^2R$.

ii $P = IV$ and $I = \frac{V}{R}$ so $P = \frac{V^2}{R}$.

2 a $E = VIt \Rightarrow I = \frac{E}{Vt} = \frac{0.8}{230 \times 10 \times 10^{-3}} =$ 0.35 A (2 s.f.)

b As the filament of the lamp heats up, its resistance increases. This will make the current flowing through it decrease.

c $P = \frac{E}{t} = \frac{0.8}{10 \times 10^{-3}} = 80$ W

3 a Chemical energy is transformed into electrical potential energy (usually we would just say electrical energy rather than the full-term electrical potential energy).

b The label '1.5 V' is the emf of the battery. This means that the battery, when connected into a circuit, will convert 1.5 J of chemical energy for every Coulomb of charge that flows through it.

c i In normal operation, $\varepsilon > V$ because the transformation of chemical energy to electrical energy always wastes some energy in the form of thermal energy in the battery itself.

ii If no current flows through the battery, then $\varepsilon = V$. (Note that this is directly from the definition of what emf is.)

4 a Chemical energy is transformed into electrical energy.

b No. The laws of thermodynamics state that, in any energy transformation, some energy is wasted as thermal energy. So, as the cell transforms chemical energy into electrical energy, some is transformed into thermal energy that heats up the cell.

5 a In a resistor, electrical energy is transformed into thermal energy.

b Since some energy is transformed into thermal energy when a cell is used in a circuit—much like the effect of a resistor – it is sensible to consider that the cell itself has a resistance.

6 a Once the stored energy in a primary cell has been used up, the cell is of no use. It is not possible to 're-energise' the primary cell. In a secondary cell, once the energy has been used up, it is possible to re-energise the cell by rearranging the distribution of charges within the cell – this is what happens when a rechargeable cell is attached to a charger. So, the secondary cell can be used many times.

b 1200 mA-hours is really a measure of how much charge can flow through the cell before it has used up all of its energy. So, 1200 mA-hours = $1200 \times 10^{-3} \times 60 \times 60 = 4.32 \times 10^3$ C.

With a constant current of 1.6 mA, the cell will last for $\frac{4.32 \times 10^3}{1.6 \times 10^{-3}} = 2.7 \times 10^6$ s (750 hours).

7 a

Resistance of variable resistor / Ω	Current in circuit / A	Power dissipated in variable resistor / W
1.0	1.25	1.56
2.0	**1.00**	**2.00**
3.0	0.83	2.08
4.0	0.71	2.04
5.0	0.63	1.95

b The power dissipated in the resistor is a maximum when the resistance is the same value as the internal resistance of the cell.

8 a i A semiconductor is a component made out of a material, or a combination of materials, that is usually an insulator, but in certain circumstances can become a conductor.

ii It is possible to change the insulating property of the semiconductor into a conducting property if charge carriers, electrons or holes, can be introduced or produced.

iii A p-type semiconductor is one that has an excess of positive charge carriers, or holes. One could also think of it as having insufficient electrons.

iv An n-type semiconductor is one that has an excess of negative charge carriers—or electrons.

v A hole is a location within the atomic structure that is devoid of an electron in an otherwise electron-filled space. This gives the hole the property of being relatively positively charged compared to what is around it.

b i Doped means that different kinds of atoms have been introduced into the atomic structure of the material.

ii Because the phosphorous atoms have more valence electrons than the silicon atoms, there is an excess of negative charge carriers, so the phosphorous-doped layer has become an n-type semiconductor.

c i Because the boron atoms have fewer valence electrons, there is an excess of holes, so the boron-doped layer has become a p-type semiconductor.

ii Any free electrons will migrate towards the relatively positively charged lower layer.

iii This will leave the phosphorous atoms short of electrons.

iv This will leave the boron atoms short of holes.

v Where the free electrons have filled up the spaces of the holes, there will be no charge carriers of either kind. The term *depletion zone* refers to the absence of charge carriers in this zone.

vi The upper layer has become relatively positively charged because it has lost electrons.

vii The lower layer has become relatively negatively charged because it has gained electrons (and lost holes).

viii The electric field in the depletion zone will be directed downwards from the upper layer towards the lower layer.

d i The free electrons will be forced upwards, and the holes will be forced downwards.

ii This will produce a potential difference, or voltage between the upper and lower layers of about 0.5 or 0.6 V.

iii The potential difference will cause a current to flow.

e i Thin: because more light can penetrate through it to the depletion zone and create more electron–hole pairs. Heavily doped: to produce more free electrons that can migrate to the depletion zone

ii Thick: to provide a larger depletion zone in which free electron–hole pairs can be created. Lightly doped: because the depletion zone is large it isn't necessary to have a large proportion of boron atoms present. So a light doping is sufficient.

iii Since each cell produces a voltage of about 1.5 or 1.6 V, many such cells are required in series—since in series voltages will add together—to produce a working voltage capable of driving a substantial current.

iv Chains of cells in series are linked in parallel to allow a substantial total current to flow, since in parallel the currents will add together.

Exercise 11.3

1 a They will all read the same current, I.

b i $V_1 = IR_1$

ii $V_2 = IR_2$

iii $V_3 = IR_3$

c $V_{\text{total}} = V_1 + V_3 + V_3$

d $V_{\text{total}} = V_1 + V_3 + V_3$

$\equiv IR_1 + IR_2 + IR_3$

$= I(R_1 + R_2 + R_3) = IR_{\text{total}}$

$\therefore R_{\text{total}} = R_1 + R_2 + R_3$

2 a $R = \rho\frac{l}{A} = 1.68 \times 10^{-8} \times \frac{0.75}{1 \times 10^{-6}}$

$= 0.013\ \Omega$

Since this value is likely to be much less than the value of any resistors in a circuit, it is acceptable to consider its resistance to be zero.

b No. If $R = 0$, then $V = IR = I \times 0 = 0$.

c i–iii They are all the same, V.

d A_1 measures $I_1 = \frac{V}{R_1}$, A_2 measures $I_2 = \frac{V}{R_2}$ and A_3 measures $I_3 = \frac{V}{R_3}$.

So, A_{total} measures $I_{\text{total}} = I_1 + I_2 + I_3 = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$

e $I_{\text{total}} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \Rightarrow \frac{I}{V} = \frac{1}{R_{\text{total}}} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$

3 a $R = \rho\frac{l}{A}$

b i No.

ii Twice as long, $2l$

iii $R_{\text{total}} = \rho\frac{2l}{A} = 2R$

iv $R_{\text{total}} = \rho\frac{nl}{A} = nR$

c i Twice the area, $2A$.

ii $R_{\text{total}} = \rho\frac{1}{2A} = \frac{1}{2}R$

iii $R_{\text{total}} = \rho\frac{1}{nA} = \frac{1}{n}R$

4 a $V_1 = IR_1$; $V_2 = IR_2$; $V_3 = IR_3$ and $V_{\text{total}} = V_1 + V_2 + V_3 = \sum(IR)_i$

Energy supplied to the circuit is from the emf source only. So, conservation of energy gives that energy supplied = energy used.

So, $\varepsilon = \sum(IR)_i$, which is Kirchhoff's second law.

b Conservation of charge gives that the charge flowing into the junction must equal the charge flowing out of the junction. If we consider the charge flowing out to be a negative flow and the charge flowing in to be a positive flow then, added together, the total charge flowing must be zero.

So, $\sum I_i = 0$, which is Kirchhoff's first law.

5 a $R_{\text{total}} + R_1 + R_2 + R_3 = 3.0 + 5.2 + 0.3 = 8.5\ \Omega$

b $\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{3} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

$\therefore R_{\text{total}} = 2\ \Omega$

c These are the possible combinations:

10 Ω	one resistor on its own OR two parallel sets of two resistors in series
20 Ω	two of the resistors in series
30 Ω	three of the resistors in series
40 Ω	all four resistors in series
15 Ω	one resistor in series with two resistors in parallel
13.3 Ω	one resistor in series with three resistors in parallel
25 Ω	two resistors in series with two resistors in parallel
5 Ω	two resistors in parallel
3.3 Ω	three resistors in parallel
2.5 Ω	all four resistors in parallel

6 a $\frac{1}{R_{\text{total}}} = \frac{1}{(12 + 8 + 4)} + \frac{1}{(2 + 4 + 2)} = \frac{1}{6} \Rightarrow$

$R_{\text{total}} = 6\ \Omega$

b $I = \frac{V}{R} = \frac{6}{6} = 1\ A$

c Total current in the circuit is 1 A.

Current flowing through 12 Ω/8 Ω/4 Ω resistors is 0.25 A.

So, power dissipated by 8 Ω resistor is $P = I^2R = 0.25^2 \times 8 = 0.5$ W.

7 Suppose a current, flows into the cube. Using Kirchhoff's first law:

$\frac{I}{3}$ flows through one of the three resistors at the first junction. $V_1 = \frac{IR}{3}$.

At the next junction, the current flowing through one of the resistors is $\frac{I}{6}$, so $V_2 = \frac{IR}{6}$.

$\frac{I}{3}$ flows through the last resistor before the opposite corner, so $V_3 = \frac{IR}{3}$.

Using Kirchhoff's second law,

$V_{total} = V_1 + V_2 + V_3 = \frac{IR}{3} + \frac{IR}{6} + \frac{IR}{3} = \frac{5IR}{6}$,

$R_{total} = \frac{V_{total}}{I} = \frac{5IR}{6I} = \frac{5}{6}R$.

Exercise 11.4

1 a i $R_{total} = R + R_A$

ii Yes. The resistance has increased. This means that the current that will now flow through this part of the circuit will be smaller than it would be without the ammeter. So, the ammeter would measure the current to be too small.

iii R_A should be as small as possible. (Ideal ammeters are usually taken to have zero resistance, although in practice this is impossible to achieve.)

b i $R_{total} = \frac{RR_A}{R + R_A}$

ii Yes. The resistance of this part of the circuit is now smaller than it had been without the ammeter. And, the ammeter now provides a path parallel to the path through the resistor for the current to flow through. So the ammeter will now not be measuring the current flowing through the resistor.

c i Ammeters are placed in series.

ii An ideal ammeter has zero resistance.

2 a $R_{total} = \frac{RR_V}{R + R_V}$

b Yes. The overall resistance is now less than it had been without the voltmeter. So, the voltmeter will now measure a value of potential difference that is smaller than it should be.

c Ideally, the resistance of the voltmeter should be infinite.

3 a i The voltmeter is in parallel with the 1 kΩ resistor, making the total resistance of this combination of the two resistors 500 Ω.

This 500 Ω is in series with the other 1 kΩ resistor.

So, the voltage across the combination is $\frac{1}{3} \times 6 = 2$ V.

ii Total resistance of the combination is $R_{total} = \frac{1 \times 100}{1 + 100} = 0.99$ kΩ.

So, the voltage across the combination is $V = \frac{0.99}{0.99 + 1} \times 6\text{ V} = 2.98$ V.

b An ideal voltmeter has such a high resistance that when it is placed in parallel with another resistor, the total resistance of the combination is no different from the resistance of the resistor. The voltmeter does not change the circuit in which it has been put.

c In the previous case, a 'perfect' voltmeter would read 3.00 V. The difference the voltmeter in **part a ii** has made is only about 1%. So, as long as a voltmeter has a resistance that is at least 100 times larger than the resistor across which it is placed, the value of the voltage it reads will be within 1% of what it ought to be. A larger ratio of resistances will reduce the difference further.

4 a *emf* means how many joules of energy are transformed by the cell/battery for every coulomb of charge that flows through the cell/battery.

b The total resistance is now:

$R_{total} = 0.4 + 5.6 = 6.0\ \Omega$.

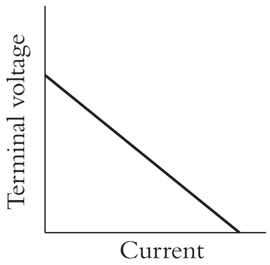
So, the current is $I = \frac{\varepsilon}{R} = \frac{1.5}{6} = 0.25$ A.

c The voltage across the internal resistance will be $V = Ir = 0.25 \times 0.4 = 0.1$ V.

So, the terminal voltage will be $1.5 - 0.1 = 1.4$ V.

d R_{total} is now $0.6 + 0.4 = 1\ \Omega$. Terminal voltage will be $0.6 \times 1.5\text{ V} = 0.9$ V.

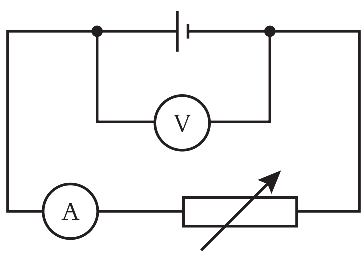
5 **a**



b The equation for this graph is $V = \varepsilon - ir$, where V is the terminal voltage, ε is the emf of the cell, i is the current in the circuit and r is the internal resistance of the cell. The internal resistance of the cell, r, is the gradient of the graph.

c The value of V when $i = 0$ is ε. So, the terminal voltage when no current flows is the emf of the cell.

6



Equipment required: cell, leads, variable resistor, ammeter, voltmeter.

Method: measure the terminal voltage using the voltmeter. Measure the current flowing in the circuit using the ammeter. Vary the current in the circuit by changing the resistance of the variable resistor. Make a table of values of current and voltage.

Plot a graph of *terminal voltage* (y-axis) against *current* (x-axis.) The internal resistance, r, can be found from the gradient of the graph. The emf of the cell, ε, is the y-axis intercept.

7 **a** $I = \frac{\varepsilon}{R_{total}} = \frac{6}{1+5} = 1$ A

b $V = IR = 1 \times 5 = 5$ V

c $P = I^2R = 1^2 \times 5 = 5$ W

8 **a** **i** $R = \frac{V}{I} = \frac{3}{0.4 \times 10^{-3}} = 7500\ \Omega$ (7.5 kΩ)

ii A = 0.2 mA; B = 0.2 mA; C = 0.6 mA; D = 0.6 mA

iii Either: $R = \frac{V}{I} = \frac{3}{0.6 \times 10^{-3}} = 5000\ \Omega$ (5 kΩ)

or $\frac{1}{R_{total}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{7500} + \frac{1}{7500 + 7500} = \frac{3}{15\,000} = \frac{1}{5000}$

$\therefore R_{total} = 5000\ \Omega$.

b **i** The total resistance in the circuit is

$R = 1 + \frac{3 \times 6}{3 + 6} = 3\ \Omega$.

So, the current flowing through the 1 Ω resistor is $I = \frac{V}{R} = \frac{6}{3} = 2$ A.

ii Either the voltage across the 6 Ω resistor is $V = 6 - (1 \times 2) = 4\ V$,

so $I = \frac{V}{R} = \frac{4}{6} = 0.67$ A,

OR

2 A splits in the ratio 1:2 for the resistors 6:3, so $I = \frac{1}{3} \times 2 = 0.67$ A for the 6 Ω resistor.

iii Either the voltage across the 3 Ω resistor is $V = 6 - (1 \times 2) = 4\ V$,

so $I = \frac{V}{R} = \frac{4}{3} = 1.33$ A,

OR

2 A splits in the ratio 1:2 for the resistors 6:3, so $I = \frac{2}{3} \times 2 = 1.33$ A for the 3 Ω resistor.

9 **a** **i** This is a Wheatstone bridge circuit – a kind of bridge in which all the components are resistors.

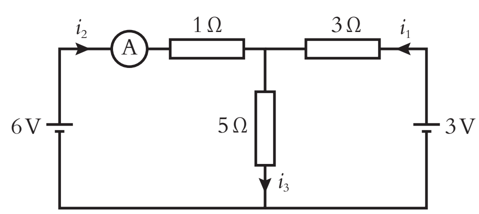
ii Voltage across $R_2 = E\frac{R_2}{R_1 + R_2}$

iii Voltage across $R_4 = E\frac{R_4}{R_3 + R_4}$

iv If the ammeter reads zero, there is no voltage across the ammeter. So,

$E\frac{R_2}{R_1 + R_2} = E\frac{R_4}{R_3 + R_4} \Rightarrow \frac{R_1}{R_2} = \frac{R_3}{R_4}$.

10 **a**



For the left-hand loop, $6 = i_2 + 5i_3$ ① (Kirchhoff's second law).

For the right-hand loop: $3 = 3i_1 + 5i_3$.

According to Kirchhoff's first law, $i_1 = i_3 - i_2$.

So, $3 = 3i_3 - 3i_2 + 5i_3 = 8i_3 - 3i_2$. ②

3 ① + ② = $18 + 3 = 3i_2 + 15i_3 + 8i_3 - 3i_2 = 23i_3 = i_3 = \frac{21}{23}$ A

And, from ①, $i_2 = 6 - 5i_3 = 6 - 5 \times \frac{21}{23} = \frac{138 - 105}{23} = \frac{33}{23}$ A (= 1.43 A)

b **i** For the right-hand loop, let the current flowing through the 3 Ω resistor be i_1, the current flowing through the 1 Ω resistor be i_2 and the current flowing through the 2 Ω resistor be i_3.

Then, $2 = 3\,i_1 + 2\,i_3$.

For the left-hand loop, $3 = i_2 + 2\,i_3$.

And $i_2 = i_3 - i_1$.

So, $3 = i_3 - i_1 + 2i_3$ and $2 = 2i_3 + 3i_1$.

$9 = 9i_3 - 3i_1$ and $2 = 2i_3 + 3i_1$

$11 = 11i_3 \Rightarrow i_3 = 1$ A

ii $2 = 3i_1 + 2 \Rightarrow i_1 = 0$

Exam-style questions

Multiple-choice questions

1 B **[1]**

2 C **[1]**

3 B **[1]**

4 A **[1]**

5 C **[1]**

6 D **[1]**

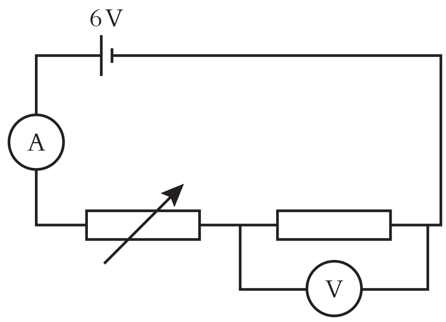
7 A **[1]**

8 C **[1]**

9 C **[1]**

Short-answer questions

10 a

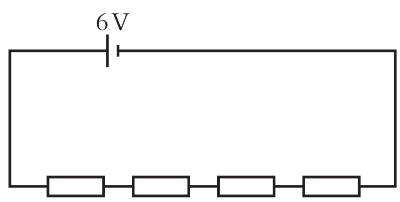


[2]

b The graph of V against I is a straight line that passes through the origin, showing that $V \propto I$, so the resistor obeys Ohm's law and is ohmic. **[1]**

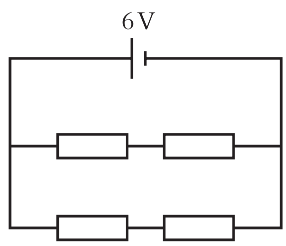
c Using the point on the graph at $I = 50$ mA and $V = 5$ V, $R = \frac{5}{50 \times 10^{-3}} = 100\ \Omega$. **[2]**

11 a i



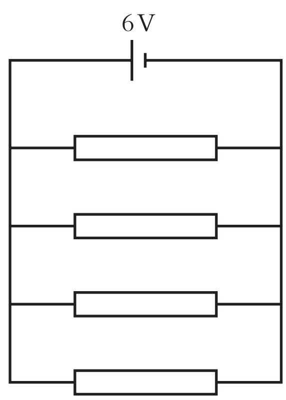
[1]

ii



[1]

iii



[1]

b The circuit in **part a i** has the highest total resistance, so the smallest current will flow. The cell will be able to provide this current for the longest time of the three circuits. **[2]**

12 a At the point $I = 1$ A and $V = 16$ V, the two components have the same resistance. **[1]**

$R = \frac{V}{I} = \frac{16}{1} = 16\ \Omega$ **[1]**

b i Since the current is the same through each component, **[1]** the value of I for which $V_A + V_B = 12$ is 0.5 A. So the ammeter reads 0.5 A. **[1]**

ii $R = \frac{V}{I} = \frac{12}{0.5} = 24\ \Omega$ **[1]**

Alternatively, $R_{total} = R_A + R_B = \frac{8}{0.5} + \frac{4}{0.5} = 24\ \Omega$.

13 a $Q = It = 250 \times 10^{-3} \times 60 = 15$ C **[1]**

b electrical potential energy → kinetic energy of electrons → thermal energy in resistor **[2]**
OR electrical potential energy → thermal energy OR kinetic energy → thermal energy **[1]**

c $E = VQ = 6 \times 15 = 90$ J (1.4 kJ 2 s.f.) **[1]**

d $P = \frac{E}{t} = \frac{90}{60} = 1.5$ W **[1]**

14 a $R_{total} = 30 + \frac{60 \times 30}{60 + 30} = 50\ \Omega$ **[2]**

b $I = \frac{V}{R} = \frac{6}{50} = 0.12$ A **[1]**

c Power dissipated in X $= I^2R = 0.12^2 \times 30 = 0.43$ W

Power dissipated in Y $= I^2R = 0.08^2 \times 30 = 0.19$ W

So, $\frac{P_X}{P_Y} = \frac{0.43}{0.19} = 2.3$. **[2]**

15 a i The same current flows through the 8 Ω resistor, so $V_{resistor} = IR = 0.375 \times 8 = 3$ V. **[1]**

So the potential difference across the bulb must be 12 – 3 = 9 V. **[1]**

ii $R = \frac{V}{I} = \frac{9}{0.375} = 24\ \Omega$. **[1]**

b The bulb is now in series with a resistance that is half of what it had been, giving the circuit a lower total resistance.

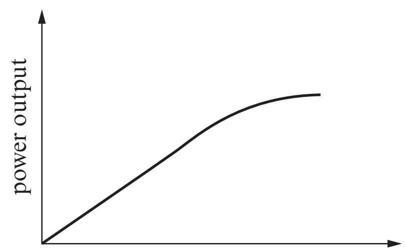
So, a larger current will flow through the bulb, OR the potential difference across the bulb will now be larger, and so IV is larger, *making the bulb brighter*. **[1]**

16 a Since emf = constant, $I \propto P$, so the graph of *current* against *light intensity* will be the same shape as the graph of power against light intensity. **[1]**

b Only simple statements are required here. Any two from the following: **[2]**

- Light is incident on P–N junction.
- Photons produce electron–hole pairs in depletion zone.
- The electric field produced by P–N junction forces freed electrons and holes to opposite sides of the junction.
- This produces a potential difference across P–N junction; p.d. causes current to flow.

c i



[1]

ii The more electron–hole pairs are produced (by the absorption of incident light photons) thse more difficult it becomes to produce even more. So, the rate of production of electron–hole pairs decreases until it reaches a maximum value. **[1]**

At this value, no more electron–hole pairs can be produced and the power output will become constant at its maximum value. **[1]**

Chapter 12

Exercise 12.1

1 a Time period, T, is the time taken to make one complete oscillation.

b Frequency, f, is the number of complete oscillations made in 1 s.

c Amplitude, A, is the maximum displacement from the equilibrium position.

d Equilibrium position is the position of the oscillator when there are no unbalanced forces acting on it.

e Displacement, x, is the distance (and direction) of the oscillator away from the equilibrium position.

f Angular frequency is the angle, in radians, turned through in 1 s.

2 a The body will experience an unbalanced force.

b The force acting on the body is directed towards the equilibrium point. This will make the body accelerate towards the equilibrium point. So, whenever the body is away from the equilibrium point, it will always be accelerating towards the equilibrium point. This, in effect, is an oscillation.

c The force acting on the body must be proportional to the displacement from the equilibrium point if the body is going to oscillate in SHM.

d Either

force is proportional to, and in the opposite direction to, the displacement from the equilibrium position

OR

acceleration is proportional to, and in the opposite direction to, the displacement from the equilibrium position.

3 a Period = $\frac{2 \times 10^{-3}}{5} = 4 \times 10^{-4}$ s

b Frequency, f = 2500 Hz

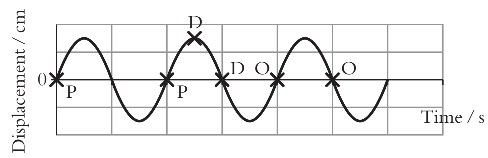
c Amplitude = maximum displacement = 3 cm

4 a i Where the gradient of the graph is a maximum—this occurs when the displacement is zero—that is when the oscillator is passing through the equilibrium position.

ii Where the gradient of the graph is zero—this occurs when the displacement is a maximum or minimum from the equilibrium position.

b The gradient of the graph of displacement against time gives the velocity of the oscillating body.

5 a, b and c



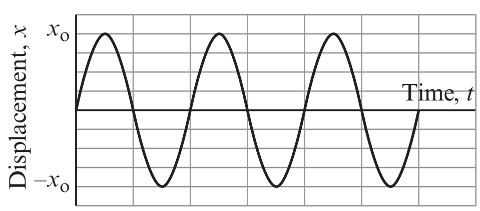
a Any two points that are a whole number of waves apart will be in phase.

b Any two points that are an odd number of half-waves apart will be out of phase.

c Two points that are a quarter of a wave out of phase will have a phase difference of $\frac{\pi}{2}$.

Exercise 12.2

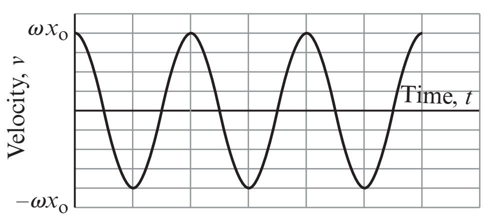
1 a



b i $v = \omega x_o \cos(\omega t)$

ii $v_{max} = \omega x_o$

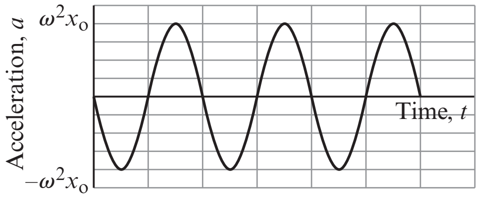
c



d **i** $a = -\omega^2 x_o \sin(\omega t)$

ii $a_{max} = \omega^2 x_o$

e



f $x = x_o \sin \omega t$ and $a = -\omega^2 x_o \sin(\omega t)$

$\therefore a = -\omega^2 x$

2 **a** The graph shows that $F \propto -x$.

Since $a = \frac{F}{m}$, the graph is also showing that $a \propto -x$, which is, effectively, the definition for SHM.

b **i** Amplitude is maximum displacement = 30 cm.

ii The gradient of the graph = $-m\omega^2$ = $\frac{6}{-0.3} = -20\ \text{Nm}^{-1} \Rightarrow \omega = \sqrt{\frac{20}{0.5}}$ = 6.3 rads^{-1}

iii $f = \frac{\omega}{2\pi} = \frac{6.3}{2\pi} = 1.0$ Hz.

iv $T = \frac{1}{f} = \frac{1}{1} = 1$ s

3 **a** p

b $q = mg \sin\theta$

c The pendulum will accelerate towards the equilibrium position.

d $mg \sin\theta = ma \Rightarrow a = -g \sin\theta$; here, the minus sign is showing that the acceleration is in the opposite direction to θ.

e The displacement from the equilibrium position, x, is given by $x = l\theta$, so

$$a = -g \sin\left(\frac{x}{l}\right)$$
$$= -g\frac{x}{l}$$
$$= -\frac{g}{l}x$$

f The definition of SHM gives $\omega^2 = \frac{g}{l} \Rightarrow$ $f = \left(\frac{1}{2\pi}\right)\left(\sqrt{\frac{g}{l}}\right)$.

g $T = \frac{1}{f} = 2\pi\sqrt{\frac{l}{g}}$

4 **a** For a stretched spring, the extension of the spring is proportional to the force used to stretch it, or $F = kx$.

b **i** The weight of the mass downwards, $w = mg$

The tension in the spring upwards, $T = kx$

ii The two forces are equal and opposite.

c **i** $F = kx_o$

ii The tension in the spring has changed. It is now larger by kx_o.

iii Upwards.

iv The upwards force will cause the mass to accelerate upwards.

v $a = \frac{F}{m} = -\frac{kx}{m}$ (The minus sign here is showing that the direction of the acceleration is opposite to the direction of the displacement.)

vi The definition of SHM is $a = -\omega^2 x$.

So, $\omega^2 = \frac{k}{m} \Rightarrow \omega = \sqrt{\frac{k}{m}}$.

vii $\omega = 2\pi f = \sqrt{\frac{k}{m}} \Rightarrow f = \left(\frac{1}{2\pi}\right)\left(\sqrt{\frac{k}{m}}\right)$

viii $T = \frac{1}{f} = 2\pi\sqrt{\frac{m}{k}}$

5 **a** $\omega = 2\pi f = 2\pi \times 250 = 1.57 \times 10^3$ rads^{-1}

b **i** Since $x = 0$ at $t = 0$, the equation is $x = 2 \times 10^{-4} \sin(1.57 \times 10^3\ t)$.

ii $x = (2 \times 10^{-4}) \times \sin((1.57 \times 10^3) \times (1.0 \times 10^{-3})) = 2.0 \times 10^{-4}$ m from its equilibrium position.

c **i** $v = ((2 \times 10^{-4}) \times (1.57 \times 10^3)) \times \cos(1.57 \times 10^3\ t)$

ii $v = ((2 \times 10^{-4}) \times (1.57 \times 10^3)) \times \cos((1.57 \times 10^3) \times (1.0 \times 10^{-3}))$
$= 0.0$ ms^{-1} (2 s.f.)

iii $v_{max} = \omega x_o$
$= (1.57 \times 10^3) \times (2 \times 10^{-4})$
$= 0.31$ ms^{-1}

d **i** $a = -((1.57 \times 10^3)^2 \times (2 \times 10^{-4})) \times \sin((1.57 \times 10^3)t)$

ii $a = -((1.57 \times 10^3)^2 \times (2 \times 10^{-4})) \times \sin((1.57 \times 10^3) \times (1.0 \times 10^{-3}))$

$= -490\ \text{ms}^{-2}$ (2 s.f.)

iii $a_{max} = -\omega^2 x_o$

$= -(1.57 \times 10^3)^2 \times (2 \times 10^{-4})$

$= -490\ \text{ms}^{-2}$ (2 s.f.)

Note: You may notice that for a frequency of 250 Hz, a time of 1 ms is $\frac{1}{4}$ of a cycle. So x will be at its maximum displacement of 0.2 mm, v will be zero, and a will be the maximum acceleration.

6 a $k = \frac{F}{x}$

$= \frac{10 \times 10}{0.2}$

$= 500\ \text{Nm}^{-1}$

$T = 2\pi\sqrt{\frac{m}{k}}$

$= 2\pi\sqrt{\frac{10}{500}}$

$= 0.89$ s

b i $v_{max} = \omega x_o$

$= \frac{2\pi}{T}\omega_o$

$= \frac{2\pi}{0.89} \times 0.2$

$= 1.4\ \text{ms}^{-1}$

ii $a_{max} = \omega^2 x_o$

$= \frac{2\pi}{0.89} \times 1.4$

$= 9.9\ \text{ms}^{-2}$

7 a $v_{max} = \omega x_o$

$= 2\pi f x_o$

$= 2\pi \times 440 \times (2.5 \times 10^{-3})$

$= 6.9\ \text{ms}^{-1}$ (2 s.f.)

b $a_{max} = \omega^2 x_o$

$= (2\pi f)^2 x_o$

$= (2\pi \times 440)^2 \times 2.5 \times 10^{-3}$

$= 1.9 \times 10^4\ \text{ms}^{-2}$ (2 s.f.)

8 a i Displacement from the equilibrium position

ii Amplitude, or maximum displacement from the equilibrium position

iii Angular frequency of the oscillations

b i $v = \omega x_o \cos\omega t$

ii $\cos\omega t = \sqrt{1 - \sin^2\omega t}$ and $\sin^2\omega t = \frac{x^2}{x_o^2}$

$\therefore v = \omega x_o\sqrt{1 - \frac{x^2}{x_o^2}} = \omega\sqrt{x_o^2 - x^2}$

c i $v_{max} = \omega x_o$

$= 2\pi \times 12 \times 0.25$

$= 19\ \text{ms}^{-1}$ (2 s.f.)

ii $v = \omega\sqrt{x_o^2 - x^2}$

$= \omega\sqrt{x_o^2 - \left(\frac{x_o}{2}\right)^2}$

$= \omega x_o\sqrt{\frac{3}{4}}$

$= 16\ \text{ms}^{-1}$ (2 s.f.)

iii $E_T = E_{K_{MAX}}$ (because when E_K is a maximum $E_P = 0$)

$\therefore E_T = \frac{1}{2}m(\omega x_o)^2 = \frac{1}{2} \times 0.25 \times (2\pi \times 12 \times 0.25)^2 = 44$ J (2 s.f.)

9 a $\omega = 3\ \text{rads}^{-1}$

$f = \frac{\omega}{2\pi}$

b $= \frac{3}{2\pi}$

$= 0.48$ Hz (2 s.f.)

c $x_o = \frac{a_{max}}{\omega^2}$

$= \frac{F_{max}}{m\omega^2}$

$= \frac{2.7 \times 10^{-2}}{0.1 \times 3^2}$

$= 0.03$ m

d $v_{max} = \omega x_o = 3 \times 0.03 = 0.09\ \text{ms}^{-1}$ or $9\ \text{cms}^{-1}$

10 a i $T = 2\pi\sqrt{\frac{l}{g}}$

$= 2\pi \times \sqrt{\frac{22.45}{9.81}}$

$= 9.51$ s (3 s.f.)

ii $T = 2\pi\sqrt{\frac{l}{g}}$

$= 2\pi \times \sqrt{\frac{22.45}{\left(\frac{9.81}{6}\right)}}$

$= 23.3$ s (3 s.f.)

OR

$T = 9.51 \times \sqrt{6} = 23.3$ s

b **i** $T = 2\pi\sqrt{\frac{m}{k}}$

$= 2\pi \times \sqrt{\frac{0.45}{20}}$

$= 0.94$ s (2 s.f.)

ii Since g does not appear in the equation for T, the time period on the moon's surface would be the same; $T = 0.94$ s.

Exercise 12.3

1 **a** At a, the mass is displaced x_o above the equilibrium position. At b, the mass is at the equilibrium position. At c, the mass is displaced x_o below the equilibrium position.

b At a and at c

c At b

d **i** When the mass moves from a to b, the gravitational potential energy transfers to the potential energy in the spring and to the kinetic energy of the moving mass.

ii When the mas moves from b to c, the gravitational potential energy and kinetic energy are transferred to the potential energy in the spring.

iii When the mass moves from c to b, the potential energy in the spring is transferred to the gravitational potential energy and kinetic energy of the moving mass.

iv When the mass moves from b to a, the kinetic energy and potential energy are transferred to the gravitational potential energy of the mass

2 **a** At a, the bob is displaced x_o to one side of the equilibrium position. At b, the bob is at the equilibrium position. At c, the bob is displaced x_o to the other side of the equilibrium position.

b At a and at c

c At b

d **i** When the bob moves from a to b, the gravitational potential energy of the bob and the potential energy in the string are transferred to the kinetic energy of the moving bob.

ii When the mas moves from b to c, the kinetic energy of the bob is transferred to the potential energy in the string and the E_p of the bob.

iii When the mass moves from c to b, the gravitational potential energy of the bob and the potential energy in the string are transferred to the kinetic energy of the moving bob.

iv When the mass moves from b to a, the kinetic energy of the bob is transferred to the potential energy in the string and the gravitational potential energy of the bob.

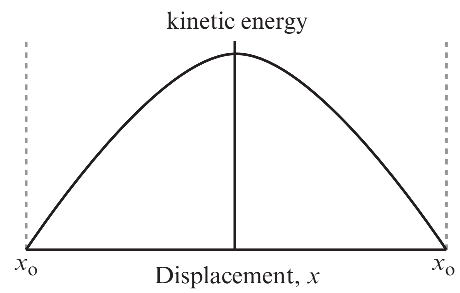
3 **a** **i** At either of the maximum displacements, x_o

ii At the equilibrium point, $x = 0$

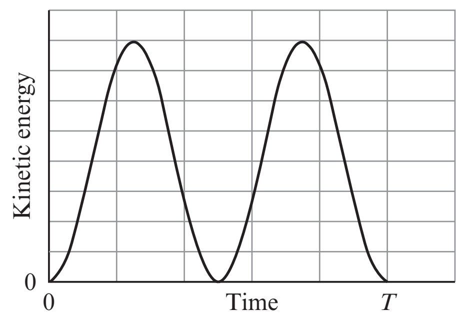
b **i** At either of the maximum displacements, x_o

ii At the equilibrium point, $x = 0$.

c



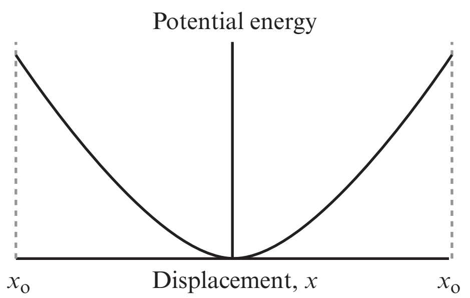
d



e **i** At the equilibrium point, $x = 0$.

ii At either of the maximum displacements, x_o

f



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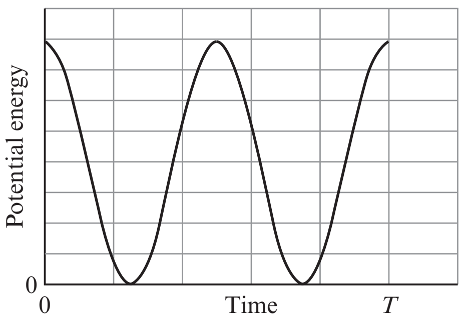


4 a i $E_K = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\omega\sqrt{x_o^2 - x^2}\right)^2 = \frac{1}{2}m\omega^2(x_o^2 - x^2)$

ii $E_{K_{max}}$ occurs when $x = 0$, so $E_{K_{max}} = \frac{1}{2}m\omega^2 x_o^2$.

b i $E_P = \frac{1}{2}kx^2$

ii $E_{P_{max}}$ occurs when $x = x_o$, so $E_{P_{max}} = \frac{1}{2}kx_o^2$.

iii $k = m\omega^2 \therefore E_{P_{max}} = \frac{1}{2}m\omega^2 x_o^2$, which is the same as $E_{K_{max}}$.

c i At the equilibrium point

ii Since $x = 0$, $E_P = 0$.

d i At x_o

ii $E_K = 0$

e Total energy of system = $E_K + E_P$ = constant.

5 a $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.75} = 8.379 = 8.4$ rads^{-1} (2 s.f.)

b $\omega^2 = \frac{k}{m} \Rightarrow k = m\omega^2 = 0.15 \times 8.379^2 = 10.53 = 11$ Nm^{-1} (2 s.f.)

c $E_{K_{max}} = \frac{1}{2}m\omega^2 x_o^2 = \frac{1}{2} \times 0.15 \times 8.379^2 \times 0.05^2 = 1.3 \times 10^{-2}$ J (2 s.f.)

6 a $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi \times \sqrt{\frac{0.45}{15}} = 1.088 = 1.1$ s (2 s.f.)

b $E_T = \frac{1}{2}m\omega^2 x_o^2 = \frac{1}{2}m\left(\frac{2\pi}{T}\right)^2 x_o^2 = \frac{1}{2} \times 0.45 \times \left(\frac{2\pi}{1.088}\right)^2 \times 0.3^2 = 0.676 = 0.68$ J

c $E_K = \frac{1}{2}m\omega^2(x_o^2 - x^2) = \frac{1}{2}m\left(\frac{2\pi}{T}\right)^2 (x_o^2 - x^2) = \frac{1}{2} \times 0.45 \times \left(\frac{2\pi}{1.088}\right)^2 \times (0.3^2 - 0.2^2) = 0.375 = 0.38$ J (2 s.f.)

d $E_P = E_T - E_K = 0.676 - 0.375 = 0.30$ J (2 s.f.)

7 a $f = \frac{1}{2\pi}\sqrt{\frac{g}{l}} = \frac{1}{2\pi}\sqrt{\frac{10}{0.2}} = 1.125 = 1.1$ Hz (2 s.f.)

b Number of oscillations in 1 minute $= \frac{60}{T} = 60 \times f = 60 \times 1.125 = 67.5 = 68$ (2 s.f.)

c $v_{max} = \omega x_o = 2\pi f x_o = 2\pi \times 1.125 \times 0.2 = 1.414 = 1.4$ ms^{-1} (2 s.f.)

d $E_T = \frac{1}{2}m\omega^2 x_o^2 = \frac{1}{2} \times 0.2 \times (2\pi \times 1.125)^2 \times 0.2^2 = 0.2$ J

8 a $\omega = 2\pi f = 2\pi \times \frac{24}{60} = 2.5$ rads s^{-1} (2 s.f.)

b $v_{max} = \omega x_o = 2.5 \times 0.04 = 0.1$ ms^{-1}

c i $E_T = \frac{1}{2}m\omega^2 x_o^2 = \frac{1}{2} \times 0.25 \times 2.5^2 \times 0.04^2 = 1.25$ mJ $= 1.3$ mJ (2 s.f.)

ii Halving the amplitude will quarter the total energy, so $E_T = \frac{1}{4} \times 1.25 = 0.3$ mJ.

9 $E_T = \frac{1}{2}m\omega^2 x_o^2 \Rightarrow x_o = \sqrt{\frac{E_T}{\frac{1}{2}m\omega^2}} = \sqrt{\frac{2.0}{\frac{1}{2} \times 1.2 \times (2\pi \times 0.4)^2}} = 0.73$ m (2 s.f.)

10 a i $E_T \propto m$

ii $E_T \propto f^2$

iii $E_T \propto \frac{1}{T^2}$

iv $E_T \propto x_o^2$

b i $E_T \propto m$

ii $E_T \propto f^2$

iii $E_T \propto \frac{1}{T^2}$

iv $E_T \propto x_o^2$

Students should notice that the relationships above apply to **all** systems undergoing SHM.

Exam-style questions

Multiple-choice questions

1 B [1]

2 A [1]

3 C [1]

4 D [1]

5 D [1]

6 D [1]

7 C [1]

8 C [1]

9 A [1]

Short-answer questions

10 a 20 cm [1]

b $\omega = \frac{2\pi}{T} = \frac{2\pi}{\left(\frac{3.0}{10}\right)} = 20.95 = 21$ radss^{-1} [2]

c $a_{max} = \omega^2 x_o = 20.95^2 \times 0.2 = 88$ ms^{-2} [2]

11 a $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.8} = 7.855 = 7.9$ radss^{-1} (2 s.f.) [1]

b $v_{max} = \omega x_o = 7.855 \times 0.06 = 0.47$ ms^{-1} [2]

c $E_{K_{max}} = \frac{1}{2}mv_{max}{}^2 = \frac{1}{2} \times 1.25 \times 0.47^2 =$ 0.14 J (2 s.f.) [2]

12 a Acceleration must be proportional [1] and in the opposite direction [1] to displacement from the equilibrium point.

b i Time period will not change. (It is independent of the amplitude.) [1]

ii $v_{max} \propto x_o$, so the maximum [1] velocity will decrease linearly with time. [1]

13 a The graph shows that

- acceleration is proportional to displacement and [1]
- gradient of line is negative (or equivalent statement). [1]

b ω^2 = magnitude of slope of graph = $\frac{6}{0.3} = 20 \Rightarrow \omega = \sqrt{20} = 4.5$ radss^{-1} (2 s.f.) [1]

c $v_{max} = \omega x_o = 4.5 \times 0.3 = 1.4$ ms^{-1} [1]

d $E_T = \frac{1}{2}m\omega^2 x_o^2 = \frac{1}{2} \times 0.3 \times 20 \times 0.3^2 =$ 0.27 J [1]

14 a $f = \frac{10\pi}{2\pi} = 5$ Hz [1]

b i $x = 0.25\cos(10\pi \times 0.3) = -0.25$ m, that is 0.25 m to the left of the equilibrium position. [1]

ii $v = 0$ [1]

iii $t = 0.05$ s, 0.15 s, 0.25 s, 0.35 s, . . . [1]

iv $v_{max} = \omega x_o = 10\pi \times 0.25 = 7.9$ ms^{-1} (2 s.f.) [1]

15 a $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi \times \sqrt{\frac{1.20}{9.81}} = 2.2$ s (2 s.f.) [2]

b $x = x_o\cos\left(\frac{2\pi}{T}t\right) = 15 \times \cos\left(\frac{2\pi}{2.2} \times 1.5\right) =$ -6.2 cm from the equilibrium position

So, the mass is 6.2 cm to the left of the equilibrium position. [1]

c $v = \frac{2\pi}{T}\sqrt{x_o^2 - x^2} = \frac{2\pi}{2.2} \times \sqrt{15^2 - 6.2^2} =$ 39 cms^{-1} [2]

OR

$v = -\frac{2\pi}{T}x_o\sin\left(\frac{2\pi}{T}t\right) = -\frac{2\pi}{2.2} \times 15 \times \sin\left(\frac{2\pi}{2.2} \times 1.5\right) = 39$ cms^{-1} [2]

16 a $T = \frac{2\pi}{\omega} = \frac{2\pi}{25} = 0.25$ s [1]

b $E_T = \frac{1}{2}m\omega^2 x_o^2 = \frac{1}{2} \times 0.05 \times 25^2 \times 0.05^2$ $= 0.039 \approx 40$ mJ [2]

c $0.01 = \frac{1}{2}m\omega^2\left(x_o^2 - x^2\right) \Rightarrow$

$x = \sqrt{0.05^2 - \frac{2 \times 0.01}{0.05 \times 25^2}} = 4.3$ cm

So, the mass is 4.3 cm from the equilibrium position. [2]

17 a $f = \left(\frac{1}{2\pi}\right)\left(\sqrt{\frac{k}{m}}\right) = \left(\frac{1}{2\pi}\right) \times \left(\sqrt{\frac{20}{0.5}}\right) =$ 1.0 HZ [1]

b $v = \omega\sqrt{x_o^2 - x^2} = 2\pi \times 1 \times \sqrt{0.25^2 - 0.1^2}$ $= 1.4$ ms^{-1} (2 s.f.) [2]

c i Zero. The angular frequency is independent of the amplitude. [1]

ii $v_{max} \propto x_o$. So the maximum speed will change by a factor of a half. [1]

Chapter 13

Exercise 13.1

1 **a** **i** Wavelength is the distance from one peak of a wave to the next peak (or trough to trough).

ii Frequency is the number of waves that pass a given point in 1 s.

iii Time period is the amount of time it takes for one complete wave to pass a given point.

iv Amplitude is the maximum displacement from the equilibrium position of whatever is oscillating in the wave.

v Wave speed is the distance that one complete wave travels in 1 s.

b $f = \frac{1}{T}$

c Amplitude

d A disturbance that transfers energy and momentum through oscillations of the particles of a medium.

2 **a** **i** Amplitude and period—and from the period, frequency.

ii Wavelength

b **i** Amplitude and wavelength

ii Frequency (or time period)

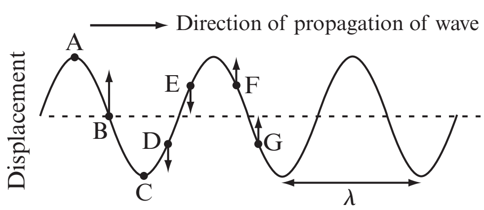
3 **a** **i** A

ii C

b **i** B

ii A and C

c and **d**



4 **a** A to C or B to D or C to E

b Frequency, f, is the number of waves that pass a point in 1 s. Time period, T, is the time it takes for one complete wave to pass a point.

c **i** λ

ii T

iii speed, $v = \frac{\text{distance travelled}}{\text{time taken}} = \frac{\lambda}{T} = f\lambda$

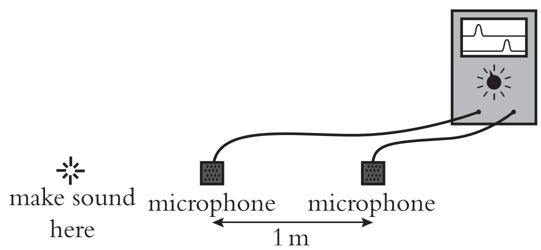
5 **a** **i** $x_o = 2.5$ cm

ii $\lambda = 10$ cm

b **i** It will take a time, $T = \frac{1}{f} = \frac{1}{2}$ s.

ii $v = f\lambda = 2 \times 10 = 20\ \text{cms}^{-1}$

6 Equipment: an oscilloscope with a dual trace and two microphones, as in Figure 13.14.



Place the two microphones a set distance apart, say, 1.0 m. Connect each microphone to the oscilloscope. When each microphone receives a sound, the oscilloscope trace will show a small blip.

Measurements: distance between the two microphones, time between the two blips on the oscilloscope screen

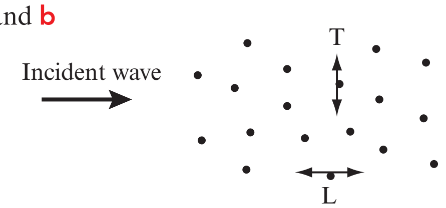
speed of sound in air =

$$\frac{\text{distance between two microphones}}{\text{time interval between the two blips on the oscilloscope}}$$

% uncertainty in speed = % uncertainty in distance + % uncertainty in time

Exercise 13.2

1 a and **b**

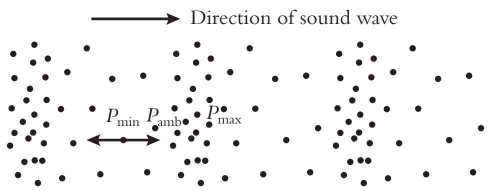


c In a transverse wave, oscillations occur perpendicularly to the direction of travel of the wave. In a longitudinal wave, oscillations occur along the same direction (and its opposite) as that of the wave.

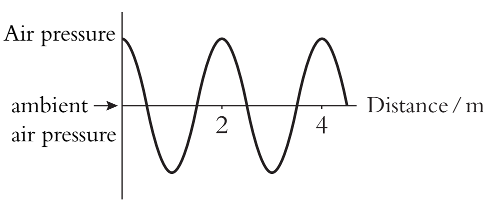
d i Ripples on some water, electromagnetic waves, waves on a string, some seismic waves

ii Sound waves, some seismic waves, compression waves on a Slinky

2 a



b and **c**



d $f = \frac{v}{\lambda} = \frac{330}{2} = 165$ Hz

3 a No.

b i 2.5 mm

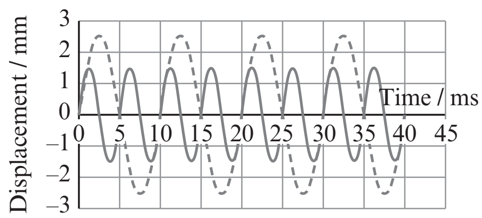
ii From the figure, $T = 10$ ms, so $f = \frac{1}{T} = \frac{1}{10 \times 10^{-3}} = 100$ Hz.

c $v = f\lambda = 100 \times 3.3 = 330$ ms^{-1}

d i The gradient of the graph gives the speed of the molecule. At $t = 2.5$ ms, the gradient os the graph is zero, so the speed of the molecule is zero.

ii The gradient of the graph is a maximum at $t = 10$ ms, so the speed of the molecule must be a maximum.

e



(Exactly twice as many waves and any amplitude that is smaller.)

4 a It will be compressed.

b The compressed spring will exert a force on the second ball, which will make the second ball accelerate towards the right.

c The moving second ball will compress the second spring.

d Each time a spring is compressed it will exert a force on the next ball, making the next ball accelerate to the right.

e The first spring will now be extended (beyond its uncompressed length). This will make the spring exert a force on the second ball to make the second ball accelerate towards the left. The leftwards moving second ball will now extend the second spring, causing the second spring to accelerate the third ball towards the left, and so on.

f The original pushing of the first ball introduced kinetic energy (and hence momentum in the left-to-right direction). As each successive ball moves, the kinetic energy and momentum is moving in the left-to-right direction. This is what a wave does; it transfers energy and momentum.

g It is a pulse of a longitudinal wave. (Because the motion of the balls is in the same direction and opposite direction to the transfer of energy along the line of balls.)

h i The spring constant of the springs and the mass of the balls

ii For the spring constant of the springs, for a given initial displacement of the first ball, the spring constant will determine how much force the compressed spring exerts on the second ball, and so on along the line. A larger spring constant means that the springs will exert larger forces on the balls. This will make the balls accelerate more, causing the speed at which the pulse travels along the line to be larger.

For the mass of the balls, a larger mass means that the balls will accelerate less for a given force from the springs. Thus, a larger mass will cause the speed of the pulse along the line to be smaller.

The springs are representing the forces that exist between atoms and molecules through which the longitudinal wave (sound) is travelling. In a steel bar these intermolecular forces are very large (they are related to the Young modulus of the material, a kind of measure of how stiff the material is), so much so that they dominate any effect of atomic mass. In the air, these forces are related to the bulk modulus of the material—a measure of how squashy the material is—and they are much smaller values (by a factor of more than 10^6). This means that the sound waves travel much faster through the steel than through the air—in fact, about 10 times faster.

5 a The movement of the first ball upwards will stretch the spring between the first ball and the second ball. This stretched spring will exert a force on the second ball, pulling it upwards. In turn, this then stretches the next spring, which pulls the third ball upwards, and so on.

When the first ball is then pulled downwards, the spring between the first ball and the second ball is once again stretched, causing it to exert a force downwards on the second ball. This accelerates the second ball downwards and so on.

b This is a model for a pulse of a transverse wave, because the motion of the balls is perpendicular to the transfer of energy and momentum.

c Yes.

6 a i $T = \frac{1}{f} = \frac{1}{100} = 0.01$ s

ii 0

b $v_{\text{average}} = \frac{\text{net displacement}}{\text{time}} = \frac{0}{1} = 0\ \text{ms}^{-1}$

c distance moved = 100 × 4 × 5 cm = 2000 cm = 20 m

d $\text{speed}_{\text{average}} = \frac{\text{distance moved}}{\text{time taken}} = \frac{20\ \text{m}}{1\ \text{s}}$ $= 20\ \text{ms}^{-1}$

e Velocity is a vector; speed is a scalar. The point on the string doesn't actually go anywhere; it's net *displacement* is zero, so its average velocity is zero. It does *move* over a distance 100 × 4 × 5 cm, so its average speed is not zero.

f There is no information about the wavelength of the wave, so we cannot use the equation $v = f\lambda$ to find v.

7 a Echolocation is a way in which animals can use the echo of a sound wave to detect objects and their distance away. It is used by toothed whales, porpoises and bats.

b i $\lambda = \frac{v}{f} = \frac{1.4 \times 10^3}{120 \times 10^3} = 0.01\ \text{m} = 1\ \text{cm}$ (2 s.f.)

ii distance away = speed × time = $1.4 \times 10^3 \times \frac{1}{2} \times 0.3 = 210$ m

iii Yes. The squid is larger than the wavelength of the sound waves, so the whale should be able to detect the squid.

Exercise 13.3

1 a An electromagnetic wave does not require a material medium for it to travel through; it can travel through a vacuum.

b An electromagnetic wave is an oscillating electric field, accompanied by an oscillating magnetic field (oscillating perpendicularly to it) moving through space.

c They all travel at the same speed; the speed of light ($3.0 \times 10^8\ \text{ms}^{-1}$ in a vacuum).

d Gamma rays, X-rays, ultraviolet light, visible light, infrared light, microwaves, radio waves

2 a Radioactive nuclei: when an unstable nucleus changes its nuclear energy level to a lower level, it will emit a gamma ray photon, because nuclear energy levels are in the order of 100 MeV or more.

Particle pair annihilation: when a sub-atomic particle and its anti-particle interact they will annihilate producing a pair of gamma rays.

Decay of pi- mesons: neutral pi- mesons (π^{o}) will decay into two gamma ray photons.

b Gamma rays contain large amounts of energy. When gamma rays interact with matter, they will lose some or all of this energy, causing ionisation of atoms. If such atoms are part of human tissue, then cells can become mutated or destroyed. This can harm human organs.

c Less than 10 pm

d i Cancerous cells can be irradiated with highly focused gamma rays, which can destroy the cells. Also, some medical equipment is made out of plastic and cannot be sterilised using superheated steam. So, such equipment is irradiated with gamma rays, which kill any bacteria present. Gamma rays are also used in positron emission tomography (PET) scans, which can provide vital information about ill-performing biological processes.

ii Packaged food can be irradiated with gamma rays to kill any fungi or bacteria present. This will lengthen the shelf life of the packaged food. (This is particularly useful in the worldwide exportation of food.)

iii Airborne and ground-based gamma ray detectors can search for the presence of trace radioactive materials, such as thorium or uranium, under the surface of the Earth. This can help with geological mapping, mineral exploitation and environmental contamination.

iv Some astronomical objects emit bursts of gamma rays. These astronomical objects can be studied by detecting and analysing the gamma rays emitted from them.

3 a X-rays are produced when highly accelerated electrons strike a dense target. The electrons lose their kinetic energy and some of this is transformed into an X-ray.

b About 1 pm to 1 nm

c Some uses are

- medical uses: for examining suspected broken bones, checking for cancerous growths with mammograms, chest x-rays for detecting pneumonia or other chest infections, dental examinations;
- industrial uses: non-destructive testing, exploring for cracks or breaks in pipes, quality control of manufactured items; and
- security: airports and other secure buildings use soft x-rays to check for illegal metal objects (like guns or other potentially lethal weapons).

d X-rays contain large amounts of energy. Some of this energy will be absorbed by our bodies, which may cause mutation or destruction of vital cells. This can lead to cancer.

4 a Excited electrons in atoms that fall from their high energy levels to (usually) the ground state will emit radiation that has enough energy (more than about 3 eV) and a low enough wavelength to be ultraviolet light.

b 1 nm to 400 nm

c Eyes: too much ultraviolet radiation can be harmful to our eyes, causing cataracts and potential blindness. (This is why we are encouraged to wear sunglasses when the sunshine is very strong.)

Skin: Too much absorption of ultraviolet by the skin can cause: sunburn, premature ageing, excessive wrinkle growth, enhanced liver spot formation and cancer.

d Vitamin D production: absorption of ultraviolet light stimulates the skin to produce vitamin D (a vital vitamin for healthy bodies). Ultraviolet radiation is also used in sun-tanning booths.

Phototherapy: treatment of acne, jaundice, eczema and psoriasis using ultraviolet lamps

Illumination: fluorescent lamps use ultraviolet light, backlighting in aeroplanes uses ultraviolet light, emergency exit signs

Disinfecting: irradiating air, water or surfaces of objects can kill bacteria and fungal presence, which has a disinfectant, germicidal or sterilisation effect

Ultraviolet curing: industrial adhesives, varnishes, inks and dental implant material (i.e. fillings) can be set when irradiated with ultraviolet. This is particularly useful in the automotive and aeronautical industries.

5 **a** Medium energy level drops by bound atomic electrons (1 or 2 eV) will produce radiation that is visible.

b The first letters of each word represent the seven major colours in the visible spectrum: Red, Orange, Yellow, Green, Blue, Indigo and Violet.

c Red: 650 nm, Orange: 600 nm, Yellow: 580 nm, Green: 540 nm, Blue: 470 nm, Indigo: 450 nm, Violet: 420 nm. (All these could vary by about ±20 nm or so.)

d The greenish side of yellow. This is roughly in the centre of the visible spectrum. It's also not far from the wavelength at which the Sun's radiation is most pronounced (red-orange).

6 **a** Small falls in electron energy levels produce infrared radiation.

b 1 μm to 1 mm

c Electrical heating, optical fibres, remote control units, security systems, thermal imaging

7 **a** In magnetron vacuum tubes, bunched groups of electrons are made to oscillate in cavities by electric and magnetic fields. This oscillatory motion causes the emission of microwaves.

b 1 mm to about 10 cm

c Perhaps most commonly, microwaves are used in cooking food. They are also used radar and speed guns, telecommunications and mobile phones.

8 **a** Oscillating electric charges, such as electrons, create radio waves.
So, an alternating current will produce radio waves.

b 10 cm to over 1000 km.

c Most commonly they are used in communication systems: radios, televisions and cellular phones. They are also used extensively in the scientific exploration of the upper atmosphere and in observations of radio sources in space by radio telescopes.

d Radio waves have long wavelengths. This enables them to be diffracted easily around large obstacles (like mountains or large buildings.)

Exercise 13.4

1 **a** $p = mv$

b **i** Diffraction

ii Less. Their paths will spread out because they collide slightly with the edges of the hole. Those particles that pass through the hole without touching the edges of the hole will be undeflected. More momentum suggests that if they collided slightly with the edges of the hole, their paths would be altered by only a small amount, and so they would spread out only a little.

iii Less. A bigger hole means that a larger proportion of the particles will pass through the hole undeflected, so fewer particles will have their paths changed.

c **i** In diffraction, waves with a larger wavelength diffract more than waves with a smaller wavelength. This suggests that $\lambda \propto \frac{1}{p}$.

ii The amount of diffraction, or spreading out of the particles, is proportional to the wavelength of the particles, and so inversely proportional to their momenta. The amount of diffraction is inversely proportional to the size of the hole. So, together these suggest that amount of spreading out $\propto \frac{1}{pb}$, where b is the 'size' of the hole.

d An electron. Its momentum will be the smallest (but not zero), giving it the largest wavelength. The stationary atom has no momentum (and so cannot exhibit wave-like properties).

e It tells us the probability of finding the particle at any given position.

2 a Accelerating large masses, such as massive stars, galaxies or black holes

b The curvature of spacetime. One can think of this as space itself stretching and compressing as the gravitational wave passes through.

c Transverse

d They travel at the speed of light, c.

e The amplitude of the oscillations of spacetime is very, very small (perhaps only one part in 10^{21}). This makes it very difficult to detect changes. Observations by LIGO, for example, have required extremely sensitive, expertly crafted laser detectors operating in different places separated by large distances. (There are other reasons too, but these delve into the very quantum nature of matter. Such physics is too advanced for an IB Physics course!)

Exam-style questions

Multiple-choice questions

1 A **[1]**

2 C **[1]**

3 D **[1]**

4 A **[1]**

5 C **[1]**

6 B **[1]**

7 C **[1]**

8 B **[1]**

Short-answer questions

9 a In water, the molecules are much closer together than in air. The energy of an oscillation is transmitted quickly from one molecule to the next. Air molecules are much farther apart, so an oscillation is transmitted much less quickly. **[2]**

b i $\lambda = \frac{v}{f} = \frac{1.5 \times 10^3}{200 \times 10^3}$
$= 7.5 \times 10^{-3}$ m (or 7.5 mm) **[1]**

ii $\lambda = \frac{v}{f} = \frac{330}{200 \times 10^3}$
$= 1.65 \times 10^{-3}$ m (or 1.65 mm) **[1]**

c No. 200 kHz is above the range of human ears, so it cannot be heard. **[1]**

10 a 0.6 m **[1]**

b 1.0 m **[1]**

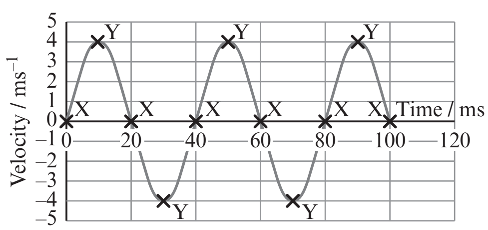
c The first peak has moved 0.25 m in a time of 100 ms, so $v = \frac{s}{t} = \frac{0.25}{0.1} = 2.5\ \text{ms}^{-1}$. **[1]**

d $f = \frac{v}{\lambda} = \frac{2.5}{1.0} = 2.5$ Hz **[1]**

e $T = \frac{1}{f} = \frac{1}{2.5} = 0.4$ s **[1]**

11 a $f = \frac{1}{T} = \frac{1}{40 \times 10^{-3}} = 25$ Hz **[2]**

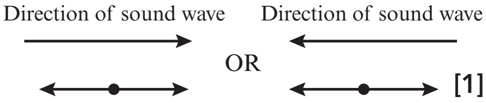
b i and **ii**



Any of the positions shown. **[2]**

c The area under the curve from $t = 0$ ms to $t = 20$ ms represents the distance travelled from a peak to a trough, so this will be twice the amplitude of the oscillations. **[1]**

12 a



[1]

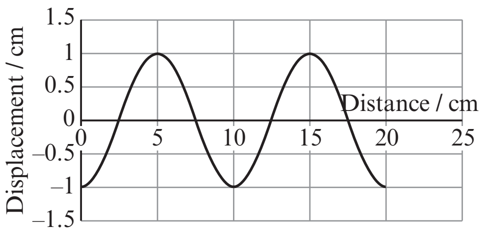
b i $f = \frac{1}{T} = \frac{1}{5 \times 10^{-3}} = 200$ Hz **[1]**

ii $v = f\lambda = 200 \times 1.65 = 330$ ms^{-1} **[1]**

c i Louder ⇒ larger amplitude ⇒ greater distance travelled in the same time ⇒ greater average speed. **[1]**

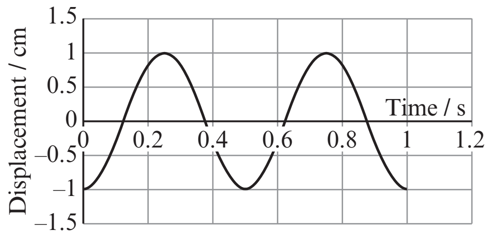
ii No change. Same frequency ⇒ same time period. **[1]**

13 a



- Correct axes (with units) **[1]**
- Correct scales **[1]**
- Negative cosine curve—at least two complete waves **[1]**

b



- Correct x-axis, with units and scale **[1]**
- Negative cosine curve **[1]**

14 a $v = f\lambda = 2.0 \times 2 \times 0.6 = 2.4$ ms^{-1} **[2]**

b 2.4 ms^{-1} **[1]**

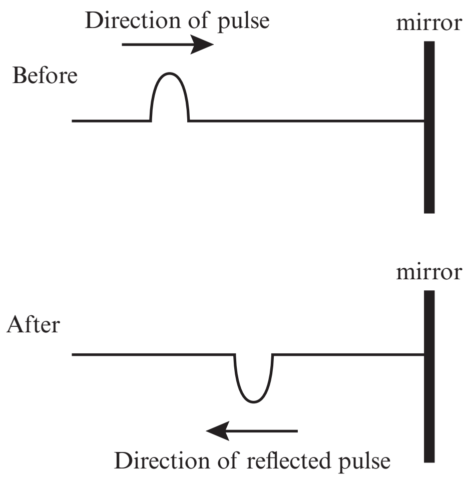
c i The speed will decrease. **[1]**

ii The wavelength will also decrease. **[1]**

Chapter 14

Exercise 14.1

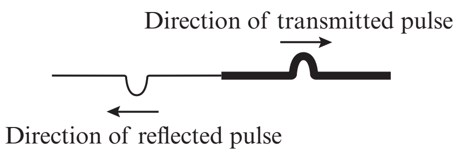
1 a



b The pulse has been reflected.

c There has been a phase change of π radians (or 180°).

2 a i



ii The transmitted pulse continues onwards with no phase change (and a small decrease in amplitude), but the reflected pulse undergoes a phase change of 180°, or π radians.

b i The transmitted pulse will move slower than the reflected pulse.

ii The speed of the pulse depends on the mass per unit length of the medium through which it is travelling. (In fact, $v \propto \sqrt{\frac{1}{\text{mass per unit length}}}$.) A greater mass per unit length means a slower speed of the pulse.

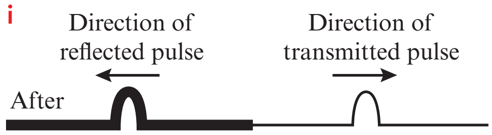
c i Same phase—no change of phase

ii Out of phase—a phase change of π radians (or 180°)

d i Smaller amplitude than the original pulse

ii Smaller amplitude than the original pulse

3 a i



ii both the transmitted and the reflected pulse do not undergo a phase change.

b i The transmitted pulse will move faster than the reflected pulse.

ii The speed of the pulse depends on the mass per unit length of the medium through which it is travelling. (In fact, $v \propto \sqrt{\frac{1}{\text{mass per unit length}}}$.) A smaller mass per unit length means a faster speed of the pulse.

c i Same phase—no change of phase

ii Same phase—no change of phase

d i Smaller amplitude than the original pulse

ii Smaller amplitude than the original pulse

4 a i A ray shows the direction in which waves are travelling.

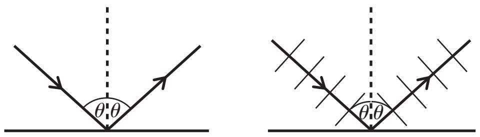
ii A wavefront is a line, or curve, on which all parts of a wave are of the same phase. The wavefront is perpendicular to the ray.

iii The normal is a constructional line that is perpendicular to the surface between two media at the point where a ray meets the surface.

iv The angle of incidence is the angle between the incident ray and the normal

v The angle of reflection is the angle between the reflected ray and the normal.

b



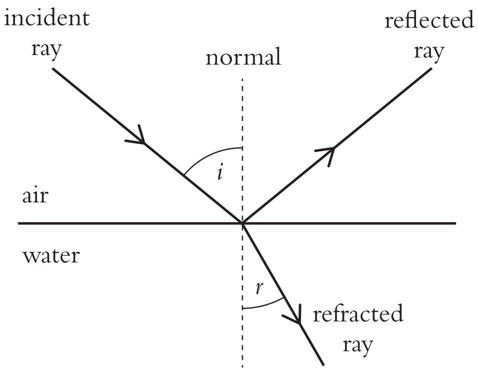
c The two angles are the same.

d Rays and wavefronts are perpendicular to each other.

5 a i Refraction is the change in wave speed that occurs when a wave moves from one medium into a different medium. (Note that although there may be a change in direction, the change in direction is not a definiton of refraction.)

ii The refractive index, n, is the ratio of the speed of electromagnetic waves in a vacuum to the speed of the same wave in a different medium; $n = \frac{v_{\text{vacuum}}}{v_{\text{medium}}} = \frac{c}{v}$.

b



c $\frac{\sin i}{\sin r} = n$, where n is the refractive index of the water.

d Snell's law

6 a $r = \sin^{-1}\left(\frac{\sin i}{n}\right) = \sin^{-1}\left(\frac{\sin 40°}{1.5}\right) = 25°$ (2 s.f.)

b $v_{\text{glass}} = \frac{c}{1.5} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8\ \text{ms}^{-1}$

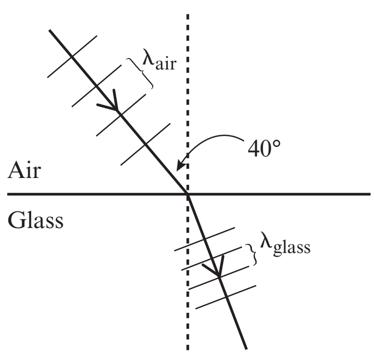
c $\lambda_{\text{air}} = \frac{c}{f} = \frac{3 \times 10^8}{6 \times 10^{14}} = 5 \times 10^{-7}\ \text{m}$

d $\lambda_{\text{glass}} = \frac{v_{\text{glass}}}{f} = \frac{2 \times 10^8}{6 \times 10^{14}} = 3.3 \times 10^{-7}\ \text{m}$

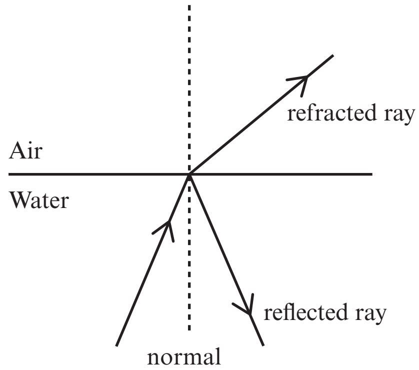
e Green

f The same (green). This is an important idea: it is the frequency of an electromagnetic wave that dictates what colour we perceive, not the wavelength.

g and **h**



7 a



b It increases until it reaches its maximum value of 90°.

c From Snell's law, the critical angle will be $\theta_c = \sin^{-1}\left(\frac{1}{n}\right)$,

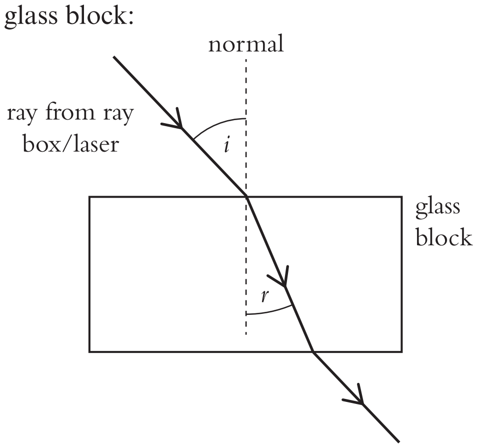
So $\theta_c = \sin^{-1}\left(\frac{1}{1.33}\right) = 49°$ (2 s.f.)

d This is called the critical angle.

e At angles greater than θ_c, the ray undergoes total internal reflection.

8 Equipment: Light box or laser to produce a thin beam of light, rectangular glass block, protractor, pencil, white paper.

Method: Place the glass block on white paper. Set up the ray box or laser so that a thin beam of light is incident on one side of the glass block:



Draw around the glass block. Make marks where the incident beam approaches and is incident on the block, and where the beam leaves the opposite side of the glass block.

Remove the glass block. Complete the paths of the incident ray and the refracted ray through the glass block. Draw in the normal where the ray is incident on the glass block. Now measure, using the protractor, the angle of incidence, r, and the angle of refraction, r.

Repeat this several times to get a range of angles of incidence.

Record values of i and r in a suitable table.

Process the values of i and r into two more columns, sin i and sin r.

Plot a graph of sin i (on the y-axis) against sin r (on the x-axis).

Obtain a best-fit line. It should be a straight line that passes through the origin, with gradient $= \frac{\sin i}{\sin r}$ = refractive index of the glass block.

9 a Speed of light in the cladding is faster than the speed of light in the glass fibre.

b i $\frac{c}{v_{\text{glass fibre}}} = n \Rightarrow v_{\text{glass fibre}} = \frac{c}{n} = \frac{3.0 \times 10^8}{1.47}$
$= 2.04 \times 10^8 \text{ ms}^{-1}$

ii $\frac{c}{v_{\text{cladding}}} = n \Rightarrow v_{\text{cladding}} = \frac{c}{n} = \frac{3.0 \times 10^8}{1.45}$
$= 2.07 \times 10^8 \text{ ms}^{-1}$

c $\frac{1}{\sin \theta_c} = \frac{v_{\text{cladding}}}{v_{\text{glass fibre}}} \Rightarrow \theta_c = \sin^{-1}\left(\frac{v_{\text{glass fibre}}}{v_{\text{cladding}}}\right)$
$= \sin^{-1}\left(\frac{2.04}{2.07}\right) = 80.2° \text{ (3 s.f.)}$

d This is the critical angle

e Actual path length $= \frac{1.0 \text{ km}}{\cos 9.8°} = 1015$ m, so time taken $= \frac{s}{v} = \frac{1015}{2.04 \times 10^8} = 4.98 \times 10^{-6}$ s.

f $\frac{1000}{2.04 \times 10^8} = 4.90 \times 10^{-6}$, so time taken is 8.0×10^{-8} s longer.

g Advantages over copper cables might include

- almost no signal loss,
- greater bandwidth (i.e. can carry more information),
- faster transfer speed,
- thinner,
    - less prone to damage and breaking,
    - cables can be longer and so transmit information over larger distances,
    - more reliable and less prone to electromagnetic interference and
    - cheaper long term than copper cables.

10 a i $\lambda = \frac{v}{f} = \frac{6000}{60} = 100$ m

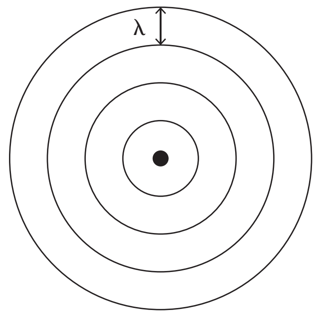
ii $\frac{\sin i}{\sin r} = \frac{v_X}{v_Y} \Rightarrow r = \sin^{-1}\left(\frac{v_Y}{v_X} \times \sin i\right) =$
$\sin^{-1}\left(\frac{4000}{6000} \times \sin 45°\right) = 28° \text{ (2 s.f.)}$

b At the layer Y/Z boundary, $\frac{\sin 28°}{\sin r} = \frac{4000}{9000} \Rightarrow r = \sin^{-1}\left(\frac{9000}{4000} \times 0.47\right) =$
$\sin^{-1}(1.06)$,

which is not possible. So the compression waves will undergo total internal reflection at the layer Y/layer Z boundary and therefore not be transmitted into layer Z.

Exercise 14.2

1 a and d



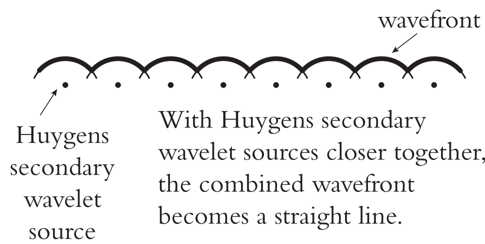
b They are all travelling radially outwards.

c All the wavefronts are circular; they have all travelled the same distance in the same amount of time; hence, they are travelling at the same speed.

d See answer to **part a**.

2 a Such sources are called Huygens secondary wavelet sources.

b



3 a *In phase* means that each source produces a peak (or a trough) at the same time.

b They will arrive in phase because they have both travelled the same distance (or path length) at the same speed, having begun in phase.

c Constructive interference

d The amplitude would be 2A.

4 **a** They will arrive out of phase, because they have both travelled the same distance (or path length) at the same speed, having begun out of phase.

b Destructive interference

c The amplitude would be zero.

5 **a** Path difference

b **i** Constructive interference

ii Constructive interference

iii Constructive interference

c **i** Destructive interference

ii Destructive interference

iii Destructive interference

6 **a** Using Pythagoras,

path difference = YP − XP = $\sqrt{(XP^2 + XY^2)}$ − XP = $\sqrt{(4^2 + 3^2)} - 4 = 1.0 = \lambda$.

So, there would be constructive interference.

b Using Pythagoras,

path difference = YP − XP = $\sqrt{(XP^2 + XY^2)}$ − XP = $\sqrt{(12^2 + 5^2)} - 12 = 1.0 = \lambda$.

So, there would be constructive interference.

c Using Pythagoras,

path difference = YP − XP = $\sqrt{(XP^2 + XY^2)}$ − XP = $\sqrt{(0.75^2 + 1^2)} - 0.75 = 0.5 = \frac{1}{2}\lambda$.

So, there would be destructive interference.

d Using Pythagoras,

path difference = YP – XP = $\sqrt{(XP^2 + XY^2)}$ − XP = $\sqrt{(6.5^2 + 2.6^2)} - 6.5 = 0.5 = \frac{1}{2}\lambda$.

So, there would be destructive interference.

e Using Pythagoras,

path difference = YP – XP = $\sqrt{(XP^2 + XY^2)}$ − XP = $\sqrt{(1.95^2 + 4.0^2)} - 1.95 = 2.5 = \frac{5}{2}\lambda$.

So there would be destructive interference.

7 **a** She will hear the music become louder until a maximum, then quieter to a minimum, then louder and so on.

b If she sits equidistant from the two speakers, the path difference of waves from the two speakers is zero, so constructive interference will occur, making the sound loud.

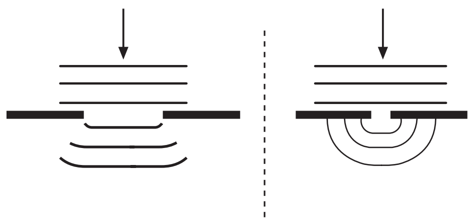
c If she moves a small distance sideways, the path difference will no longer be zero. As the path difference approaches half a wavelength the sound will become quieter and quieter. Since the loudenss of the sound is proportional to the square of the amplitude of the superposed waves, any reduction in amplitude results in a significant reduction in intensity, or loudness.

d With the two speakers closer together, a small sideways movement of the fan does not produce such a large change in path difference. So the waves continue to superpose constructively (or nearly so as she moves) over a larger sideways distance.

e Probably not as good an idea as putting the two speakers close together. Closer speakers give a larger lateral range over which waves can superpose constructively and so make the listening better.

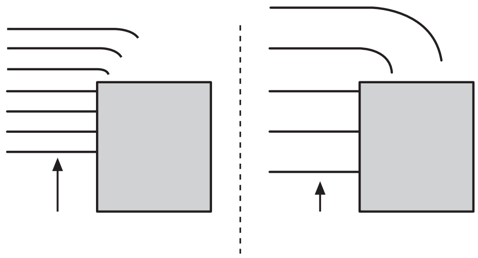
Exercise 14.3

1 **a**



b The amount of diffraction depends on the size of the aperture. Small apertures cause more diffraction than large apertures.

2 a



b Longer wavelengths diffract more than shorter wavelengths.

3 a At B, waves from the two slits are arriving in phase. They add together constructively and so produce a wave with a large amplitude, hence a bright spot.

b At D, waves from the two slits are arriving out of phase. They add together destructively and so produce a wave with little or no amplitude, hence a dark spot.

c At C, you would expect to see a bright region. This is because the waves from the two slits have to travel exactly the same distance to get to C. If these waves began in phase, they will still be in phase when they get to C and will add together constructively to produce a bright region.

d Red light has a longer wavelength than green light. So, in order for the path difference between the two rays to satisfy the same criteria for constructive and destructive interference, B, C and D will be farther apart, although the postion of C, directly opposite the two slits will not change.

4 a The distance is given by YZ.

b Since XYZ is a right-angled triangle, YZ = $d \sin \theta$.

c λ

d $\lambda = d \sin \theta$

e $2\lambda = d \sin \theta_2$

f $n\lambda = d \sin \theta_n$

5 a Triangles XYZ, XCP and YCP are similar triangles. $\frac{s}{D} = \tan \theta \approx \sin \theta$ (since θ is a small angle).

So, $s = D \sin \theta$.

b $\sin \theta = \frac{\lambda}{d} = \frac{s}{D} \Rightarrow s = \frac{\lambda D}{d}$

c $s_n = \frac{n\lambda D}{d}$

d $s_{n+1} = \frac{(n+1)\lambda D}{d}$

e $s = (s_{n+1}) - s_n = \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d} = \frac{\lambda D}{d}$

6 a $s = \frac{\lambda D}{d} = \frac{450 \times 10^{-9} \times 8}{0.15 \times 10^{-3}} = 2.4$ cm

b Since $s \propto \lambda$, if the new λ is 1.5 times larger, then the separation of the maxima will be 1.5 times larger.

So, $s_{new} = 1.5 \times 2.4 = 3.6$ cm.

c $\frac{\lambda}{d} = \frac{450 \times 10^{-9}}{0.15 \times 10^{-3}} = 3 \times 10^{-3}$

Since the maximum value of $\sin \theta$ is 1, $n = \frac{1}{3 \times 10^{-3}} = 333$.

So, there would be $2 \times 333 + 1 = 667$ maxima. (Assuming you could see them all!)

7 a At the central maximum the path difference is zero. This is true for all wavelengths. So all wavelengths present in the white light will interfere constructively, making the central maximum white.

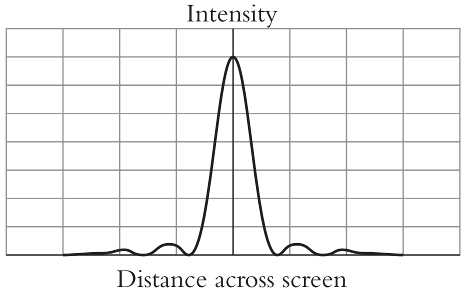
b For all other maxima either side of the central maximum, the path difference condition for constructive interference depends on the wavelength. So, light of different wavelengths will interfere constructively at slightly different angles, making the maxima on the interference pattern coloured (as a spectrum of colours).

c Red light has a longer wavelength than blue light, so the angles at which red light will interfere constructively will be larger than those for blue light.

d $s = \frac{\lambda D}{d} = \frac{500 \times 10^{-9} \times 5}{0.2 \times 10^{-3}} = 1.25$ cm.

Exercise 14.4

1 a



b **i** Smaller separation of the maxima

ii Larger separation of the maxima

2 **a** violet

b The first minimum occurs when $\sin\theta = \frac{\lambda}{b} = \frac{400 \times 10^{-9}}{5 \times 10^{-6}} \Rightarrow \theta = 8.0 \times 10^{-2}$ radians or 4.6°.

So, the angular width of the central maximum will be twice this value: = 9.2°.

c Width of central maximum = 2 × 4 tan 4.6° = 64 cm.

d **Part a** will be red; **parts b** and **c** will both be double their violet values, so 18.4° and 1.28 m, respectively.

3 Sound waves of frequency 300 Hz have a wavelength in air of $\lambda = \frac{v}{f} = \frac{330}{370} = 0.89$ m.

So, the angle at which the first minimum of the single-slit diffraction pattern would occur is

$\sin^{-1}\left(\frac{\lambda}{b}\right) = \sin^{-1}\left(\frac{0.89}{0.9}\right) = 90°$.

This means that the central maximum of the single-slit diffraction pattern covers all angles possible between −90° and +90°. All pupils within the classroom will be within this range and can hear the sound from the shoes.

4 **a** **i** $\theta = \sin^{-1}\left(\frac{\lambda}{b}\right) = \sin^{-1}\left(\frac{2 \times 10^{-2}}{5 \times 10^{-2}}\right) = 23.6°$ (0.412 radians)

ii Second-order minimum occurs at $\theta = \sin^{-1}\left(\frac{2\lambda}{b}\right) = \sin^{-1}\left(\frac{2 \times 2 \times 10^{-2}}{5 \times 10^{-2}}\right)$ = 53.1° (0.927 radians).

So, at a distance of 1.5 m away, these minima will be separated by (0.927 − 0.412) × 1.5 = 0.77 m (77 cm).

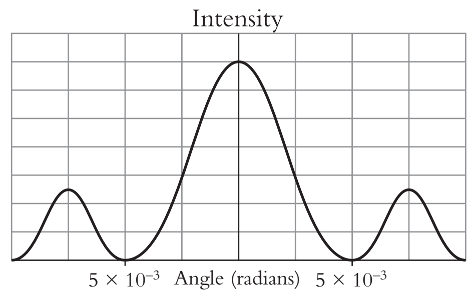
b The third-order minimum should occur at an angle

$\theta = \sin^{-1}\left(\frac{3\lambda}{b}\right) = \sin^{-1}\left(\frac{3 \times 2 \times 10^{-2}}{5 \times 10^{-2}}\right)$
$= \sin^{-1}(1.2)$

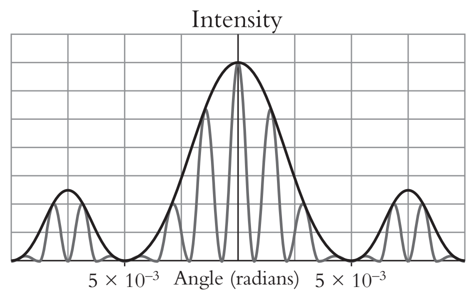
Since this is not possible, there will be four minima altogether: two on either side of the central maximum.

Exercise 14.5

1 **a**



b



c The single-slit diffraction pattern forms an 'envelope' inside which the interference pattern must occur. This is why, if each of the two slits is considered to be infinitesimally thin, the maxima of the interference pattern will be equally bright—they are all inside the central maximum of the diffraction pattern.

2 **a** In the single-slit diffraction pattern, $n\lambda = b \sin\theta_n$ tells us where the minima of the diffraction pattern occur.

b In the two-slit interference pattern, $m\lambda = d \sin\theta_m$ tells us where the maxima of the interference pattern occur.

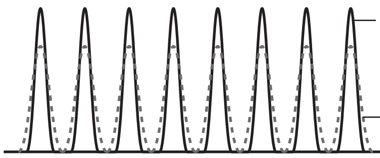
c $\sin\theta_n = \frac{n\lambda}{b} = \sin\theta_m = \frac{m\lambda}{d}$

Therefore, $\frac{n}{b} = \frac{m}{d}$.

So, $\frac{d}{b} = \frac{m}{n}$.

d The missing maximum from the interference pattern is given when n = 1, so it is the mth maximum that is missing; this is the value of $\frac{d}{b}$. When n = 2, it will be the (m × 2)th maximum that is missing: $\frac{2d}{b}$.

e The number of maxima of the interference pattern inside the central maximum of the diffraction pattern is given by $\frac{2d}{b} - 1$.

3 **a** 

b The intensity of the maxima has increased. There are more slits, so more energy is able to pass through onto the screen.

c The spacing of the maxima has remained the same, because the spacing of the slits is the same.

d The width of the maxima has decreased. With more slits, it is less likely that waves can arrive in phase over a small range of distances on the screen. Waves that do arrive in phase—and therefore produce a maximum—only do so for a limited region on the screen, leading to thinner maxima.

4 **a** A diffraction grating is a set of many, very narrow, slits. Each slit is separated from the one next to it by a very small distance. Light passing through a diffraction grating will then produce an interference pattern similar to that shown in the answer to **question 3 part a**.

b The light would be a different colour: blue rather than orange. The spacing of the maxima would be smaller (by a factor of $\frac{450}{590}$).

c $s = \frac{10^{-3}}{600} = 1.7 \times 10^{-6}$ m.

The angle between the central maximum and the first-order maximum is given by $\sin^{-1}\frac{\lambda}{s} = \sin^{-1}\frac{630 \times 10^{-9}}{1.7 \times 10^{-6}} = 21.8°$ (0.38 radians).

Therefore, the separation on the screen = $0.38 \times 5 = 1.9$ m.

d $s \sin\theta = 2\lambda \Rightarrow \lambda = \frac{s \times \sin\theta}{2} = \frac{10^{-3}}{2 \times 600} \times \sin\left(\tan^{-1}\left(\frac{9.5}{8}\right)\right) = 640$ nm (2 s.f.)

5 **a** The first maximum of an interference pattern would occur at an angle given by

$\theta = \sin^{-1}\left(\frac{\lambda}{s}\right) = \sin^{-1}\left(\frac{1.0 \times 10^{-10}}{\left(\frac{10^{-3}}{600}\right)}\right) = \frac{1.0 \times 10^{-10}}{1.7 \times 10^{-6}} =$

5.9×10^{-5} radians ($\approx 3.4 \times 10^{-3}$ degrees).

This angle is too small to be resolved by an X-ray detector.

b The spacing of the atoms is now much smaller, so now the angle for the first maximum of an interference pattern would occur at

$\theta = \sin^{-1}\left(\frac{\lambda}{s}\right) = \sin^{-1}\left(\frac{1.0 \times 10^{-10}}{0.3 \times 10^{-9}}\right) = 19.5°$.

This is easily resolved.

c Using X-rays with crystals produces interference patterns that can be easily observed. This allows scientists to make accurate measurements of the spacing and orientation of atoms within a crystal.

Exam-style questions

Multiple-choice questions

1	C	[1]
2	C	[1]
3	D	[1]
4	B	[1]
5	C	[1]
6	A	[1]
7	D	[1]
8	D	[1]
9	C	[1]
10	C	[1]

Short-answer questions

11 **a** A line, or surface, showing parts of a wave that are of the same phase as each other. **[1]**

b **i** $v = f\lambda = 3.0 \times 60 \times 10^{-3} = 0.18$ ms^{-1} **[2]**

ii $\lambda = \frac{v}{f} = \frac{0.12}{3.0} = 0.04$ m (= 40 mm) **[1]**

iii $n = \frac{0.18}{0.12} = 1.5$ **[1]**

12 **a** $\lambda = \frac{v}{f} = \frac{330}{880} = 0.375$ m **[1]**

b **i** $\frac{v_{water}}{v_{air}} = \frac{1}{\sin\theta_c} \Rightarrow v_{water} = \frac{1}{\sin 13°} \times 330 = 1467 = 1470$ ms^{-1} (3 s.f.) **[2]**

ii $n = \frac{v_{air}}{v_{water}} = \frac{330}{1467} = 0.22$ **[1]**

c The sound waves will be partly reflected and partly transmitted. **[1]**

13 a $\frac{\sin i}{\sin r} = n$ Therefore, $r = \sin^{-1}\left(\frac{\sin i}{n}\right) = \sin^{-1}\left(\frac{\sin 40°}{1.5}\right) = 25°$. (2 s.f.) **[2]**

b $v_{glass} = \frac{c}{1.5} = \frac{3 \times 10^8}{1.5} = 2 \times 10^8\ \text{ms}^{-1}$ **[1]**

c $\lambda_{air} = \frac{c}{f} = \frac{3 \times 10^8}{6 \times 10^{14}} = 5 \times 10^{-7}$ m **[1]**

d $\lambda_{glass} = \frac{V_{glass}}{f} = \frac{2 \times 10^8}{6 \times 10^{14}} = 3.3 \times 10^{-7}$ m **[1]**

14 a $\lambda = \frac{c}{f} = \frac{3.0 \times 10^8}{4.57 \times 10^{14}} = 656$ nm **[1]**

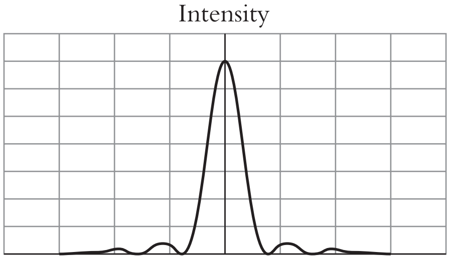
b $x = \frac{\lambda D}{d} = \frac{656 \times 10^{-9} \times 3.6}{60 \times 10^{-6}} = 3.9$ cm **[2]**

c The bright fringes will now be

closer together. **[1]**

brighter/more intense. **[1]**

15 a



[2]

b The blue regions (as opposed to the previous red regions) will be closer together. **[1]**

c With a smaller slit, the intensity of the pattern would be reduced (because less light is able to pass through), and the spacing of the bright regions would increase (more diffraction). **[2]**

16 a Angular width of the central maximum

$= 2 \times \sin^{-1}\left(\frac{\lambda}{b}\right)$

$= 2 \times \sin^{-1}\left(\frac{530 \times 10^{-9}}{1 \times 10^{-4}}\right)$

$= 1.06 \times 10^{-2}$ radians **[1]**

So, the width of the central maximum on the screen $= 1.06 \times 10^{-2} \times 8 = 8.5$ cm. **[1]**

b $\frac{650}{530} = 1.23$

So, the width of the new central maximum will be 1.23×8.5 cm = 10.5 cm. **[1]**

c With white light,

- the central maximum will be white. **[1]**
- the higher-order maxima will each be a spectrum of colours, with violet and blue closest to the central order maximum and red farthest away from the central maximum. **[1]**

(However, beyond about the second order, it is likely that these maxima will overlap, and so other colours will be seen where this occurs.)

17 a 300 lines mm^{-1} means that the spacing of the slits is $\frac{1}{300\ \text{mm}^{-1}} = 3.3 \times 10^{-6}$ m. **[1]**

b $\theta = \sin^{-1}\left(\frac{\lambda}{s}\right) = \sin^{-1}\left(\frac{590 \times 10^{-9}}{3.3 \times 10^{-6}}\right) = 10.3°$ (0.18 radians) **[2]**

c Maxima would also be produced as long as $\frac{n\lambda}{s} \le 1.0$.

So, there would be a second-order maximum at $\sin^{-1} 2\frac{\lambda}{s} = 21°$.

A third-order maximum at $\sin^{-1} 3\frac{\lambda}{s} = 32.4°$.

A fourth-order maximum at $\sin^{-1} 4\frac{\lambda}{s} = 45.7°$.

A fifth-order maximum at $\sin^{-1} 5\frac{\lambda}{s} = 63.4°$.

But, there is no sixth-order maximum because $6\frac{\lambda}{s} > 1$. **[2]**

18 a The angle at which the first-order maximum for the blue light occurs is

$\theta_{blue} = \sin^{-1}\left(\frac{\lambda}{s}\right) = \sin^{-1}\left(\frac{467 \times 10^{-9}}{\frac{1}{5 \times 10^5}}\right) = 13.5°$. **[1]**

The angle at which the first-order maximum for the red light occurs is

$\theta_{red} = \sin^{-1}\left(\frac{\lambda}{s}\right) = \sin^{-1}\left(\frac{700 \times 10^{-9}}{\frac{1}{5 \times 10^5}}\right) = 20.5°$. **[1]**

So, the angle between the two is $20.5 - 13.5 = 7.0°$. **[1]**

b The angle at which the third-order maximum for the blue light occurs is

$$\theta_{\text{blue}} = \sin^{-1}\left(\frac{3\lambda}{s}\right) = \sin^{-1}\left(\frac{3 \times 467 \times 10^{-9}}{\frac{1}{5 \times 10^5}}\right) = 44°.$$

The angle at which the second-order maximum for the red light occurs is

$$\theta_{\text{red}} = \sin^{-1}\left(\frac{2\lambda}{s}\right) = \sin^{-1}\left(\frac{2 \times 700 \times 10^{-9}}{\frac{1}{5 \times 10^5}}\right) = 44°.$$ **[1]**

Since these two angles are the same, the two maxima overlap. **[1]**

19 a i $s = \frac{\lambda}{\sin\theta} = \frac{550 \times 10^{-9}}{\sin 26} = 1.25\ \mu\text{m}$ **[2]**

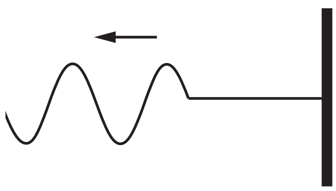
ii Number of slits $\text{mm}^{-1} = \frac{1000}{1.25} =$ 800 **[1]**

b $\lambda = s \sin\theta = 1.25 \times 10^{-6} \times \sin 23.5 =$ 498 nm **[2]**

Chapter 15

Exercise 15.1

1 **a**



b

		Figure				
		i	ii	iii	iv	v
Point	1	0	−1.4A	−2A	−1.4A	0
	2	0	0	0	0	0
	3	0	1.4A	2A	1.4A	0
	4	0	0	0	0	0
	5	0	−1.4A	−2A	−1.4A	0
	6	0	0	0	0	0
	7	0	1.4A	2A	1.4A	0
	8	0	0	0	0	0
	9	0	−1.4A	−2A	−1.4A	0

c **i** Destructive interference occurs **at all times** at these points, causing zero net displacement.

ii Points where the net displacement is always zero are known as nodes.

d **i** Values in the table for points: 2, 4, 6 and 8 will not change; they will all be zero.

Values in the table for points: 1, 5 and 9 will become 1.4A, 2A, 1.4A, 0.

Values in the table for points: 3 and 7 will become −1.4A, −2A, −1.4A, 0.

ii The net displacement at these points oscillates between ±2A.

iii These points are known as antinodes.

e These pairs of points are all half a wavelength apart.

f No, the standing wave does not transfer any energy.

2 **a** A standing wave is formed when two similar waves, travelling in opposite directions, superpose.

b The standing wave does not transfer energy from one place to another; the progressive wave does.

c **i** Yes, for example, standing waves on a string.

ii Yes, for example, standing waves in a pipe.

3 **a** **i** A point on the string where the amplitude of oscillation is always zero.

ii A point on the string where the amplitude of oscillation is a maximum.

iii The maximum displacement of a point on the string from the equilibrium point.

iv The number of complete oscillations in 1 s.

b **i** $v_{ave} = \frac{\text{distance travelled}}{\text{time taken}} = \frac{0\text{ cm}}{1\text{ s}}$
$= 0\text{ cm}s^{-1}$

ii $v_{ave} = \frac{\text{distance travelled}}{\text{time taken}} = \frac{8\times\ 4\times 12\text{ cm}}{1\text{ s}}$
$= 384\text{ cm}s^{-1}$

c **i** $0\text{ cm}s^{-1}$

ii $0\text{ cm}s^{-1}$

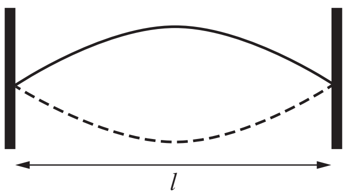
d $\lambda = 2 \times 30\text{ cm} = 60\text{ cm}$

e $v = f\,\lambda = 8 \times 60 = 480\text{ cm}s^{-1}$

Exercise 15.2

1 **a** **i** The first harmonic mode is the mode in which the lowest frequency of a standing wave can be formed.

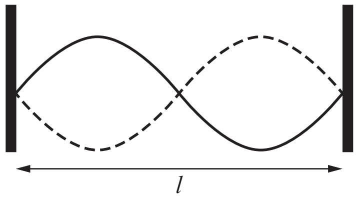
ii



iii $\lambda = 2l$

b **i** $f_2 = 2f_o$

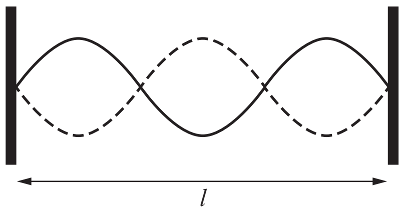
ii



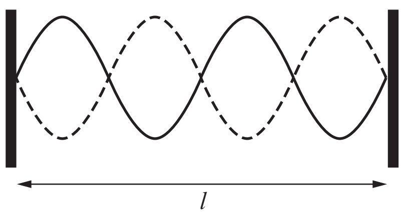
iii $\lambda = \frac{2l}{2} = l$

c **i** $f_3 = 3f_o; f_4 = 4f_o; f_5 = 5f_o$

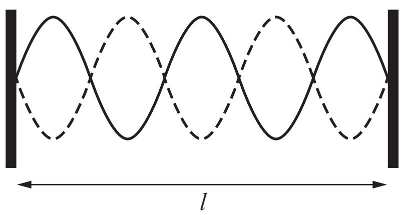
ii Third harmonic:



Fourth harmonic:



Fifth harmonic:



iii $\lambda_n = \frac{2l}{n}; f_n = \frac{v}{\lambda_n} = \frac{nv}{2l}$

2 **a** $v = f\lambda = 440 \times 2 \times 1.2 = 1056$
$= 1100\ \text{ms}^{-1}$ (2 s.f.)

b **i** $\lambda_2 = \frac{2l}{n} = \frac{2 \times 1.2}{2} = 1.2\ \text{m}; f = 2f_o$
$= 2 \times 440 = 880\ \text{Hz}$

ii $\lambda_3 = \frac{2l}{n} = \frac{2 \times 1.2}{3} = 0.8\ \text{m}; f = 3f_o$
$= 3 \times 440 = 1320\ \text{Hz}$
$= 1300\ \text{Hz}$ (2 s.f.)

iii $\lambda_n = \frac{2l}{n} = \frac{2.4}{n}\text{m}; f = nf_o$

c $\lambda = \frac{v}{f_o} = \frac{330}{440} = 0.75\ \text{m}$

3 **a** 6 cm

b 3 cm

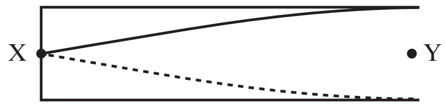
c Eighth harmonic

Exercise 15.3

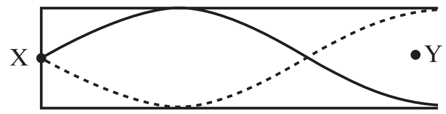
1 **a** **i** Node

ii Antinode

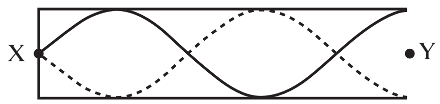
b **i**



ii



iii



c Even-numbered harmonics would suggest that there would be a node at point Y. This cannot occur, since the pipe is open here. Only odd-numbered harmonics will produce antinodes at point Y.

d $\lambda_n = \frac{4l}{n}$, where n is an odd integer.

2 **a** Nodes

b In the first harmonic mode, half of a wave will be present. So, the wavelength will be twice the length of the pipe. $\lambda = 2l$.

c $\lambda_n = \frac{2l}{n}$

3 **a** Antinodes

b In the fundamental mode, half of a wave will be present. So, the wavelength will be twice the length of the pipe. $\lambda = 2l$.

c $\lambda_n = \frac{2l}{n}$

d They are the same. The equation $\lambda_n = \frac{2l}{n}$ applies to pipes with two open or two closed ends.

e No. All harmonics are possible.

4 **a** $\lambda = \frac{v}{f} = \frac{330}{220} = 1.5$ m. In the second harmonic, one complete wave will be present, so the length of the pipe must be 1.5 m.

b $f_3 = \frac{3}{2}f_2 = \frac{3}{2} \times 220 = 330$ Hz

c $\lambda_4 = \frac{1}{2}\lambda_2 = \frac{1}{2} \times 1.5 = 0.75$ m

d A pipe that is closed at one end can produce the same first harmonic frequency in only half the length. So the organ pipes don't need to be so long.

5 **a** Pushing the piston inwards makes the length of the pipe of the whistle smaller. This means that standing waves that are set up inside the pipe will have to have a smaller wavelength. Since the speed of sound in air is fixed, a smaller wavelength of standing wave means there will be a larger frequency—hence a higher-pitched sound.

b Pulling the piston outwards makes the length of the pipe of the whistle longer. This means that standing waves that are set up inside the pipe will have to have a longer wavelength. Since the speed of sound in air is fixed, a longer wavelength of standing wave means there will be a smaller frequency—hence a lower-pitched sound.

Exercise 15.4

1 **a** *free oscillation*: when a system is displaced from its equilibrium point and left to oscillate without the effect of any additional forces.

b *forced oscillation*: when a system is acted upon by a periodic external force.

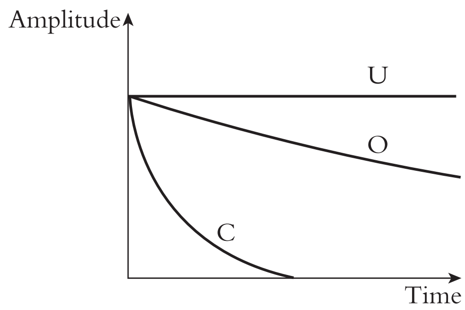
c *resonance*: when the frequency of an external periodic force applied to a system is the same as the natural (or resonant) frequency of the system. This will result in large amplitudes of oscillation.

2 **a** Damping means that energy is removed from the oscillator, leading to a reduction in the oscillator's amplitude.

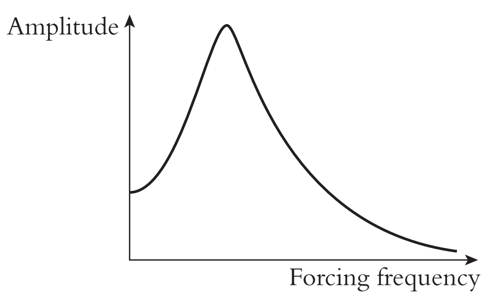
b All mechanical systems are subject to friction in one form or another. The effect of friction on an oscillating system is to transfer energy from the oscillating system into the internal energy of the system and its surroundings. This loss of energy means that the amplitude of oscillation will decrease; the oscillations are being damped.

c A system that is overdamped will move towards its equilibrium position over a long period of time, but it will not oscillate. An underdamped system will continue to oscillate but will move towards its equilibrium position over a long period of time. A critically damped system will move towards its equilibrium position before the oscillator has been able to make another oscillation.

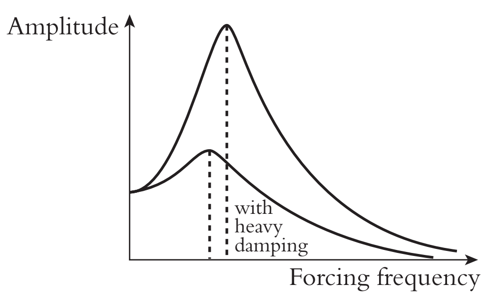
3



4 **a**



b



c It will occur at a slightly lower frequency.

d **i** The oscillating body is in phase with the forcing frequency.

ii The oscillating body lags the forcing frequency by $\frac{\pi}{2}$.

iii The oscillating body lags the forcing frequency by π.

5 **a** When the frequency of sound being produced by a speaker matches the natural frequency of the speaker cabinet, the cabinet itself will vibrate at the forcing frequency, producing an unwanted sound that is often just described as noise. High-quality speaker cabinets are designed such that their natural frequency is in a range of values significantly lower than audible frequencies. Usually, this is affected by making the speaker cabinets very heavy.

b The microwave oven emits electromagnetic waves at the same frequency as the natural frequency at which water molecules oscillate. The water molecules are then forced to oscillate in resonance, thus producing large amplitudes, which we associate with the water molecules getting very hot.

c When a substantial number of people walked along the bridge, they tended to walk 'in step' with each other.
Two thousand people stepping, each in phase with each other, forced the bridge to oscillate. The frequency of the oscillations was close to the resonant frequency of the bridge, causing the oscillations of the bridge to be dangerously large.

The redesign of the bridge used the idea of damping to reduce the amplitude of oscillations. Large viscous fluid dampers were installed to extract energy from horizontal oscillations of the bridge, essentially preventing any effects of resonance.

d Modern skyscrapers are designed so that their foundations are not directly part of the building itself. The foundations and the main structure are joined by sets of base isolation systems, which heavily damp any oscillations caused by seismic activity. In this way, the structure itself cannot then oscillate with a large amplitude.

e The electrical circuits of the radio receiver allow oscillating currents to flow.
The natural frequency of such alternating currents is determined by the values of the capacitance, inductance and impedance of the receiver's components. When the values of these components are set so that the natural frequency of allowable alternating currents matches the frequency of the radio waves from any particular radio station's broadcasts, resonance occurs and the alternating currents that flow have a large amplitude. These larger amplitude alternating currents are then used to produce sound via amplifiers with speakers or via crystal oscillators.

Exam-style questions

Multiple-choice questions

1 D **[1]**

2 A **[1]**

3 B **[1]**

4 A **[1]**

5 B **[1]**

6 C **[1]**

7 D **[1]**

8 B **[1]**

9 C **[1]**

Short-answer questions

10 **a** A progressive wave transfers energy from one place to another.
A standing wave does not transfer energy from one place to another. **[1]**

b **i** There will have to be a node at both ends of the string, so the only wavelengths possible will be given by $\frac{2 \times 4\text{ m}}{\text{n}}$ (where n is an integer) Since $f = \frac{\text{speed of waves}}{\text{wavelength}}$, f will occur only at certain discrete values. **[1]**

ii The minimum frequency will occur at the fundamental mode, which will have a wavelength of $2 \times 4 = 8$ m.
So $f = \frac{760}{8} = 95$ Hz.

iii $f_5 = \frac{5 \times 760}{8} = 475$ Hz. Or, more simply, $f_5 = 5f_1 = 5 \times 95 = 475$ Hz.

11 a A node is a place where there is no oscillation of the medium (the amplitude is zero). An antinode is where the amplitude of the oscillation of the medium is a maximum. **[2]**

b The wavelength of the waves is twice the node separation, so $\lambda = 2 \times 8$ cm = 16 cm. **[1]**

c In the fundamental mode (first harmonic) there will be $\frac{1}{2}$ a wavelength present. In the second harmonic there will be one complete wavelength present and in the third harmonic there will be 1.5 wavelengths present. Since the string is 24 cm long (= 1.5 λ), it is the third harmonic that is present. **[1]**

d Since the tension in the string and the mass per unit length of the string have not changed, the speed of the waves along the string remains constant. So, if the frequency of the standing wave has increased, then the wavelength must have decreased so that $f \times \lambda$ = constant. So, the separation of the nodes and antinodes will decrease. **[1]**

12 a There are 1.25 wavelengths in the pipe. **[1]**

So, $1.25\,\lambda = 3 \Rightarrow \lambda = \frac{3}{1.25} = 2.4$ m. **[1]**

b $f = \frac{v}{\lambda} = \frac{330}{2.4} = 137.5$ Hz
(140 Hz to 2 s.f.) **[1]**

c Only odd-numbered harmonics can occur. The first harmonic has 1 node and 1 antinode. The third harmonic has 2 nodes and 2 antinodes.
The fifth harmonic has 3 nodes and 3 antinodes. This is what the diagram shows, so it is the fifth harmonic that is present. **[2]**

13 a Half a wave is present, so the wavelength of the standing wave is 2×2.4 m = 4.8 m. **[1]**

b i It will be the same as before. **[1]**

ii The speed of wave $\propto \sqrt{\text{Tension}}$,

so the frequency will increase **[1]**
by a factor of $\sqrt{2}$. **[1]**

c The mass per unit length of the string **[1]**

14 a In this context the term, *exponential* refers to the fact that the amplitude of the oscillations decreases by the same factor, or fraction, on each oscillation. **[1]**

b Use the constant ratio rule, that is
$\frac{A_1}{A_2} = \frac{A_2}{A_3} = \frac{A_3}{A_4}$ . . . and so on. **[2]**

c $0.5 = 0.9^n$

$\log 0.5 = n \log 0.9 \Rightarrow n = \frac{\log 0.5}{\log 0.9} = 6.6.$ **[1]**

So, it would take seven oscillations for the amplitude to have fallen below a half of its initial value. **[1]**

15 a Energy $\propto$ Amplitude^2 **[1]**

b i $0.001 = 0.9^n$

$\log 0.001 = n \log 0.9 \Rightarrow$

$n = \frac{\log 0.001}{\log 0.9} = 66$

So, 66 oscillations would be required for the energy to be 0.1% of its initial value. **[2]**

ii $Q = 2\pi \times \frac{\text{E}}{\text{E lost per cycle}} = 2\pi \times \frac{E}{0.1E}$
$= 2\pi \times 10 = 63$

Since this value is about the same as the answer to **part i**, the Q factor could be described as the number of oscillations required for the energy of the oscillator to fall to 0.1% of its initial value. (In practice, engineers will assume that when the energy in an oscillating system has fallen to such a low amount compared to its initial value, then the oscillations have ceased.) **[1]**

c 1 **[1]**

16 a There are five oscillations in 20 s. So $T = 4$ s; therefore, $f = \frac{1}{4} =$ 0.25 Hz. **[2]**

b Any form of resistive force that causes an oscillating body to lose energy and so makes the amplitude of its oscillations smaller. **[1]**

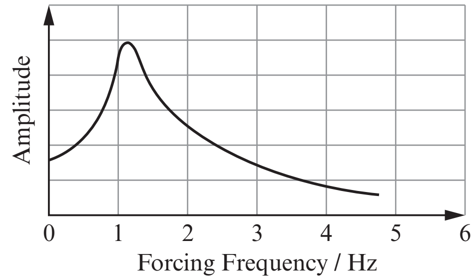
c Using the constant ratio rule: Successive positive amplitudes are 9.5, 7.8, 6.4, 5.2, 4.3. **[1]**
(or any other sensible sequence)

So, $\frac{9.5}{7.8} = 1.2$, $\frac{7.8}{6.4} = 1.2$, $\frac{6.4}{5.2} = 1.2$, and $\frac{5.2}{4.3} = 1.2$, therefore exponential. **[1]**

17 a The condition in which a body is being forced to oscillate at its natural frequency. **[1]**

b i $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{20}{0.4}} = 1.1$ Hz **[2]**

ii



- Correct shape curve **[1]**
- Max. amplitude at 1.1 Hz **[1]**
- Non-zero amplitude at $f = 0$ **[1]**

Chapter 16

Exercise 16.1

1 a i f wavefronts

ii $T = \frac{1}{f}$

iii $\lambda = \frac{c}{f} = cT$

b i Less time

ii During the time it takes for a wavefront to travel the distance, λ, the observer has moved towards the source a distance of $\frac{v}{f}$. So the distance that the next wavefront has to travel before it is received by the observer is less. Since the waves travel at a constant speed, this means that it will take less time for the next wavefront to arrive at the observer.

iii A larger frequency

iv A smaller wavelength

2 a i f wavefronts

ii $\mathrm{T} = \frac{1}{f}$

iii $\lambda = \frac{c}{f} = cT$

b i More time

ii During the time it takes for a wavefront to travel the distance, λ, the observer has moved away from the source a distance of $\frac{v}{f}$. So the distance that the next wavefront has to tavel before it is received by the observer is more than before. Since the waves travel at a constant speed, this means that it will take more time for the next wavefront to arrive at the observer.

iii A smaller frequency

iv A larger wavelength

3 a No. The relative motion between the two is the same, whether it is the observer moving or the source moving.

b i Smaller frequency

ii Larger frequency

iii For relative motion away from each other, the wavelength will be larger. For relative motion towards each other, the wavelength will be smaller.

4 a i $\lambda = \frac{c}{f}$

ii $t = \frac{1}{f} = \frac{\lambda}{c}$

iii $vt = \frac{c\lambda}{c} = \frac{v}{f}$

iv $\lambda' = \lambda - \frac{v}{f}$

v $\lambda' = \lambda - \frac{v}{f} = \lambda - \frac{v\lambda}{c} = \lambda\left(1 - \frac{v}{c}\right)$

vi $\Delta\lambda = \lambda - \lambda' = \lambda - \lambda\left(1 - \frac{v}{c}\right) = \lambda\left(1 - 1 + \frac{v}{c}\right) \Rightarrow \frac{\Delta\lambda}{\lambda} = \frac{v}{c}$

b i $\lambda = \frac{c}{f}$

ii $t = \frac{1}{f} = \frac{\lambda}{c}$

iii $vt = \frac{v\lambda}{c} = \frac{v}{f}$

iv $\lambda' = \lambda + \frac{v}{f}$

v $\lambda' = \lambda + \frac{v}{f} = \lambda + \frac{v\lambda}{c} = \lambda\left(1 + \frac{v}{c}\right)$

vi $\Delta\lambda = \lambda' - \lambda = \lambda\left(1 + \frac{v}{c}\right) - \lambda = \lambda\left(1 + \frac{v}{c} - 1\right) \Rightarrow \frac{\Delta\lambda}{\lambda} = \frac{v}{c}$

c i $\lambda' = \lambda - \frac{v}{f}$

ii $\Delta\lambda = \lambda - \lambda' = \lambda - \lambda\left(1 - \frac{v}{c}\right) = \lambda\left(1 - 1 + \frac{v}{c}\right) \Rightarrow \frac{\Delta\lambda}{\lambda} = \frac{v}{c}$

d $\frac{\Delta\lambda}{\lambda} = \frac{\Delta f}{f} = \frac{v}{c}$

e No, they will be the same, providing v is the *relative* speed at which the source and the observer are moving towards/away from each other.

5 a *Blue-shift* refers to the change in wavelength to a *smaller wavelength*, because the relative motion between source and observer is *towards* each other. This is because blue light is at the short wavelength end of the visible part of the electromagnetic spectrum. *Red-shift* refers to the change in wavelength to a *larger wavelength*, because the relative motion between source and observer is *away from* each other. This is because red light is at the long wavelength end of the visible part of the electromagnetic spectrum.

b By measuring the red-shifts exhibited by distant galaxies, Edwin Hubble and other cosmologists and astrophysicists have been able to confirm that the universe is expanding. Moreover, Hubble's law has shown that the farther away a galaxy is from us, the faster it is receding from us.

c **i** Since the measured wavelength is larger than the actual wavelength, the distant star must be moving away from the Earth.

ii $\frac{\Delta\lambda}{\lambda} = \frac{v}{c} \Rightarrow v = \frac{c \times \Delta\lambda}{\lambda} = \frac{3 \times 10^8 \times (580.9 - 527.0)}{527.0} =$ $3.1 \times 10^7\ \text{ms}^{-1}$ away from the Earth (i.e. about one-tenth of the speed of light).

6 $\frac{\Delta\lambda}{\lambda} = \frac{v}{c} = \frac{620 - 590}{620} = 0.048$

So the motorist would have had to be driving at a speed of $0.048c = 1.45 \times 10^7\ \text{ms}^{-1}$.

This is $1.45 \times 10^7 \times 60 \times 60 = 5.22 \times 10^7\ \text{kmhr}^{-1}$!

This is not possible for a vehicle on the road. So the motorist was telling a lie.

7 **a** A Doppler radar gun measures small changes in the frequencies of electromagnetic waves—usually microwaves. Waves are transmitted at a known frequency. These waves reflect from a moving car to the receiver in the Doppler gun. The received waves show a change in frequency if there is a relative motion between the observer and the car. Since the speed of electromagnetic waves is known, the Doppler gun uses the expression $\frac{v}{c} = \frac{\Delta f}{f}$ to calculate an accurate value for v, the speed of the moving car.

b **i** Microwaves

ii $v = c\frac{\Delta f}{f} = 3.0 \times 10^8 \times \frac{2 \times 10^3}{24 \times 10^9} =$ $25\ \text{ms}^{-1}$ ($90\ \text{kmhr}^{-1}$)

c Ultrasonic waves will suffer too much absorption, too much scattering by the air and too much diffraction. They also travel significantly slower.

Exercise 16.2

1 **a** vt

b $u_s t$

c ft

d $vt - u_s t = (v - u_s)t$

e $\lambda' = \frac{\text{distance in which wavefronts are located}}{\text{number of wavefronts}} = \frac{(v - u_s)t}{ft} = \frac{v - u_s}{f}$

f $f' = \frac{v}{\lambda'} = \frac{v}{\left(\frac{v - u_s}{f}\right)} = f\left(\frac{v}{v - u_s}\right)$

g Larger frequency

h Yes. The speed of the sound waves through the air is determined by the properties of the air itself (such as temperature, pressure, composition etc.).

i $\lambda' = \frac{v}{f'} = \frac{v}{f\left(\frac{v}{v - u_s}\right)} = \lambda\frac{v - u_s}{v} = \lambda\left(1 - \frac{u_s}{v}\right)$

j Smaller wavelength.

2 **a** vt

b $u_s t$

c ft

d $vt - u_s t = (v + u_s)t$

e $\lambda' = \frac{\text{distance in which wavefronts are located}}{\text{number of wavefronts}} = \frac{(v + u_s)t}{ft} = \frac{v + u_s}{f}$

f $f' = \frac{v}{\lambda'} = \frac{v}{\left(\frac{v + u_s}{f}\right)} = f\frac{v}{v + u_s}$

g Smaller frequency

h Yes. The speed of the sound waves through the air is determined by the properties of the air itself (such as temperature, pressure, composition etc.).

i $\lambda' = \frac{v}{f'} = \frac{v}{f\left(\frac{v}{v + u_s}\right)} = \lambda\left(\frac{v + u_s}{v}\right) = \lambda\left(1 + \frac{u_s}{v}\right)$

j Larger wavelength

3 **a** $v + u_o$

b The same

c $f' = \frac{v'}{\lambda} = \frac{v + u_o}{\lambda} = \frac{v + u_o}{\left(\frac{v}{f}\right)} = f\frac{v + u_o}{v} = f\left(1 + \frac{u_o}{v}\right)$

d Larger frequency

4 **a** $v - u_o$

b The same

c $f' = \frac{v'}{\lambda} = \frac{v - u_o}{\lambda} = \frac{v - u_o}{\left(\frac{v}{f}\right)} = f\frac{v - u_o}{v} = f\left(1 - \frac{u_o}{v}\right)$

d A smaller frequency

5 **a** As the train approaches the bridge, the train-spotter will hear the whistle from the train as a higher frequency sound than normal—it will also get louder as it approaches. When the train has passed under the bridge and recedes from the train-spotter, the train-spotter will hear the whistle from the train at a lower frequency than normal—and the sound will become quieter.

b **i** $f' = f\left(\frac{v}{v + u}\right) = 800\left(\frac{330}{330 - 60}\right) =$ 978 Hz (980 Hz to 2 s.f.)

ii $f' = f\left(\frac{v}{v + u}\right) = 800\left(\frac{330}{330 + 60}\right) =$ 677 Hz (680 Hz to 2 s.f.)

6 **a** $f = \frac{v}{\lambda} = \frac{330}{0.5} = 660$ Hz

b **i** $f' = f\frac{v}{v - u_s} = 660 \times \frac{330}{330 - 20} = 703$ Hz (3 s.f.)

ii $f' = f\frac{v}{v + u_s} = 660 \times \frac{330}{330 + 20} = 622$ Hz (3 s.f.)

iii $f' = f\frac{v + u_o}{v} = 660 \times \frac{330 + 20}{330} = 700$ Hz

iv $f' = f\frac{v - u_o}{v} = 660 \times \frac{330 - 20}{330} = 620$ Hz

c The frequency observed from a moving source is slightly larger than the frequency observed by a moving observer.

7 **a** 40 kHz is above the range of human hearing; it is ultrasonic.

b **i** $f' = f\frac{v}{v - u_s} = 40 \text{ kHz} \times \frac{330}{330 - 6} =$ 40.74 kHz = 40.7 kHz (3 s.f.)

ii $f'' = f'\frac{v + u_o}{v} = 40.74 \times \frac{330 + 6}{330} =$ 41.45 kHz = 41.5 kHz (3 s.f.)

8 **a** $f' = f\frac{v + u_o}{v} = 800 \times \frac{330 + 10}{330} = 824$ Hz

b $f'' = f'\frac{v}{v - u_o} = 824 \times \frac{330}{330 - 10} = 850$ Hz

9 **a** $f_{\text{towards}} = f\left(\frac{v}{v - u_s}\right)$ and $f_{\text{away}} = f\left(\frac{v}{v + u_s}\right)$.

So, $\frac{f_{\text{towards}}}{f_{\text{away}}} = \frac{f\left(\frac{v}{v - u_s}\right)}{f\left(\frac{v}{v + u_s}\right)} = \frac{v + u_s}{v - u_s} = k$.

Therefore, $u_s = \left(\frac{k - 1}{k + 1}\right) v$ and $k = \frac{480}{400} = 1.2$.

So, $u_s = \frac{1.2 - 1}{1.2 + 1} \times 330 = 30 \text{ ms}^{-1}$.

b $f_{\text{towards}} = f\left(\frac{v}{v - u_s}\right) \Rightarrow f = 480 \times \frac{300}{330} =$ 436 Hz = 440 Hz (2 s.f.)

Alternatively,

$f_{\text{away}} = f\left(\frac{v}{v + u_s}\right) \Rightarrow f = 400 \times \frac{360}{330} =$ 436 Hz = 440 Hz (2 s.f.)

Exam-style questions

Multiple-choice questions

1	D	[1]
2	D	[1]
3	C	[1]
4	A	[1]
5	C	[1]
6	B	[1]
7	B	[1]
8	C	[1]

Short-answer questions

9 **a** $f = \frac{c}{\lambda} = \frac{3.0 \times 10^8}{575 \times 10^{-9}} = 5.2 \times 10^{14}$ Hz **[1]**

b Since the received frequency is less than the emitted frequency **[1]**, the star is exhibiting red-shift, so it is moving away from the Earth. **[1]**

c $\frac{\Delta f}{f} = \frac{v}{c} \Rightarrow v = c\frac{\Delta f}{f} = 3.0 \times 10^8 \times \frac{(5.2 - 4.8) \times 10^{14}}{5.2 \times 10^{14}} = 2.3 \times 10^7 \text{ ms}^{-1}$ away from the Earth **[2]**

10 **a** The change in frequency **[1]** when there is *relative motion* between a source and an observer. **[1]**

b The star is moving away from the Earth. **[1]**

The star's speed is about 5% of the speed of light. **[1]**

c The speed of the person is far too small compared to the speed of light for the light to be detectably blue-shifted.

11 a The cars have a relative motion that is towards each other **[1]** making the received frequency higher **[1]**. (OWTTE) **[1]**

b $f' = f\frac{v+u_o}{v-u_s} =$ **[1]**

$200 \times \frac{300+20}{300-20} = 230$ Hz (2 s.f.) **[2]**

12 a i $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{5.0 \times 10^3} = 6.0 \times 10^4$ m (60 km) **[1]**

ii $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{4.85 \times 10^3} =$ 6.186×10^4 m (62 km) **[1]**

b Away from the Earth **[1]**

(A lower received frequency—or longer wavelength—means the signal has been red-shifted, so the spacecraft must be travelling away from the Earth.)

c $\frac{\Delta\lambda}{\lambda} = \frac{v}{c} \Rightarrow v = c\frac{\Delta\lambda}{\lambda} = 3 \times 10^8 \times \frac{(61.86-60) \times 10^3}{60 \times 10^3} = 9.0 \times 10^6\ \text{ms}^{-1}$ **[2]**

OR

$\frac{\Delta f}{f} = \frac{v}{c} \Rightarrow v = c\frac{\Delta f}{f} = 3 \times 10^8 \times \frac{(5.0-4.85) \times 10^3}{5.0 \times 10^3} = 9.0 \times 10^6\ \text{ms}^{-1}$ **[2]**

13 The frequency received by blood cells travelling at 20 cms^{-1} is

$f' = f\frac{v+u_o}{v} = 10.000 \times \frac{1500+0.2}{1500} =$ 10.00133 MHz. **[1]**

So, the frequency received by the doctor would be

$f'' = f'\frac{v}{v-u_o} = 10.00133 \times \frac{1500}{1500-0.2} =$ 10.00266 MHz. **[2]**

Since the doctor receives a frequency higher than this, the speed of the blood cells must be higher than 20 cms^{-1}. **[1]**

This suggests that there is a blockage. **[1]**

14 a $f' = f\frac{v+u_o}{v} = 1250 \times \frac{330+25}{330} =$ 1345 Hz **[2]**

b $f' = f\frac{v-u_o}{v} = 1250 \times \frac{330-9}{330} =$ 1216 Hz **[1]**

c The answer to **part a** will be smaller. **[1]**
The answer to **part b** will be larger. **[1]**

Warmer air means a larger speed of sound. This will give a smaller received frequency.

15 a i Red-shift is the change in wavelength (an increase), or frequency (a decrease), caused by an emitter moving away from an observer. **[1]**

ii Blue-shift is the change in wavelength (a decrease), or frequency (an increase), caused by an emitter moving towards an observer. **[1]**

b $f_{\text{emitted}} = \frac{c}{\lambda} = \frac{3 \times 10^8}{620 \times 10^{-9}} =$ 4.8×10^{14} Hz **[1]**

$\Delta f = f_{\text{emitted}} \times \frac{v}{c} = 4.8 \times 10^{14} \times \frac{4.5 \times 10^6}{3 \times 10^8} = 7.2 \times 10^{12}$ Hz **[1]**

Smaller **[1]**

Chapter 17

Exercise 17.1

1 a i A gravitational field is a region in space in which a mass will experience a gravitational force.

ii A uniform gravitational field is a gravitational field in which the gravitational force on a given mass is the same at every point in the field. This could also be expressed as a gravitational field in which the field strength is constant at all points in the field.

iii The radius of the Earth is so large compared to the height above the Earth's surface in which we live. So the field strengths at the Earth's surface and at a height, say, 1 km, above the Earth's surface will both be almost exactly the same.

2 a $R^3 \propto T^2$, where R is the orbital radius and T is the orbital period. (At the time of publishing, Kepler did not know any more than this.)

b $a = \frac{v^2}{R}$

c $v = \frac{2\pi R}{T}$

d $a = \frac{v^2}{R} = \frac{\left(\frac{2\pi R}{T}\right)^2}{R} = \frac{4\pi^2 R^2}{T^2 R} = \frac{4\pi^2 R^3}{T^2} \times \frac{1}{R^2}$

e Since $\frac{4\pi^2 R^3}{T^2}$ = constant, then $a = \frac{\text{constant}}{R^2}$

$\Rightarrow a \propto \frac{1}{R^2}$.

And since $F = m\,a$, and F must be provided by the gravitational force exerted by the Sun on the planet, $F \propto \frac{1}{R^2}$

3 a A point mass is a theoretical object of no size having mass.

b i Since the point mass has no 'size', its density must be infinite.

ii Nothing real can have an infinite density, so the point mass has to be theoretical only.

c The size of the Sun and the size of the planets is very much smaller than their separation. This allowed Newton to model the Sun and the planets as point masses.

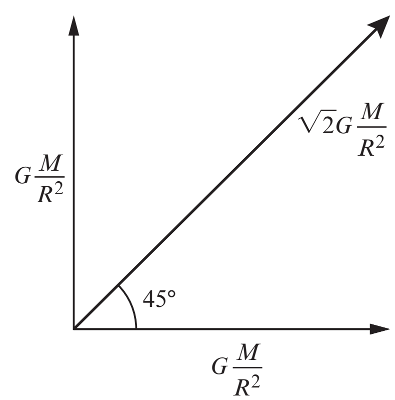
The Sun and the planets are spherical—or very nearly so in the case of the planets—and behave, at a distance larger than their size, as if all their mass is concentrated at the centre of mass—the centre of the sphere.

4 a i $g = G\frac{M}{R^2}$

ii Towards M

b Since the gravitational force on a unit test mass at X due to the left-hand mass is directed horizontally to the left and the gravitational force on the same unit test mass due to the right-hand mass is directed horizontally to the right—and of the same magnitude, the total force on the unit test mass is zero. So, the new gravitational field strength at X is zero. (The two vectors representing the gravitational field strengths due to the left-hand and the right-hand masses are equal and opposite, so adding them gives a zero result.)

c



5 a Gravitational field strength, g: the amount of gravitational force experienced by a unit mass in the field.

b $g = \frac{F}{m}$

c **i** $g = -G\frac{M}{R^2} = 6.67 \times 10^{-11} \times \frac{6 \times 10^{24}}{(6.4 \times 10^6)^2} = -9.8\ \text{Nkg}^{-1}$ (or $9.8\ \text{Nkg}^{-1}$ towards the centre of the Earth)

ii The Earth is not a perfect sphere; it is a sphere that is 'squashed' from top to bottom. So, its radius is not the same everywhere. We usually quote g as a globally averaged value to eliminate the difficulty caused by this changing radius.

iii g is largest at the poles, because the radius of the Earth is smallest there—and since the gravitational field strength is an inverse square law, a smaller radius gives a bigger field strength.

6 **a** $g_{\text{Sun}} = -G\frac{M}{R^2} = 6.67 \times 10^{-11} \times \frac{2 \times 10^{30}}{(1.5 \times 10^{11})^2}$

$= -5.93 \times 10^{-3}\ \text{Nkg}^{-1}$

b $g_{\text{Moon}} = -G\frac{M}{R^2} = 6.67 \times 10^{-11} \times \frac{7.3 \times 10^{22}}{(3.8 \times 10^8)^2}$

$= -3.37 \times 10^{-5}\ \text{Nkg}^{-1}$

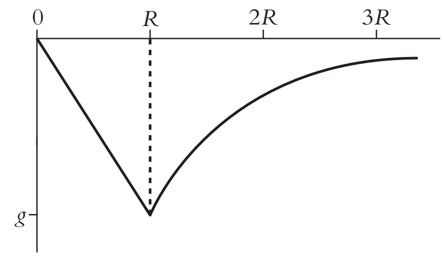
c The gravitational field strength of the Sun where the Earth is in its orbit is greater than the gravitational field strength of the Moon where the Earth is. However, if one considers the relatively small distance that the Earth is away from the Moon, the size of the force acting on the face of the Earth closest to the Moon is significantly greater than the size of the force acting on the face of the Earth farthest from the Moon. It is this difference in force from the gravitational field of the Moon that causes the tides.

The extra distance that the opposite face of the Earth adds to the distance between the Earth and the Sun means that there is little difference in the gravitational force from the Sun on the opposite faces of the Earth.

7 **a** **i** $g = -G\frac{M}{R^2}$ and $M = \frac{4\pi R^3\rho}{3}$ so

$g = -G\frac{4\pi R^3\rho}{3R^2} = -G\frac{4\pi\rho R}{3}$

ii



iii $g = G\frac{4\pi\rho R}{3} \Rightarrow G = \frac{3g}{4\pi R\rho}$

b $G = \frac{3 \times 9.81}{4\pi \times 6.4 \times 10^6 \times 5.5 \times 10^3} = 6.65 \times 10^{-11}\ \text{N m}^2\text{kg}^{-2}$

So, percentage difference is $\frac{6.67 - 6.65}{6.67} \times 100\% = 0.3\%$.

8 **a** $\frac{M_{\text{Earth}}v^2}{R} = G\frac{M_{\text{Sun}}M_{\text{Earth}}}{R^2}$

b Rearranging the previous equation:

$\frac{M_{\text{Earth}}v^2}{R} = G\frac{M_{\text{Sun}}M_{\text{Earth}}}{R^2} \Rightarrow v = \sqrt{\frac{GM_{\text{Sun}}}{R}}$.

c $v = \sqrt{\frac{GM_{\text{Sun}}}{R}} = \sqrt{\frac{6.67 \times 10^{-11} \times 2 \times 10^{30}}{1.5 \times 10^{11}}} = 3 \times 10^{-4}\ \text{ms}^{-1}$ (1 s.f.)

19 miles $\text{s}^{-1} = 19 \times 1603 = 30457\ \text{ms}^{-1} = 3 \times 10^4\ \text{ms}^{-1}$

The song's claim is quite right!

9 a When $g = 0$, the two gravitational field strengths from the two bodies have to be equal:

$G\frac{M_{\text{Earth}}}{x^2} = G\frac{M_{\text{Moon}}}{(R-x)^2}$, where x is the distance from the Earth to point X and R is the separation of the Earth and the Moon.

So, $\frac{M_{\text{Earth}}}{x^2} = \frac{M_{\text{Moon}}}{(R-x)^2} \Rightarrow M_{\text{Earth}}(R-x)^2 = M_{\text{Moon}}x^2$.

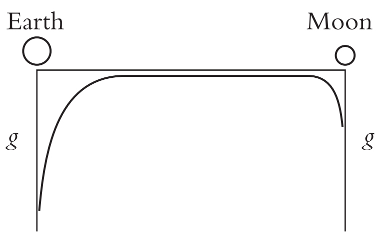
Therefore, $(M_{\text{Earth}} - M_{\text{Moon}})x^2 - 2M_{\text{Earth}}Rx + M_{\text{Earth}}R^2 = 0$.

Solving: $x = \frac{2M_{\text{Earth}}R \pm \sqrt{(2M_{\text{Earth}}R)^2 - 4(M_{\text{Earth}} - M_{\text{Moon}})M_{\text{Earth}}R^2}}{2(M_{\text{Earth}} - M_{\text{Moon}})} =$

$\frac{2 \times 6 \times 10^{24} \times 3.8 \times 10^{8} \pm \sqrt{(2 \times 6 \times 10^{24} \times 3.8 \times 10^{8})^2 - 4(6 \times 10^{24} - 7.3 \times 10^{22}) \times 6 \times 10^{24} \times (3.8 \times 10^{8})^2}}{2(6 \times 10^{24} - 7.3 \times 10^{22})} =$

3.5×10^8 m (~92% of the distance between the Earth and the Moon).

b



10 a 1 Planets orbit the Sun in elliptical orbits with the Sun at one focus.

2 Planets sweep out equal areas in equal times

3 The square of a planet's orbital period is proportional to the cube of its orbital radius

$T^2 \propto R^3$.

b i Kepler's third law; a bigger orbital period suggests a larger orbital radius. Saturn's orbital radius is larger than Jupiter's.

ii Kepler's first law; the moon will appear larger when it is closer to the earth. This is implied by the orbit being elliptical.

iii Kepler's second law; In order for a planet to sweep out an equal area in a given time, it must travel faster when it is closer to the Sun.

11 a $M_{\text{Earth}} = \frac{4\pi^2}{G} \times \frac{R_{\text{Moon}}{}^3}{T_{\text{Moon}}{}^2} = \frac{4\pi^2}{6.67 \times 10^{-11}} \times \frac{(3.8 \times 10^8)^3}{(27.3 \times 8.64 \times 10^4)^2} = 5.84 \times 10^{24}$ kg

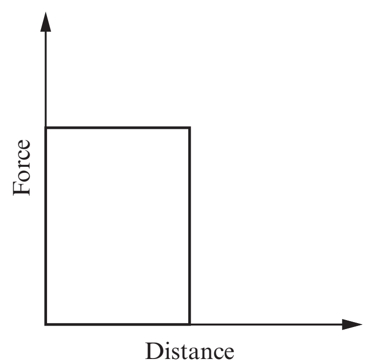
b $G\frac{M_{\text{Mars}}}{(R_{\text{Mars}})^2} = 0.38\, G\frac{M_{\text{Earth}}}{(R_{\text{Earth}})^2} \Rightarrow M_{\text{Mars}} = 0.38\frac{9.81 \times (3.4 \times 10^6)^2}{6.67 \times 10^{-11}} = 6.5 \times 10^{23}$ kg.

Exercise 17.2

1 a i mg (the same as the weight of the mass)

ii mgh (work done = force × distance moved)

iii



iv The work done is the area under the graph.

v The work done has been transferred into gravitational potential energy of the mass.

vi Yes; it now has more E_p than it had on the ground.

b **i** The area under the curve represents the work done in moving the 1-kg mass from the Earth's surface to an infinite distance away from the Earth.

ii $\int_R^\infty F\,dR = \int_R^\infty G\frac{Mm}{R^2}\,dR = \left[-G\frac{Mm}{R}\right]_R^\infty$
$= -\left(0 - G\frac{Mm}{R}\right) = G\frac{Mm}{R}$

iii work done $= G\frac{Mm}{R} = 6.67 \times 10^{-11} \times \frac{6 \times 10^{24} \times 1}{6.4 \times 10^{6}} = 62.5$ MJ

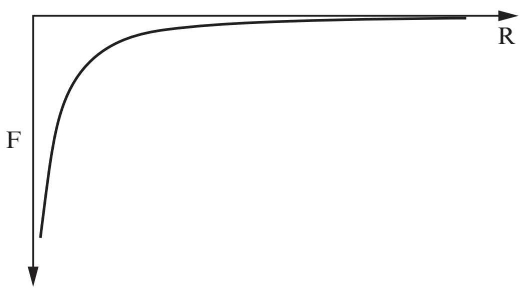
iv More

v It suggests that the value of the E_p at the Earth's surface, or above it, must be less than zero; that is it must be a *negative* quantity.

vi $E_p = -G\frac{Mm}{R}$ (The minus sign is necessary, because the E_p must be a negative quantity and G, M, m and R are all positive quantities.)

2 **a** $F = -G\frac{Mm}{R^2}$

b



c **i** Zero

ii It will make m accelerate towards M.

iii E_p is being transferred into kinetic energy. (This seems strange, since initally, the E_p of the satellite was zero.)

iv It would represent the work done by the gravitational force due to M in moving the mass, m, from infinity to a distance R^* away from M. This is the gain in kinetic energy, or the loss in E_p.

v $E_p = -G\frac{Mm}{R^*}$

vi E_p is the work done in moving a mass, m, from an infinite distance away to a distance, R, away from M.

3 **a** **i** Gravitational potential, V_g, is the gravitational potential energy of a unit mass.

ii V_g is the work done to move a unit mass from infinity to a place in a gravitational field.

iii $\mathrm{Jkg^{-1}}$

iv $V_g = \frac{E_p}{m}$ or $E_p = mV_g$

v Scalar (E_p and m are both scalars, so V_g must also be a scalar.)

b **i** $V_{g,Earth} = -G\frac{M}{R} = -6.67 \times 10^{-11} \times \frac{6 \times 10^{24}}{6.4 \times 10^{6}} = -6.25 \times 10^{7}\ \mathrm{Jkg^{-1}}$

ii $V_{g,Sun} = -G\frac{M}{R} = -6.67 \times 10^{-11} \times \frac{2 \times 10^{30}}{1.5 \times 10^{11}} = -8.89 \times 10^{8}\ \mathrm{Jkg^{-1}}$

iii $V_{g,total} = 6.25 \times 10^{7} + 8.89 \times 10^{8}$
$= -9.5 \times 10^{8}\ \mathrm{Jkg^{-1}}$

iv $E = m|V_g| = 5 \times (9.5 \times 10^{8}) = 4.75 \times 10^{9}\ J = 4.8\ GJ$ (2 s.f.)

4 **a** **i** $F = -G\frac{Mm}{R^2}$

ii $E_p = -G\frac{Mm}{R}$

iii Although we can often write 'work done = force × distance moved in direction of force', this is only true if the size of the force is independent of the distance, that is if the force is constant. Since, in the case of a gravitational field, F is a function of R, F is not constant as R varies, and so we cannot say that $E_p = F \times R$.

iv The area under the graph of F against R will give E_p.

v To make the two equations mutually consistent, we must write $E_p = -\int F\,dr$.

b **i** To find F_g from the graph of E_p against R, one would first determine the *gradient* of the graph at the value of R—and then make the answer negative.

ii $F_g = -\frac{d(E_p)}{dR}$

c **i** One rectangle of the graph = 10 Mm × −2 N = −20 MJ

Area under graph from R = 10 Mm to $R = \infty$ is a little over 1 square—maybe about 1.2 squares. This would represent E_p = 1.2 × −20 MJ = −24 MJ.

ii From the graph, at R = 10 Mm, E_p = −24 MJ.

iii They are the same.

iv The gradient is about $\frac{40\ \text{MJ}}{26\ \text{Mm}} \approx 1.5$ N. So the force is −1.5 N.

v From the graph in Figure 17.2.2, $F_g \approx 1.5$ N.

vi They are the same.

vii Yes, they have confirmed the mathematical relationships between F_g and E_p.

5 **a** The gravitational force acting on a unit mass is the gravitational field strength, g.

b The E_p of a unit mass is the gravitational potential, V_g.

c **i** $F_g = -\frac{d(E_p)}{dR}$ and $E_p = -\int F_g dR$

ii $g = -\frac{d(V_g)}{dR}$ and $V_g = -\int g\, dR$

d F_g = m g and $E_p = mV_g$, so m occurs on both sides of the equation, making the mathematical relationship between F_g and E_p and between g and V_g the same.

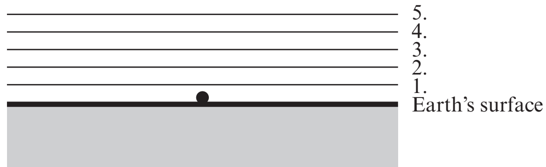
6 **a** work done = force × distance = mgh = 60 × 9.81 × 556 = 3.3 × 10^5 J

b The observer's E_p at the surface is $|E_p| = G\frac{Mm}{R} = 6.67 \times 10^{-11} \times \frac{6 \times 10^{24} \times 60}{6400 \times 10^3} =$ 3.75 × 10^9 J.

Alternatively, $|E_p| = mV_g$ = 60 kg × 62.5 MJkg^{-1} = 3.75 × 10^9 J.

So the work done = $\frac{3.3 \times 10^5}{3.75 \times 10^9}$ = 8.8 × 10^{-3} % of the observer's E_p at the Earth's surface.

7 **a** and **b**



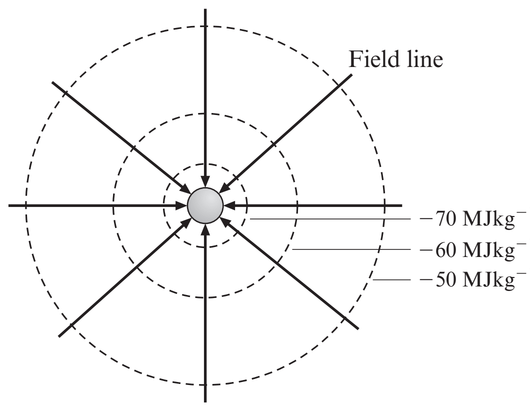
c Equipotentials

d The equipotentials are equally spaced.

e Since a higher equipotential requires work to be done against the gravitational field, and since g = constant, we can write: work done = 1.0 × 10^4 J = g × distance, and taking g = 10 Nkg^{-1}.

So separation of equipotentials = $\frac{1.0 \times 10^4}{10}$ = 1 km

8 **a** and **b**

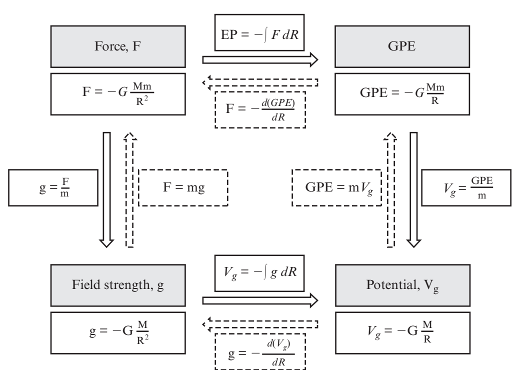


Field lines are solid lines, equipotentials are dashed lines.

c The spacing of the equipotentials is increasing as the distance away from the planet increases.

d Equipotentials are perpendicular to field lines.

9



Exercise 17.3

1 a The gravitational force exerted by M on the satellite.

b $G\frac{Mm}{R^2} = \frac{mv^2}{R} \Rightarrow E_K = \frac{1}{2}mv^2 = G\frac{Mm}{2R}$

c $E_P = -G\frac{Mm}{R}$

total energy $= E_K + E_P = G\frac{Mm}{2R} - G\frac{Mm}{R} = -G\frac{Mm}{2R}$

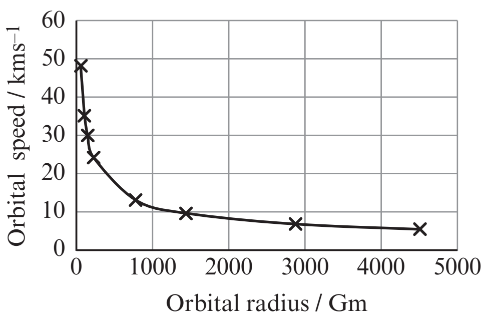
d i $E_{total} = -G\frac{Mm}{2R} = -6.67 \times 10^{-11} \times \frac{2 \times 10^{30} \times 6 \times 10^{24}}{2 \times 1.5 \times 10^{11}} = -2.7 \times 10^{33}$ J

ii $E_{total} = -G\frac{Mm}{2R} = -6.67 \times 10^{-11} \times \frac{6 \times 10^{24} \times 7.3 \times 10^{22}}{2 \times 3.85 \times 10^{8}} = -3.8 \times 10^{28}$ J

2 a

Planet	Orbital / Radius Gm	Orbital speed / kms^{-1}
Mercury	58	48
Venus	108	35
Earth	150	30
Mars	228	24
Jupiter	779	13
Saturn	1434	10
Uranus	2873	7
Neptune	4495	5

b



c First, take data from the graph and draw a new graph of v^2 against $1/R$.

This will give a straight line through the origin showing that $v^2 \propto 1/R$.

The gradient of this graph will be GM.

Divide the gradient by G to give M.

3 a $v = \frac{2\pi R}{T}$

b $a = \frac{v^2}{R} = \frac{\left(\frac{2\pi R}{T}\right)^2}{R} = \frac{4\pi^2 R}{T^2}$

c $a = g \Rightarrow \frac{4\pi^2 R}{T^2} = G\frac{M}{R^2} \therefore R = \sqrt[3]{\frac{GMT^2}{4\pi^2}}$

d $R = \sqrt[3]{\frac{GMT^2}{4\pi^2}} =$

$\sqrt[3]{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times (24 \times 60 \times 60)^2}{4\pi^2}} =$

4.2×10^7 m

So distance above the Earth's surface $= (42 - 6.4) \times 10^6$ m $= 3.6 \times 10^7$ m

This is 36 000 km.

e The orbit must be over the equator.

f An orbit this high above the Earth's surface can "see" about 1/3 of the surface of the Earth. This makes a satellite in such an orbit ideal for telecommunications or for large scale meteorological observations.

4 a Using Kepler's third law:

$R = \sqrt[3]{\frac{GM}{4\pi^2}T^2} =$

$\sqrt[3]{\frac{6.67 \times 10^{-11} \times 2 \times 10^{30}}{4\pi^2} \times (25 \times 24 \times 60 \times 60)^2} =$

2.5×10^{10} m

(Interestingly, this is nearly 36 solar radii away from the centre of the Sun.)

b This would be inside the orbit of Mercury.

5 a $v = \frac{2\pi R}{T} = \frac{2\pi \times (6400 + 420) \times 10^3}{93 \times 60} =$

$7.68\ kms^{-1} = 7.7\ kms^{-1}$ (2 s.f.)

b $g = \frac{v^2}{R} = \frac{(7.68 \times 10^3)^2}{(6400 + 420) \times 10^3} = 8.65\ Nkg^{-1} =$

$8.7\ Nkg^{-1}$

c $g = G\frac{M}{R^2} = \frac{v^2}{R} \Rightarrow M = \frac{Rv^2}{G} =$

$\frac{6820 \times 10^3 \times (7.68 \times 10^3)^2}{6.67 \times 10^{-11}} = 6.0 \times 10^{24}$ kg

d The total energy of the ISS is

$E_{total} = -G\frac{M_{Earth}M_{ISS}}{2R} = -6.67 \times 10^{-11} \times \frac{6 \times 10^{24} \times 4.2 \times 10^5}{2 \times 6820 \times 10^3} = -1.2 \times 10^{13}$ J.

So to escape the Earth's gravitational field, the ISS will require 1.2×10^{13} J.

6 a total energy = E_P + chemical energy = $-62.5\ \text{MJkg}^{-1} + 62.5\ \text{MJkg}^{-1} = 0$ J

b i E_K will decrease—it is being transformed into E_P.

ii E_P will become less negative (i.e. approach zero).

iii 0 J

iv It will escape the Earth's gravitational field completely, at which point its E_K will be zero.

v $\frac{1}{2}v^2 = 62.5\ \text{MJ} \Rightarrow v = \sqrt{2 \times 62.5 \times 10^6}$ $= 1.1 \times 10^4\ \text{ms}^{-1}$ (11 kms^{-1})

vi This is called the escape speed.

vii It would be parabolic.

c Now, the total energy of the rocket is less than zero, so it cannot escape the Earth's gravitational field. It will travel upwards and then remain in an elliptical orbit around the Earth.

d Now the total energy of the rocket is greater than zero. So, it will be able to leave the Earth's gravitational field completely and keep moving away. Its trajectory will now be hyperbolic.

7 a The gravitational potential energy of a unit mass at the surface of the body is $-G\frac{M}{R}$.

If this unit mass were given E_K of $G\frac{M}{R}$ then it would be able to escape to infinity.

$\frac{1}{2}v_{esc}^2 = G\frac{M}{R} \Rightarrow v_{esc} = \sqrt{\frac{2GM}{R}}$

b From $v_{esc} = \sqrt{\frac{2GM}{R}}$, and given $v_{esc} = c$, then $R = \frac{2GM}{c^2}$.

c i $R = \frac{2GM}{c^2} = \frac{2 \times 6.67 \times 10^{-11} \times 2 \times 10^{30}}{(3 \times 10^8)^2} =$

$2.96 \times 10^3\ \text{m} \approx 3$ km

ii $R = \frac{2GM}{c^2} = \frac{2 \times 6.67 \times 10^{-11} \times 6 \times 10^{24}}{(3 \times 10^8)^2} =$

$8.89 \times 10^{-3}\ \text{m} \approx 9$ mm

iii $R = \frac{2GM}{c^2} =$

$\frac{2 \times 6.67 \times 10^{-11} \times 21.2 \times 2 \times 10^{30}}{(3 \times 10^8)^2} =$

$6.28 \times 10^4\ \text{m} \approx 63$ km

d A black hole

8 a The frictional forces will decrease the total energy of the satellite.

b Decreasing the total energy $\left(E_{total} = -\frac{Gmm}{2R}\right)$ means that the total energy becomes more negative.

This makes the orbital radius of the satellite smaller—the satellite moves closer to the Earth's surface.

c Since the orbital speed is given by $v = \sqrt{\frac{GM}{R}}$, a smaller value for R means that the speed of the satellite will increase.

d Energy is being transferred from an E_P store to a E_K store (+ thermal energy, as the satellite starts to heat up).

e The rate at which E_P is being transferred is increasing.

f The increased heating of the satellite may make it burn up before it reaches the Earth's surface.

9 a $c = \sqrt{\frac{2GM}{R}} \Rightarrow R = \frac{2GM}{c^2} =$

$\frac{2 \times 6.67 \times 10^{-11} \times 2.4 \times 2.0 \times 10^{30}}{(3 \times 10^8)^2} =$

7.1×10^3 m (7.1 km)

b The actual size of the black hole will be smaller than this. The value of the gravitational field strength at the surface of the black hole will be too great to allow radiation to leave, but the field strength reduces with distance, so there will be a distance at which the gravitational field strength is just strong enough to prevent radiation from escaping. At any distance greater than this, radiation can escape.

c The Schwarzschild radius

Exam-style questions

Multiple-choice questions

1	B	[1]
2	B	[1]
3	C	[1]
4	A	[1]
5	B	[1]
6	C	[1]

7 B [1]

8 D [1]

9 B [1]

10 D [1]

11 C [1]

12 D [1]

13 D [1]

14 A [1]

15 B [1]

16 C [1]

17 D [1]

Short-answer questions

18 a Either: The gravitational force between two point masses is proportional to the product of their masses and inversely proportional to the square of their separation.

Or $F = -G\frac{Mm}{R^2}$, where M and m are the two masses, R is their separation and G is the universal gravitational constant. [2]

b $E_P = -\int F\,dR = -\int -G\frac{Mm}{R^2}\,dR = -G\frac{Mm}{R}$ [2]

c $V_g = \frac{E_P}{m} = -G\frac{M}{R} = -6.67 \times 10^{-11} \times \frac{6 \times 10^{24}}{6.4 \times 10^6} = 6.25 \times 10^7 \text{ Jkg}^{-1} =$ 62.5 MJkg^{-1} [2]

19 a It shows that the force between M and m is attractive. [1]

b $a = \frac{F}{m} = -G\frac{Mm}{R^2 m} = -G\frac{M}{R^2}$ [1]

c $a = \frac{v^2}{R} = G\frac{M}{R^2} \Rightarrow v = \sqrt{G\frac{M}{R}}$ [2]

d $v = \sqrt{G\frac{M}{R}} = \sqrt{6.67 \times 10^{-11} \times \frac{2 \times 10^{30}}{1.5 \times 10^{11}}}$

$= 2.98 \times 10^4 \text{ ms}^{-1} \approx 30 \text{ kms}^{-1}$ [1]

20 a The gravitational force **[1]** acting on a unit mass. [1]

b $\frac{\text{N}}{\text{kg}} = \frac{\text{J}}{\text{m kg}} = \frac{\text{kg m}^2}{\text{m kgs}^2} = \frac{\text{m}}{\text{s}^2}$ [1]

c $g = G\frac{M}{R^2} = 6.67 \times 10^{-11} \times \frac{6 \times 10^{24}}{(6.4 \times 10^6)^2} =$ 9.8 Nkg^{-1} [2]

21 a First law: Planets orbit the Sun in elliptical paths with the Sun at one focus. [1]

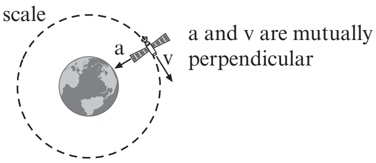
Second law: Planets sweep out equal areas in equal times. [1]

Third law: $R^3 \propto T^2$, where R is the orbital radius and T is the orbital period. [1]

b i $F = -G\frac{Mm}{R^2}$, so as the comet approaches the Sun, R decreases and the gravitational force increases, accelerating the comet with an increasing acceleration until it reaches its point of closest approach (perihelion). [1]

ii When R is smallest, the distance the comet travels in a given period must be largest (i.e. its speed must be greatest) so that the area swept out by the comet is equal to that when R is very large and its speed is small. [1]

22 a i and ii



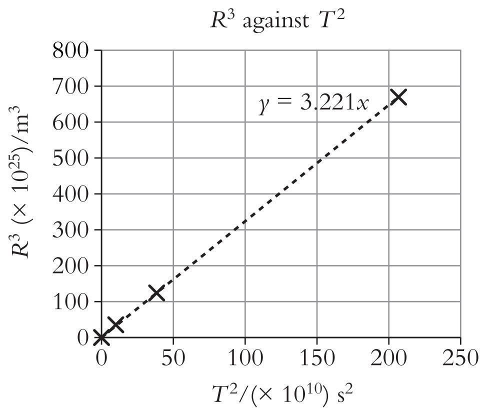
[2]

From Kepler's third law: $T = \sqrt{\frac{4\pi^2}{GM}R^3} = \sqrt{\frac{4\pi^2}{6.67 \times 10^{-11} \times 6 \times 10^{24}} \times (6.8 \times 10^6)^3} =$ 5.57 × 10^3 s (about 93 minutes) [3]

Alternatively: $v = \sqrt{G\frac{M}{R}} = \sqrt{6.67 \times 10^{-11} \times \frac{6 \times 10^{24}}{6.8 \times 10^6}} = 7.67 \times 10^3 \text{ ms}^{-1}$

and $T = \frac{2\pi R}{v} = \frac{2\pi \times 6.8 \times 10^6}{7.67 \times 10^3} = 5.57 \times 10^3$ s.

23 a



- Axes **[1]**
- Correct points **[1]**
- Straight line **[1]**

b The gradient of this graph is $\frac{R^3}{T^2} = \frac{GM}{4\pi^2} = 3.221 \times 10^{15}\ m^3s^{-2}$ **[1]**

So, $M = \frac{4\pi^2}{G} \times 3.221 \times 10^{15} = \frac{4\pi^2}{6.67 \times 10^{-11}} \times 3.221 \times 10^{15} =$ 1.91×10^{27} kg. **[1]**

24 a $v = \frac{2\pi R}{T} = \frac{2\pi \times 4.2 \times 10^8}{42 \times 60 \times 60} =$ $17.46\ kms^{-1} = 17\ kms^{-1}$ (2 s.f.) **[1]**

b $g = \frac{v^2}{R} = \frac{(17.46 \times 10^3)^2}{4.2 \times 10^8} = 0.73\ Nkg^{-1}$ **[2]**

c $|g| = G\frac{M}{R^2} \Rightarrow M = \frac{gR^2}{G} = \frac{0.73 \times (4.2 \times 10^8)^2}{6.67 \times 10^{-11}}$ $= 1.93 \times 10^{27}\ kg \approx 1.9 \times 10^{27}$ kg. **[2]**

25 a The speed of a body that will allow it to escape from the gravitational field it is in so that when it reaches an infinite distance **[1]** away, its kinetic energy will be zero. **[1]**

b The escape speed of a body is $v_{esc} = \sqrt{\frac{2GM}{R}}$. **[1]**

For the Sun, this is $v_{esc} =$ $\sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 2 \times 10^{30}}{7 \times 10^8}}$.

$= 6.17 \times 10^5\ ms^{-1}$ ($620\ kms^{-1}$ 2 s.f.) **[1]**

Since this is more than the initial speed of the solar wind, the solar wind particles will not be able to escape the solar system. **[1]**

26 a The work done, per unit mass, in moving a body from infinity to where it is in the gravitational field. **[1]**

b Gravitational potential $= -G\frac{M}{R}$

For Jupiter, $\frac{M}{R} = \frac{318}{11.2} = 28.4$ times that for the Earth.

Therefore, for Jupiter, $V_g =$ $28.4 \times 62.5\ MJkg^{-1} = -1775\ MJkg^{-1} \approx -1.8\ GJkg^{-1}$. **[2]**

c Escape speed $= \sqrt{\frac{2GM}{R}} =$ $\sqrt{2 \times 1.8 \times 10^9} = 6 \times 10^4\ ms^{-1}$ ($= 60\ kms^{-1}$) **[2]**

27 a The gravitational force between the satellite and the Earth. **[1]**

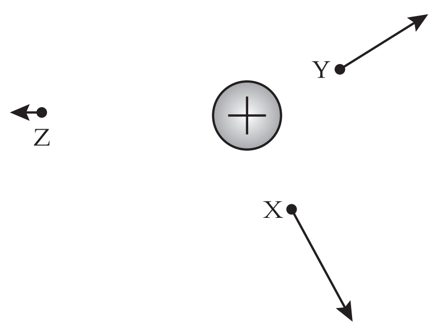
b $\frac{mv^2}{r} = \frac{GM_Em}{r^2} \Rightarrow v = \sqrt{\frac{GM_E}{r}}$ **[2]**

c $v = \frac{2\pi r}{T} \Rightarrow \frac{4\pi^2 r^3}{T^2} = GM_E \Rightarrow r = \sqrt[3]{\frac{GM_ET^2}{4\pi^2}} =$ $\sqrt[3]{\frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times (24 \times 60 \times 60)^2}{4\pi^2}} =$ 4.2×10^7 m **[2]** (This is about six Earth radii from the Earth's surface.)

Chapter 18

Exercise 18.1

1 a i



ii $F \propto \frac{1}{r^2}$, where r is the distance between the two charges.

b i F is towards B (because the charges are opposite).

ii Yes, sphere B experiences the same sized force, but in the direction towards A.

iii Newton's third law.

2 a i $F \propto q$

ii $F \propto Q$

iii $F \propto \frac{1}{r^2}$

b i Coulomb's law: the electrical force between two charged particles, q_1 and q_2, separated by a distance, r, is proportional to the product of their charges and inversely proportional to the square of their separation, r.

ii $F = k\frac{q_1 q_2}{r^2}$,

where k is Coulomb's constant

($k = \frac{1}{4\pi\varepsilon_o} = 9 \times 10^9\ Nm^2\,C^{-2}$

in air or in a vacuum), q_1 and q_2 are the charges on two charged particles and r is the separation of the charged particles.

c i $F = k\frac{Qq}{r^2} = 9 \times 10^9 \times \frac{50 \times 10^{-9} \times -20 \times 10^{-9}}{(30 \times 10^{-2})^2} = -1.0 \times 10^{-4}$ N

(Note that the minus sign is showing this is an attractive force.)

ii $F = k\frac{Qq}{r^2} = 9 \times 10^9 \times \frac{30 \times 10^{-6} \times 20 \times 10^{-6}}{(5 \times 10^{-3})^2} = 2 \times 10^5$ N (1 s.f.)

(Note that the positive value shows this is a repulsive force.)

iii $F = k\frac{Qq}{r^2} = 9 \times 10^9 \times \frac{-1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{(100 \times 10^{-12})^2} = -2.3 \times 10^{-8}$ N

iv $F = k\frac{Qq}{r^2} = 9 \times 10^9 \times \frac{6.4 \times 10^{-19} \times 1.26 \times 10^{-17}}{(3 \times 10^{-15})^2} = 8.1 \times 10^3$ N

3 a $F = k\frac{Qq}{r^2} = 9 \times 10^9 \times \frac{50 \times 10^{-3} \times 20 \times 10^{-3}}{(2 \times 10^{-3})^2} = 2.3 \times 10^{12}$ N (2 s.f.)

The sign of the force is positive, so the force is repulsive.

b $F = k\frac{Qq}{r^2} = 9 \times 10^9 \times \frac{-50 \times 10^{-3} \times 20 \times 10^{-3}}{(2 \times 10^{-3})^2} = -2.3 \times 10^{12}$ N

The sign of the force is negative, so the force is attractive.

c A positive force is repulsive, and a negative force is attractive.

4 a $F = k\frac{Q_1 q}{r_1^2} - k\frac{Q_2 q}{r_2^2} = kq\left(\frac{Q_1}{r_1^2} - \frac{Q_2}{r_2^2}\right) = 9 \times 10^9 \times 1 \times \left(\frac{12 \times 10^{-6}}{(2 \times 10^{-2})^2} - \frac{-12 \times 10^{-6}}{(2 \times 10^{-2})^2}\right) =$

5.4×10^8 N towards the $-12\,\mu C$ charge.

b $F = k\frac{Q_1 q}{r_1^2} - k\frac{Q_2 q}{r_2^2} = kq\left(\frac{Q_1}{r_1^2} - \frac{Q_2}{r_2^2}\right) = 9 \times 10^9 \times 1 \times \left(\frac{36 \times 10^{-6}}{(6 \times 10^{-2})^2} + \frac{-4 \times 10^{-6}}{(2 \times 10^{-2})^2}\right) = 0$ N.

c Since this is a Pythagoras triangle:

$F = k\frac{Q_1 q}{r_1^2} - k\frac{Q_2 q}{r_2^2} = kq\left(\frac{Q_1}{r_1^2} - \frac{Q_2}{r_2^2}\right) = 9 \times 10^9 \times 1 \times \frac{3}{5}\left(\frac{4 \times 10^{-6}}{(5 \times 10^{-2})^2} - \frac{-4 \times 10^{-6}}{(5 \times 10^{-2})^2}\right) =$

1.7×10^7 N vertically upwards. (2 s.f.)

d This is also a Pythagoras triangle, so,

$F = k\frac{Q_1 q}{r_1^2} - k\frac{Q_2 q}{r_2^2} = kq\left(\frac{Q_1}{r_1^2} - \frac{Q_2}{r_2^2}\right) = 9 \times 10^9 \times 1 \times \frac{4}{5}\left(\frac{4 \times 10^{-6}}{(5 \times 10^{-2})^2} - \frac{-4 \times 10^{-6}}{(5 \times 10^{-2})^2}\right) =$

2.3×10^7 N horizontally to the right (2 s.f.)

5 a Electric field strength: the force that acts on a unit positive test charge in the field.

b E has units of NC^{-1}.

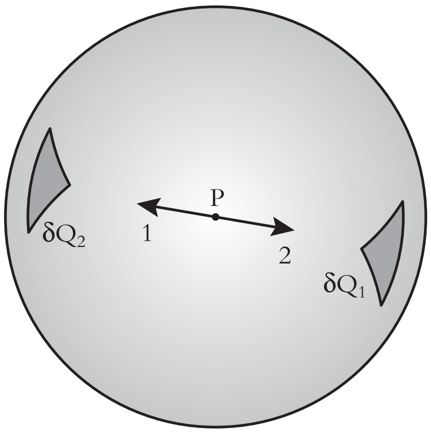
c $E = \frac{F}{q}$

6 a $E = k\frac{Q}{r^2} = 9 \times 10^9 \times \frac{6 \times 1.6 \times 10^{-19}}{(7 \times 10^{-10})^2} =$
$1.8 \times 10^{10}\ NC^{-1}$

b $F = E\,q = 1.8 \times 10^{10} \times -1.6 \times 10^{-19} =$
-2.9×10^{-9} N

c $a = \frac{F}{m} = \frac{2.9 \times 10^{-9}}{9.1 \times 10^{-31}} = 3.2 \times 10^{21}\ ms^{-2}$

7 a and b



c For each section, dQ_1, the opposite section, dQ_2, would have an opposite contribution to the total electric field strength at P. This suggests that the total field strength at the centre of the sphere would be zero.

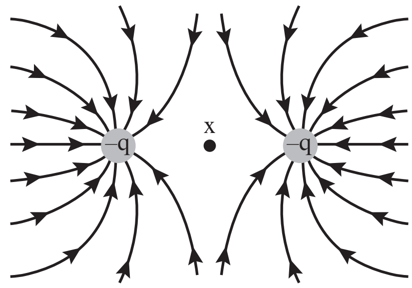
d All opposing sections of charge on the surface of the sphere would create a total field strength at P of zero. Therefore, the electric field strength inside a hollow sphere is zero in all places.

8 Using the same arguments for small charged volumes within the inside of the sphere, we can conclude that the electric field strength inside a solid charged sphere is also zero. (This assumes that the density of charge is the same throughout the sphere.)

9 a i Zero

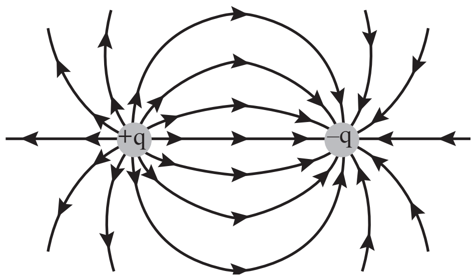
ii No.

iii



iv The arrows would point outwards, not inwards.

b



c i No

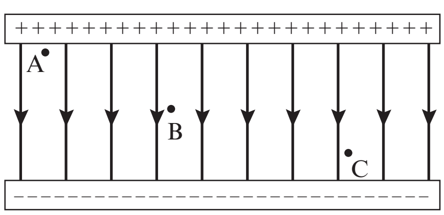
ii The electric field strength is large when the electric field lines are close together.

10 a i Electrons have been removed from the top plate, leaving it positively charged.

ii Electrons have been added to the bottom plate, making it negatively charged.

b Both plates are made from metallic conductors. Metallic conductors contain a large number of free electrons. These free electrons can move freely around. They will arrange themselves equally spaced apart, which will make the density of charge constant across the plate.

c



d Within the region between the two plates, the electric field lines are equally spaced. This shows a uniform electric field strength.

e Since the electric field strength is uniform, the size $\left(\frac{Vq}{d}\right)$ and direction (vertically downwards) of the force on the charge, q, will be the same at points, A, B and C.

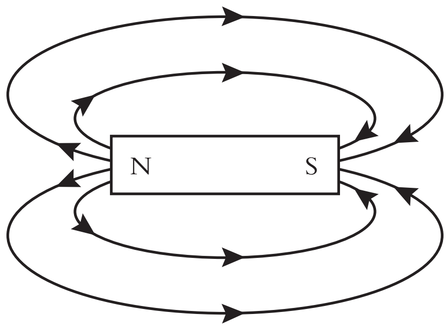
f **i** work done = Fd

ii work done = Vq

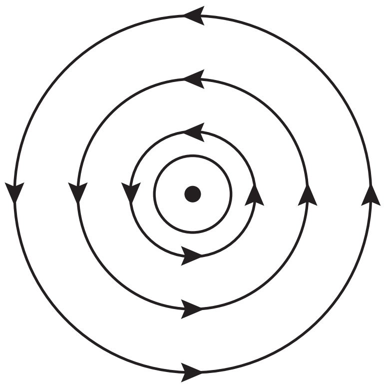
iii $Fd = Vq \therefore E = \frac{F}{q} = \frac{Vq}{dq} = \frac{V}{d}$

Exercise 18.2

1



2 **a**



b Use the right-hand grip rule: point the thumb in the direction of the current and allow the fingers to naturally curl. The direction in which the fingers curl shows the direction of the magnetic field lines.

Alternatively, if a plotting compass is placed in several positions around the wire it will align itself to the magnetic field and show the direction of the force on the north pole of the compass; this will be the magnetic field direction.

c More current means that the magnetic field around the wire will be stronger. To show this, the diagram should have more field lines, and these field lines will be closer together.

d $B \propto I$

e $B \propto \frac{1}{r}$, where r is the radial distance from the wire.

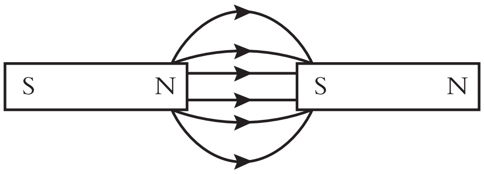
3 **a** Tesla

b **i** $B = \mu_o \frac{I}{2\pi r} = 4\pi \times 10^{-7} \times \frac{120 \times 10^{-3}}{2\pi \times 5 \times 10^{-2}} = 4.8 \times 10^{-7}\ T$

ii $B = \mu_o \frac{I}{2\pi r} = 4\pi \times 10^{-7} \times \frac{2}{2\pi \times 8 \times 10^{-2}} = 5.0 \times 10^{-6}\ T$

iii $B = \mu_o \frac{I}{2\pi r} = 4\pi \times 10^{-7} \times \frac{0.5}{2\pi \times 25 \times 10^{-2}} = 4.0 \times 10^{-7}\ T$

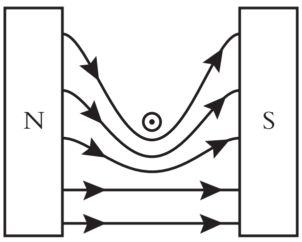
4 **a**



b **i** Out of the page—towards your eye.

ii Replace the small point inside the circle with a cross (X). This would show the current flowing into the page, away from your eye.

c **i**



ii A catapult field.

iii The effect of this catapult field is to exert a force on the conductor (in this case in the upwards direction).

iv Fleming's left-hand rule: point your first finger ahead, then point your thumb and your second finger perpendicular to the first finger. The first finger represents the field, the second finger represents the current and the thumb represents the force on the conductor.

v The force can be made larger by using stronger magnets, a larger current in the conductor, or a longer length of the conductor in the catapult field. (Also, make sure that the angle between the conductor carrying the current and the magnetic field of the two magnets is a right angle.)

5 a $F = B\,I\,l\sin\theta$

b i $F = B\,I\,l\sin\theta = 4 \times 10^{-5} \times 250 \times 10^{-3} \times 5 \times 10^{-2} \times \sin 90° = 5 \times 10^{-7}$ N

ii $F = B\,I\,l\sin\theta = 4 \times 10^{-5} \times 250 \times 10^{-3} \times 5 \times 10^{-2} \times \sin 30° = 2.5 \times 10^{-7}$ N

6 B is defined using the equation $F = B\,I\,l\sin\theta \Rightarrow B = \frac{F}{Il}$; for $\theta = 90°$, the force exerted on a 1-m length of the conductor carrying a 1 A current perpendicular to the magnetic field.

7 If the catapult force is sufficient to balance out the weight of the wire then the wire will be suspended in mid-air.

So, $B\,I\,l = mg$, where $m = \pi r^2 l\rho$.

So, $I = \frac{\pi r^2 l\rho g}{Bl} = \frac{\pi \times (1.0 \times 10^{-3})^2 \times 8900 \times 9.8}{0.2} = 1.4$ A (2 s.f.)

8 a i $B = \mu_o \frac{I}{2\pi r}$

ii B will be directed into the page.

b i $F = B\,I\,l\,(\sin\theta = 1)$

ii $F = B\,I\,l = \mu_o \frac{I}{2\pi r} \times I\,l = \mu_o \frac{I^2 l}{2\pi r}$

iii Using Fleming's left-hand rule, the force on the second wire will be directed leftwards, towards the first wire.

c The magnetic field caused by the current in the second wire is $B = \mu_o \frac{I}{2\pi r}$ and will be directed out of the page on the second wire's left-hand side (where the first wire is.) So, the force on the first wire will be $F = B\,I\,l = \mu_o \frac{I^2 l}{2\pi r}$ and will, again using Fleming's left-hand rule, be directed towards the right, towards the second wire. So the two forces experienced by the two current-carrying wires are equal in magnitude and opposite in direction, as Newton's third law states.

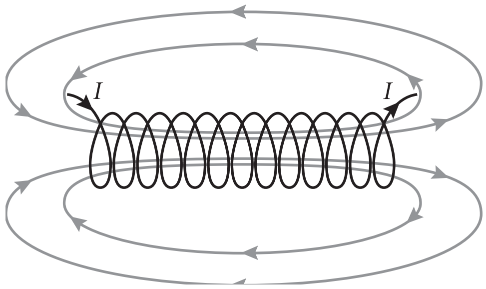
9 a The two wires would move towards each other. Each wire produces a magnetic field that interacts with the current flowing in the other wire to produce a catapult force. The direction of these two catapult forces is such that each wire experiences a force acting towards the other wire.

b With opposite currents, the two catapult forces exert a force on each wire away from the other wire. The two wires move away from each other.

c The SI unit of current, the ampere, is defined by the force experienced by each wire due to the currents flowing in the two wires:

An ampere is that constant current that flows in two infinitely long, straight, parallel conductors, of negligible cross section, 1 m apart in a vacuum, such that the force experienced by each wire is 2×10^{-7} N per metre length of the wire.

10 a



b i B would be larger: $B \propto \frac{N}{l}$.

ii B would be larger: $B \propto I$.

iii No change to B

c $B = \mu_o \frac{NI}{l}$

11 a $B = \mu_o \frac{NI}{l} = 4\pi \times 10^{-7} \times \frac{150 \times 30 \times 10^{-3}}{20 \times 10^{-2}} = 2.8 \times 10^{-5}$ T

b At the edge of the solenoid, the field is half the strength of the field in the centre.

So, $B = 1.4 \times 10^{-5}$ T at the edge.

12 a $t = \frac{l}{v}$

b $I = \frac{q}{t}$

c $F = BIl = B\frac{q}{t}l = B\frac{q}{\left(\frac{l}{v}\right)}l = Bqv$

d The force depends on the perpendicular component of the charged particle's velocity, so $F = Bqv\sin\theta$.

13 a $F = B\,q\,v = 0.5 \times 1.6 \times 10^{-19} \times 2 \times 10^{6} = 1.6 \times 10^{-13}$ N

b The direction of the force is perpendicular to the velocity, v, **and** perpendicular to the magnetic field, B.

c The electron will follow a circular path.

14 a Both paths will be circular.

b The paths will differ because

- the proton's path will be in the opposite direction to that of the electron (because their charges are opposite, so the force acting on them will be opposite) and
- the proton's path will have a much larger radius (by a factor of about 1800, because the mass of the proton is this much larger, meaning that its acceleration—and hence how much it curves—is much less).

Exercise 18.3

1 a $E = k\frac{Q}{r^2}$

b E will be zero.

c From Coulomb's law, the force is inversely proportional to the radial distance, r, away. So, as we move towards the point charge the force we need to apply increases.

d We have to do more and more work, in increasing increments, as we approach the point charge.

e The work we would do is given by the area under the graph of E against r.

f Work done = area under graph = $-\int E dr = -\int k\frac{Q}{r^2}dr = k\frac{Q}{r}$

Note that the minus sign shows that the force we have to apply to the unit test charge is in the opposite direction to the force exerted by the point charge on the unit test charge.

g This work done will be the electrical potential at a radial distance r from the point charge, Q.

h It has more energy. (Since we have done work on the test charge—and that work hasn't been transferred into kinetic energy—then the work must have been transferred into electrical potential energy.)

i The electrical force from electric field caused by the point charge will push the test charge away from Q and the test charge will lose its E_p until it is once again an infinite distance away from Q (where its E_p will be zero).

j Now we will have to do q times the work; that is $k\frac{Qq}{r}$.

k i $V_e = k\frac{Q}{r}$

ii $E_p = k\frac{Qq}{r}$

l i Volts, V—or Joules per Coulomb ($1\text{ V} \equiv 1\text{ JC}^{-1}$)

ii Joules, J

2 a $E = Vq = 50 \times 1 = 50$ J

b $E = Vq = 10 \times 1 = 10$ J

c Work done = $q\Delta V = 1 \times (50 - 10) = 40$ J

d No. The path does not matter, only the start and end points matter.

3 a An electrical equipotential is a line or surface on which the electrical potential is the same at all places.

b i Zero. X and Y are on an equipotential, so no work is required to move along from X to Y.

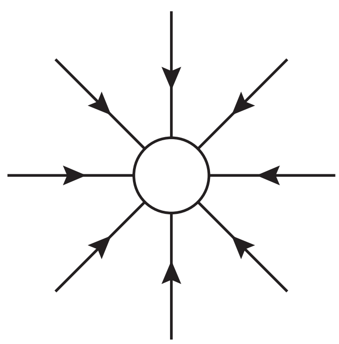
ii $E = q\,\Delta V = 4 \times 10^{-6} \times (12 - 3) = 3.6 \times 10^{-5}$ J

iii 3.6×10^{-5} J

c i No.

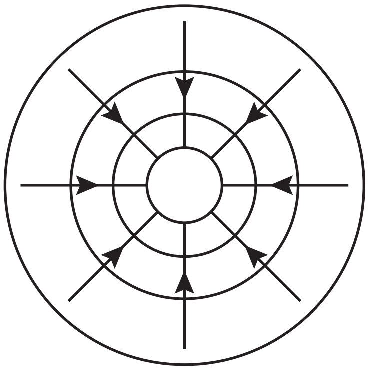
ii No.

4 a



b The potential varies inversely with increasing distance from the surface of the sphere.

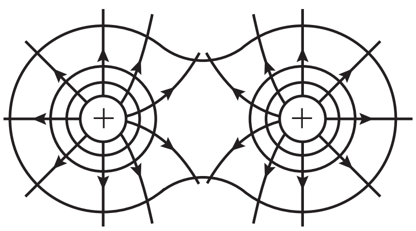
c



d When the electric field strength is large, the equipotentials are close together. When the electric field strength is small, the equipotentials are far apart.

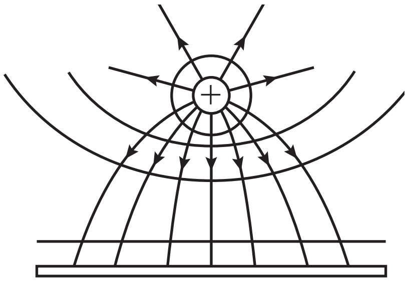
e The equipotentials and the field lines are perpendicular to each other.

5 **a** and **b**



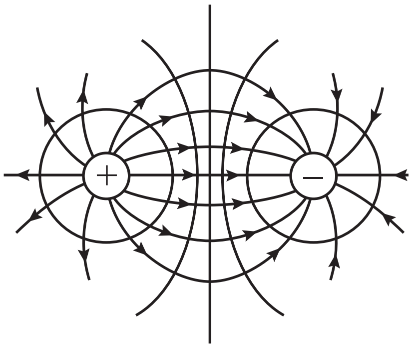
c The electric field strength is smallest where the equipotentials are farthest apart.

6 **a** and **b**



negatively charged plate

7 **a** and **b**



Field lines have arrows to show the direction of the field. Equipotentials have no direction, since potential is a scalar quantity.

c The electric field strength is greatest where the equipotentials are closest together.

8 **a** $\Delta V = kQ\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = 9 \times 10^9 \times 6 \times 10^{-3}$

$\left(\frac{1}{5 \times 10^{-2}} - \frac{1}{10 \times 10^{-2}}\right) = 5.4 \times 10^8$ V

b $E = q\,\Delta V = 2.0 \times 10^{-6} \times 5.4 \times 10^8 = 1.1$ kJ

c Since the force acting on it will be to repel it from the 6-mC charge, the 2.0-μC charge will be repelled, and it will accelerate radially away.

9 **a** $F = \frac{kQe}{r^2}$

b $\frac{mv^2}{r} = \frac{kQe}{r^2} \Rightarrow \frac{1}{2}mv^2 = \frac{kQe}{2r}$

c E_K of electron $= \frac{kQe}{2r}$ and E_P of the electron $= -\frac{kQe}{r}$ because the electron's charge, e, is negative.

So, $E_{total} = \frac{kQe}{2r} - \frac{kQe}{r} = -\frac{kQe}{2r}$, which is less than zero.

d $r = -\frac{kQe}{2 \times (-13.6\,eV)} = \frac{9 \times 10^9 \times (1.6 \times 10^{-19})^2}{2 \times 13.6 \times 1.6 \times 10^{-19}}$

$= 5.3 \times 10^{-11}$ m

10 **a** Decreased

b The total energy of the electron is now more negative than it had been. This means that it must be in an orbit that is closer to the nucleus. So, its kinetic energy has increased, since its E_K is inversely proportional to r.

c The E_P of the electron has become more negative.

d Closer

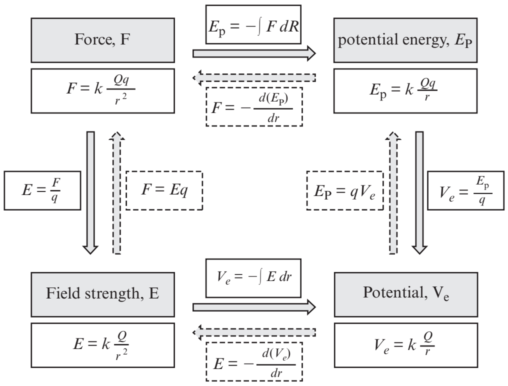
11 **a** $F = E\,q$

b $E_P = q\,V_e$

c $E = -\frac{dV_e}{dr}$

d $F = -\frac{d(E_P)}{dr}$

12



Exam-style questions

Multiple-choice questions

1 B **[1]**

2 A **[1]**

3 B **[1]**

4 A **[1]**

5 D **[1]**

6 C **[1]**

7 B **[1]**

8 A **[1]**

9 A **[1]**

10 D **[1]**

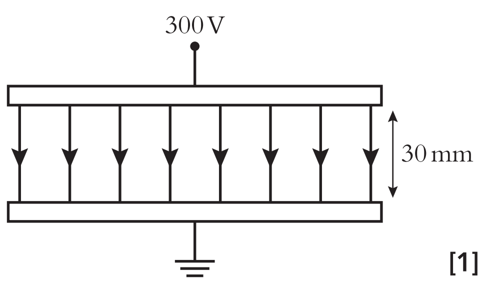
11 C **[1]**

12 D **[1]**

Short-answer questions

13 a $E = \frac{V}{d} = \frac{300}{30 \times 10^{-3}} =$
1.0×10^4 Vm^{-1} **[2]**

b



[1]

c $F = Eq = 1.0 \times 10^4 \times 1.6 \times 10^{-19}$
$= 1.6 \times 10^{-15}$ N **[1]**

d Towards the top plate **[1]**

14 a $E = \frac{V}{d} = \frac{100}{20 \times 10^{-2}} = 500$ Vm^{-1} **[2]**

b $F = Eq = 500 \times 6 \times 10^{-6} =$
3×10^{-3} N **[1]**

c Work done $= q\Delta V = 6 \times 10^{-6} \times$
$(100 - 0) = 6 \times 10^{-4}$ J **[2]**

15 a $F = k\frac{Qq}{r^2} = 9 \times 10^9 \times \frac{(1.6 \times 10^{-19})^2}{(5.3 \times 10^{-11})^2} =$
8.2×10^{-8} N **[2]**

b $a = \frac{F}{m} = \frac{8.2 \times 10^{-8}}{9.1 \times 10^{-31}} = 9.0 \times 10^{22}$ ms^{-2} **[1]**

c $v = \sqrt{ar} = \sqrt{9.0 \times 10^{22} \times 5.3 \times 10^{-11}}$
$= 2.2 \times 10^6$ ms^{-1} (2s.f.) **[2]**

16 a The left-hand point charge exerts a force $F = k\frac{Q}{r^2}$ **to the right** on the unit test charge. **[1]**

The right-hand point charge exerts a force $F = k\frac{Q}{r^2}$ **to the left** on the unit test charge. So the sum of these two forces is zero. **[1]**

b Since V_e is a scalar quantity, the net electrical potential at the position of the unit charge is $V_{e,\,net} = 2\,k\frac{Q}{r}$. **[1]**

c Suppose the test charge is displaced to the left. The net force on it will now be greater than zero and directed towards to the right, making the test charge accelerate to the right. **[1]** Similarly, if the test charge is displaced to the right, the net force on it will now be greater than zero and directed to the left, making the test charge accelerate towards the left. **[1]** This is rather like simple harmonic motion. This will result in the test charge undergoing a periodic oscillatory motion about the position equally distant from the two point charges. **[1]**

17 a $B = \mu_o\frac{NI}{l} = 4\pi \times 10^{-7} \times \frac{20 \times 50 \times 10^{-3}}{10 \times 10^{-2}} =$
$1.3 \times 10^{-5}\ T$ (2 s.f.) **[2]**

b The magnetic flux density at the two ends of the solenoid will be half the value at the centre. **[1]**

c Each pair of turns of the solenoid is behaving like a pair of parallel conductors, each with identical currents. **[1]** Each turn of the solenoid then experiences a force due to the turn next to it in the direction towards the next turn. In this way, all the turns of the solenoid are forced closer together. **[1]**

18 a 2×10^{-5} N **[1]** (From the definition of B)

b $N = nAl = 8.4 \times 10^{28} \times 1 \times 10^{-6} \times 1 = 8.4 \times 10^{22}$ electons **[2]**

c $F_{total} = NBqv = 8.4 \times 10^{22} \times 2 \times 10^{-5} \times 1.6 \times 10^{-19} \times 7.44 \times 10^{-5}$
$= 2.0 \times 10^{-5}\ N$ – the same as **part a**) **[2]**

Alternatively, this could be answered algebraically:

In **part a**, $F = BIl$

In **part c**, $F = NBqv = nAlBqv$ and $I = nAqv$ so $F = BIl$ as given by a) **[2]**

19 a An inverse-square law is a law where $Y = \frac{k}{X^2}$, **[1]**, where k is a constant (i.e. if X doubles in value, then Y becomes $\frac{1}{4}$ of the value). **[1]**

b Since Y = $\frac{k}{X^2}$, then $YX^2 = k$.
So, if pairs of values Y and X^2 are multiplied together, they should all come to the same constant, k. **[1]**

c $302 \times 2^2 = 1208$, $72 \times 4^2 = 1216$, $34 \times 6^2 = 1224$, $19 \times 8^2 = 1216$, $12 \times 10^2 = 1200$, $8 \times 12^2 = 1152$.

So, to two significant figures, all these give $k = 1200$. This confirms that X and Y are related by an inverse-square law.

Chapter 19

Exercise 19.1

1 a i $E = \frac{V}{d} = \frac{12}{4 \times 10^{-2}} = 300\ \text{Vm}^{-1}$

ii $a = \frac{F}{m} = \frac{qV}{md} = \frac{4 \times 10^{-3}}{1.5 \times 10^{-3}} \times 300$
$= 800\ \text{ms}^{-1}$

iii From $s = \frac{1}{2}at^2$, $t = \sqrt{\frac{2s}{a}} =$
$\sqrt{\frac{2 \times 4 \times 10^{-2}}{800}} = 1.0 \times 10^{-2}\ \text{s}$

iv $v = at = 800 \times 1 \times 10^{-2} = 8.0\ \text{ms}^{-1}$

v $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 1.5 \times 10^{-3} \times 8^2$
$= 48\ \text{mJ}$

b i work done $= qV = 4 \times 10^{-3} \times 12$
$= 48\ \text{mJ}$

ii The two answers are the same.
The work done by the electric field transfers to kinetic energy gained by the charged particle.

2 a v_h will not be affected by the electric field. Since the electric field can only exert a force on the charged particle in the vertical direction, the horizontal component of the particle's velocity will remain constant.

b Vertically downwards

c $v_v = at = \frac{F}{m}t = \frac{Eq}{m}t$

d $v = \sqrt{v_h^2 + v_v^2} = \sqrt{v_h^2 + \left(\frac{Eq}{m}t\right)^2}$

e $s = \left(v_h t, \frac{1}{2}\frac{Eq}{m}t^2\right)$

f Since the expression in **part e** is that of a parabola, the trajectory will be parabolic.

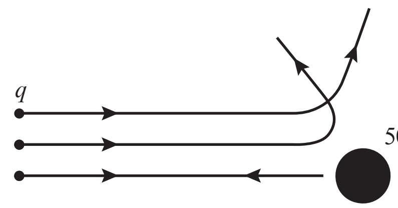
g Both examples will have a trajectory that is parabolic, because in both cases the acceleration of the body is in the vertical direction only, leaving the horizontal component of the velocity unchanged throughout. For the electric field case, the acceleration of the particle is $a = \frac{Eq}{m}$ downwards, and for the gravitational field case, the acceleration is g downwards, both of which are independent of the initial velocity of the particle.

3 If the electron is to pass through the plates, its vertical distance travelled must be less than 2.5 cm. So,

$s = \frac{1}{2}at^2$ with $a = \frac{eV}{dm} = \frac{1.6 \times 10^{-19} \times 6}{5 \times 10^{-2} \times 9.1 \times 10^{-31}} =$
$2.1 \times 10^{13}\ \text{ms}^{-2}$ and $t = \frac{30\ \text{cm}}{6\ \text{Mms}^{-1}} = 5 \times 10^{-8}\ \text{s}$
$\therefore s = \frac{1}{2} \times 2.1 \times 10^{13} \times (5 \times 10^{-8})^2 = 2.625\ \text{cm}.$

This is greater than 2.5 cm. So, the electron cannot pass through the two plates; it will hit the top plate just before it exits the electric field between the two plates.

4 a to **c**



The closer the q charge is to the $50q$ charge, the larger the electrical repulsive force becomes. This means that the acceleration of the q charge will be larger. Larger acceleration means a larger deflection from its original path.

d Yes; it should! This is very similar to what Lord Rutherford did (with Geiger and Marsden in 1909) in bombarding the nuclei of gold atoms with alpha particles in order to explore the structure of the atom.

e As the q charge approaches the $50q$ charge, its E_K is transferred to electrical potential energy, E_P. As the q charge then recedes from the $50q$ charge, its E_P is transferred back to E_K.

f This is the distance of closest approach.

g i Smaller initial E_K means that the deflection of the q charge will be greater—and it will occur at larger distances from the $50q$ charge.

ii Larger initial E_K will decrease the deflection caused by the electrical repulsion from the $50q$ charge.
Also, the distance of closest approach will become smaller—there will be more E_K to transfer to E_P, hence a smaller distance from the $50q$ charge.

5 $E_K \rightarrow E_P$ so $k\frac{Qq}{r} = E_K$

$\therefore r = k\frac{Qq}{E_K} = 9 \times 10^9 \times \frac{1.12 \times 10^{-18} \times 1.6 \times 10^{-19}}{1.6 \times 10^{-14}} =$

1.0×10^{-13} m

Exercise 19.2

1 a $F_m = Bqv$ $(\sin\theta = 1)$

b i Perpendicular to B

ii Perpendicular to v

c For a particle to travel in a circular path, it must have a centripetal force that is perpendicular to its velocity. In this case, the centripetal force is provided by the magnetic force, F_m, which is perpendicular to its velocity, v.

d $F_{\text{centripetal}} = \frac{mv^2}{r} = F_m = Bqv \therefore r = \frac{mv}{Bq}$

e i r increases.

ii r increases.

iii r decreases.

iv r decreases.

2 a i $F = Bqv = 4 \times 10^{-3} \times 1.6 \times 10^{-19} \times 5 \times 10^6 = 3.2 \times 10^{-15}$ N

ii $F = Bqv = 4 \times 10^{-3} \times 1.6 \times 10^{-19} \times 5 \times 10^6 = 3.2 \times 10^{-15}$ N

N.B. Same force because same charge but in opposite direction because charge is opposite sign.

b i $r = \frac{mv}{Bq} = \frac{1.67 \times 10^{-27} \times 5 \times 10^6}{4 \times 10^{-3} \times 1.6 \times 10^{-19}} = 13$ m

ii $r = \frac{mv}{Bq} = \frac{9.1 \times 10^{-31} \times 5 \times 10^6}{4 \times 10^{-3} \times 1.6 \times 10^{-19}} = 7.1$ mm

c i $T = \frac{2\pi r}{v} = \frac{2\pi \times 13}{5 \times 10^6} = 16\ \mu s$

ii $T = \frac{2\pi r}{v} = \frac{2\pi \times 7.1 \times 10^{-3}}{5 \times 10^6} = 8.9\ ns$

d i Since $F_m \propto v$, if v doubles, so does F_m.

ii Since $r \propto v$, if v doubles, so does r.

iii T depends on the ratio of $\frac{r}{v}$, so if both double, then the value of T remains the same.

iv This is an important idea. $T = T =$

$\frac{2\pi r}{v} = \frac{\left(2\pi\frac{mv}{Bq}\right)}{v} = \frac{2\pi m}{Bq}$

v T is independent of v.

3 a qV

b A circular path

c It will accelerate towards the left-hand semi-circle, gaining qV of kinetic energy so that its total kinetic energy is now $2qV$. Its speed has increased.

d It is travelling faster, so its radius of circular path will be larger.

e The charged particle will spiral outwards, because its radius of circular path is increasing each time it moves from one dee to the other.

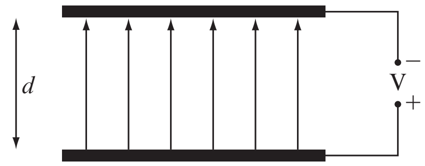
f i Quite close to the centre of the straight edge of one of the dees.

ii Near to the outer part of the straight edge of one of the dees.

g Since the time it takes to make half a circle is always the same—no matter what the speed or radius of the particle is—the frequency of the alternating supply to the two dees can remain constant and the charged particle will always arrive at the straight edge of one of the dees as the potentials of the two dees swaps over.

Exercise 19.3

1 a i



ii $F_e = \frac{qV}{d}$

iii The force is upwards and is perpendicular to the horizontal component of the charged particle's velocity, so it cannot alter the horizontal speed.

b i $F_m = Bqv$

ii The force will have to be in the downwards direction. So, using Fleming's left-hand rule, the magnetic field will have to be directed out of the page—towards one's eye.

c **i** Dynamic equilibrium means that there are no net forces acting on the charged particle; that is it will be travelling with a constant velocity.

ii The charged particle will follow a straight line horizontally through the two parallel plates.

iii The electric and magnetic forces act in opposite directions and cancel each other out, giving no net force, so

$\frac{qV}{d} = Bqv \Rightarrow B = \frac{V}{vd}$.

iv The charge, q, of the charged particle appears in both expressions for force, both electric and magnetic. So, mathematically, the charge cancels out. In addition, there is no term for mass in the expressions for force.

2 **a** The charged particle will now be deflected in the upwards direction.

b The charged particle will now be deflected in the downwards direction.

c The charged particle will now be deflected in the downwards direction.

d No deflection

e No deflection

f The charged particle will now be deflected in the downwards direction.

3 The electric force will be $F_e = \frac{qV}{d} = q \times \frac{80}{0.4} = 200\ q$.

The magnetic force will be $F_m = Bqv = 5 \times 10^{-6} \times q \times 4 \times 10^7 = 200\ q$.

Since these two force are equal, and opposite, the charged particle will pass through the region undeflected.

Exam-style questions

Multiple-choice questions

1 B **[1]**

2 D **[1]**

3 C **[1]**

4 D **[1]**

5 A **[1]**

6 C **[1]**

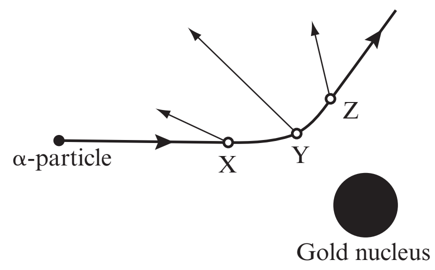
7 A **[1]**

8 B **[1]**

9 C **[1]**

Short-answer questions

10 **a** **i**



ii Y

b The alpha-particle slows down as it moves past X towards Y. Its minimum speed is at Y. **[1]** Then it speeds up again as it moves towards Z and beyond. **[1]**

c $F = k\frac{Qq}{r^2}$

$= 9 \times 10^9 \times \frac{1.26 \times 10^{-17} \times 3.2 \times 10^{-19}}{(2 \times 10^{-13})^2}$

$= 0.91$ N **[2]**

11 **a** $F = Bqv = 4 \times 10^{-6} \times 1.6 \times 10^{-19} \times 3 \times 10^6 = 1.9 \times 10^{-18}$ N **[2]**

b $a = \frac{F}{m} = \frac{1.9 \times 10^{-18}}{1.67 \times 10^{-27}} = 1.1 \times 10^9\ \text{ms}^{-2}$ **[1]**

c $r = \frac{mv}{Bq} = \frac{1.67 \times 10^{-27} \times 3 \times 10^6}{4 \times 10^{-6} \times 1.6 \times 10^{-19}} = 7.8 \times 10^3\ \text{m} \equiv 7.8\ \text{km}$ **[2]**

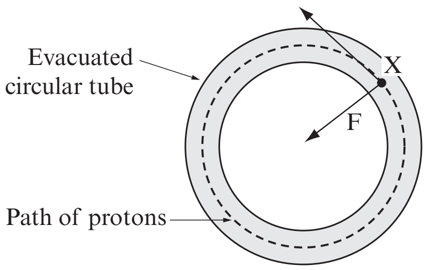
12 **a** $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 6.64 \times 10^{-27} \times (2 \times 10^7)^2 = 1.328 \times 10^{-12}$ J **[1]**

This is $\frac{1.328 \times 10^{-12}}{1.6 \times 10^{-13}} = 8.3$ MeV. **[1]**

b At the closest approach, the alpha particle's E_K will be zero, since all E_K transfers to E_P. **[1]**

So, $k\frac{Qq}{r} = E_K \Rightarrow r = k\frac{Qq}{E_K} = 9 \times 10^9 \times \frac{1.26 \times 10^{-17} \times 3.2 \times 10^{-19}}{1.328 \times 10^{-12}} = 2.7 \times 10^{-14}$ m. **[2]**

13 a **i** and **ii**



Velocity must be tangential to circular path; the force vector must be directed towards the centre of circular path. **[2]**

b $Bqv = \frac{mv^2}{r} \Rightarrow \frac{q}{m} = \frac{v}{Br} = \frac{2.88 \times 10^5}{3 \times 10^{-3} \times 1}$
$= 9.6 \times 10^7$ Ckg^{-1} **[2]**

(N.B. Look out for unit penalty here!)

c Since the specific charge of an electron is greater than that of a proton, for the same values of v and B, r would have to be smaller **[1]**—by a factor of about 1840.

14 a Out of the page **[1]**

b $F_e = \frac{qV}{d} = \frac{3.2 \times 10^{-19} \times 50}{0.25}$
$= 6.4 \times 10^{-17}$ N **[2]**

c For undeflected path, $F_e = F_m \Rightarrow$
$B = \frac{F_e}{qv} = \frac{6.4 \times 10^{-17}}{3.2 \times 10^{-19} \times 2 \times 10^7} =$
1.0×10^{-5} T. **[2]**

Chapter 20

Exercise 20.1

1 **a** The needle on the galvanometer will kick to one side of the scale and then return to zero.

b The needle on the galvanometer will kick to the other side of the scale (same amount as before) and then return to zero.

c The needle will not move.

d The needle will kick farther.

e The needle will not move.

2 **a** Towards the left (Fleming's left-hand rule)

b The left-hand side of the conductor will have a build-up of electrons, making it negatively charged. The right-hand side of the conductor will have lost some of its electrons, making it positively charged.

c Yes, there will be an electric field across the conductor because one end of the conductor is negatively charged and the other end is positively charged.

d In equilibrium, the electrical force acting on each electron is the same magnitude as the magnetic force acting on each electron but in the opposite direction.

e $F_E + F_m = 0 \Rightarrow \frac{\Delta Ve}{l} = Bev \Rightarrow \Delta V = Blv$

3 $\varepsilon = Blv = 4.0 \times 10^{-2} \times 12 \times 10^{-2} \times 2.0$
$= 9.6 \times 10^{-3} = 9.6$ mV

4 $\varepsilon = Blv = 0.14 \times 5 \times 10^{-2} \times 60 \times 10^{-2}$
$= 4.2 \times 10^{-3} = 4.2$ mV

5 $\varepsilon = Blv \sin\theta \Rightarrow \theta = \sin^{-1}\left(\frac{\varepsilon}{Blv}\right)$
$= \sin^{-1}\left(\frac{97 \times 10^{-3}}{0.3 \times 25 \times 10^{-2} \times 1.5}\right) = 60°$ (2 s.f.)

6 Magnetic flux, Φ, is the product of the magnetic flux density, B, and the perpendicular area through which it passes, A:

$\Phi = B\,A \cos\theta$, where θ is the angle between the perpendicular to the area and the field lines.

Magnetic flux linkage is the product of the magnetic flux and the number of turns of conductor: $N\Phi$.

7 $N\Phi = B\,A \cos\theta = 4.87 \times 10^{-5} \times 91.4 \times 55 \times \cos 24° = 0.22$ Wb

8 **a** ε is the induced emf (measured in volts), $\frac{d}{dt}$ is the operator that signifies the rate of change, N is the number of turns of the conductor and Φ is the magnetic flux associated with the conductor (measured in webers).

b $\varepsilon \propto N\frac{d(\Phi)}{dt}$ and $\Phi = BA$,
so $\varepsilon \propto N\left(B\frac{dA}{dt} + A\frac{dB}{dt}\right)$.

c An emf can be induced by changing A; this usually means moving something so that an area A is swept out in 1 s. Or, an emf can be induced by changing B; this usually means producing a magnetic field using an electromagnet and changing the strength of the electromagnet by changing the current flowing in it.

9 **a** The moving wire sweeps out an area, A, per second. This induces an emf in the wire, according to Faraday's law. Since the wire is connected to a galvanometer, there is a complete electrical circuit and so the induced emf causes a current to flow.

b The kinetic energy of the moving wire.

c Since the wire has lost kinetic energy, the speed of the wire must be reduced.

10 **a** Lenz's law: the direction of the induced emf is such that its effect will be to oppose the flux change that caused it.

b Conservation of energy

11 **a** There is an induced emf in the coil, which causes a current to flow.

b Lenz's law states that the direction of the induced emf (and hence the direction of the current) is such that its effect will be to oppose the original flux change. This means that the left-hand side of the coil will have to be a north pole so that it repels the north pole of the approaching magnet.

12 a When the switch is pressed, the current in the coil tries to change from zero to its proper value in a very short time. The changing current during this time causes a changing magnetic field in the solenoid. Faraday's law means that an induced emf occurs in the solenoid itself. But Lenz's law tells us that the effect of this induced emf must be to oppose its cause—that is to oppose the change from 0 to 10 V. So, the actual voltage across the solenoid will be less than 10 V. However, as soon as there is a potential difference across the solenoid, the rate of change of flux becomes less and so the induced emf becomes less, the voltage across the solenoid increases and so the current flowing through the solenoid increases. In this case, after about 100 ms, the voltage across the solenoid will be 10 V and the current flowing through the solenoid will be 1.0 A.

b Twice the number of turns means twice the induced voltage. So, the trace would take twice as much time to rise to 1.0 A.

13 a The iron rod will become magnetised, but the direction of the magnetic field in the iron rod will be changing periodically with the frequency of the supply.

b The changing magnetic field around the aluminium ring will induce an emf in the ring, which will make a current flow in the ring. However, since the rate of change of the magnetic field is changing (because the field itself is changing sinusoidally) the induced emf—and hence the current—is also changing.

c The current in the aluminium ring creates a magnetic field that opposes the magnetic field that is causing it. So, two opposing magnetic fields occur.

d The opposing magnetic fields exert a repulsive force on the ring, which pushes the ring upwards.

e The change of magnetic flux experienced by the ring is not as great as it had been when the switch was first pressed. So, the opposing magnetic fields do not exert as large a force on the ring. In fact, the ring then floats because the upwards force from the opposing magnetic fields is balancing the weight of the ring.

f For the ring with a cut through it, no induced current can flow around the ring and so no magnetic field is produced by the ring. This means no repulsive forces will occur and the ring will stay where it is.

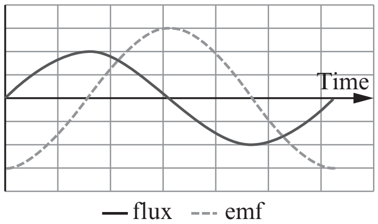
14 a As the coil moves, it leaves the magnetic field, making the magnetic flux linked to its decrease. So, the induced current must oppose this decrease and produce a magnetic field that is in the same direction as the one the coil is leaving. Therefore the induced current must flow clockwise.

b In this example the area enclosed by the coil is in a plane that is parallel to the magnetic field from the north pole of the magnet. This means there is no magnetic flux linkage. So, if the coil moves as shown, there is no change to the magnetic flux linkage, so there is no induced current in the coil.

c As the coil rotates, the area it encloses that is perpendicular to the magnetic field of the magnet changes. A change in the magnetic flux linkage will, therefore, produce an induced current in the coil. When the perpendicular area is increasing, the induced current must produce a magnetic field that opposes the north pole. When the perpendicular area is decreasing, the induced current must produce a magnetic field that is in the same direction as the north pole of the magnet. So, the induced current will be an alternating current.

Exercise 20.2

1 a



b $\Phi = BA \sin \omega t$

c i Induced emf, current will flow from A to B ii No induced emf (BC is parallel to the magnetic field, B) but current will flow from B to C

iii Induced emf, current will flow from C to D

iv No induced emf (DA is parallel to the magnetic field), but current will flow from D to A, making a complete circuit.

d i $\varepsilon = -N\frac{d\Phi}{dt} = -\frac{d}{dt}(BA \sin \omega t) = -BA\,\omega \cos \omega t$ ($N = 1$)

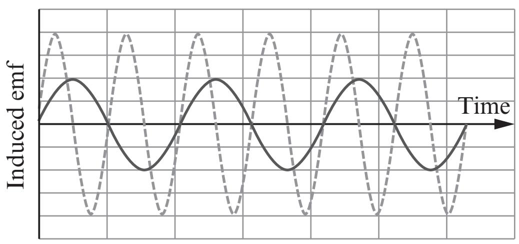
ii $BA\omega$

iii See figure in answer to **part a**.

iv They are 90° or $\pi/2$ out of phase.

e It is an a.c. generator.

2



3 a $\varepsilon \propto \omega$, so if ω is doubled, then ε will double; so, $\varepsilon = 120$ V.

b $\varepsilon \propto N$, so if N is reduced to 150 from 200 (i.e. it is now $\frac{3}{4}$), then ε will now be $\frac{3}{4}$; so, $\varepsilon = 45$ V.

4 a Zero

b Zero

c $P = IV = (I_o \sin \omega t) \times (E_o \sin \omega t) = I_o E_o \sin^2 \omega t$

d Since the average value of $\sin^2 \omega t$ is $\frac{1}{2}$, the mean power $= \frac{1}{2} I_o E_o$.

5 a The RMS value of an alternating current is the value of a *direct current* that would have the same power dissipation.

b $I_{RMS} = \frac{I_o}{\sqrt{2}}$

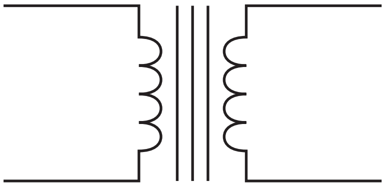
c $V_o = V_{RMS} \times \sqrt{2} = 240 \times 1.414 = 339$ V

6 $P_{average} = I_o V_o = 600 \times 10^{-3} \times 5.0 = 3$ W

7 a $V_{RMS} = \frac{V_o}{\sqrt{2}} = \frac{18}{\sqrt{2}} = 13$ V (2 s.f.)

b $P = \frac{1}{2}\frac{(V_{RMS})^2}{R} = \frac{1}{2}\frac{13^2}{12} = 7.0$ W

8 a



b Alternating current

c Iron has a high value of relative permeability (about 1000) and so is very receptive to magnetic fields. Iron is a magnetically soft material, so it will magnetise and de-magnetise quickly and easily.

d To prevent eddy currents from flowing in the core. Faraday's laws state that an induced emf will occur in **any** conductor present where there is a changing magnetic flux. So, there will be an induced emf in the iron core. Without laminating (which increases its resistance), this induced emf would make a current flow, which wastes energy heating the core.

e The alternating current in the primary coil sets up an alternating magnetic field in the iron core. The iron core links the secondary coil with the primary coil via the changing magnetic flux. Faraday's law states that the changing flux induces an emf in the secondary coil. The value of the induced emf depends on the number of turns on the coils. If the number of turns on the two coils are different, then the voltage across them will be different.

9 a A step-up transformer has more turns on the secondary coil than on the primary coil, so the voltage across the secondary coil is higher than the voltage across the primary coil.

b A step-down transformer has more turns on the primary coil than on the secondary coil, so the voltage across the secondary coil is lower than the voltage across the primary coil.

c An ideal transformer does not waste any energy (in the form of heat); the power supplied to the primary coil is the same as the power delivered by the secondary coil.

d This kind of transformer is called a *parity transformer*. It is used to isolate the actual power supply from the device that is going to be used. It is a safety feature.

10 a Step-down transformer

b $\frac{V_s}{V_p} = \frac{N_s}{N_p} \Rightarrow V_s = V_p \times \frac{N_s}{N_p} = 110 \times \frac{45}{360}$
$= 14\text{ V}$

c This may be used in several household appliances where a working voltage of 14 V is required, such as a power supply for a computer or a children's train set.

11 a $\frac{V_s}{V_p} = \frac{N_s}{N_p} \Rightarrow N_s = N_p \times \frac{V_s}{V_p} = 250 \times \frac{400\text{ kV}}{2\text{ kV}}$
$= 5 \times 10^4$ turns

b i $P = IV = 150 \times 2 \times 10^3 = 3 \times 10^5\text{ W}$

ii $I_s = \frac{P}{V_s} = \frac{3 \times 10^5}{400 \times 10^3} = 0.75\text{ A}$

12 a i $I = \frac{P}{V} = \frac{20}{240} = 83\text{ mA}$

ii $I_s = 20\, I_p = 20 \times 83\text{ mA} = 1.7\text{ A}$

b $Q = It = 1.7 \times 10 \times 60 = 1.0\text{ kC}$

13 The power loss in transmission lines depends on I^2. Reducing the current reduces energy losses. The step-up transformer changes the output voltage of the power station from about one thousand volts up to about 400 kV. In turn, this makes the current in the transmission wires 10^3 times smaller—and so the power losses will be 10^6 times smaller than without the transformer.

14 a i A

ii Out of the page—towards one's eye.

iii D

iv The magnetic field moves outwards at a finite speed (actually, c) from the wire.

v It is travelling radially outwards from the wire.

vi They are perpendicular to each other.

b i It will be changing direction at the same frequency as the a.c. supply to the wire.

ii Yes

iii An observer would see the plotting compass needle oscillating backwards and forwards, showing that the magnetic field was oscillating between out of the page and into the page.

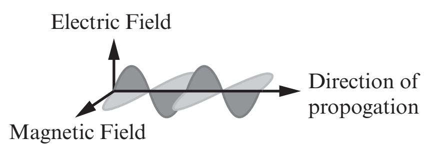
iv Yes. As already established in question 20.2.14, a changing magnetic field induces an electric field.

v Yes. The rate of change of magnetic field is also changing, so the induced electric field must also be changing.

vi They are all mutually perpendicular.

vii An electromagnetic wave is a varying magnetic field perpendicular to a varying electric field, both of which are moving in a mutually perpendicular direction.

viii



Exam-style questions

Multiple-choice questions

1 D **[1]**

2 D **[1]**

3 C **[1]**

4 A **[1]**

5 C **[1]**

6 C **[1]**

7 D **[1]**

8 C **[1]**

9 C **[1]**

Short-answer questions

10 a An induced emf will occur in any conductor linked with a changing magnetic flux **[1]** and the magnitude of the induced emf is proportional to the rate of change of magnetic flux linkage. **[1]**

(Must have both parts.)

b $\varepsilon = -N\frac{d\Phi}{dt}$ so $\varepsilon = -200 \times$ gradient of graph **[1]** $= -200 \times \frac{0.9}{6} = -30$ V **[2]**

11 a The rate of change of magnetic flux linkage is increasing. This is due to the speed of the magnet increasing, and the magnetic flux linked to the coil is increasing because the magnet is getting closer. **[2]**

b The magnet is leaving the bottom of the coil and so the change in magnetic flux is opposite to what it had been when the magnet was approaching the coil. **[1]**

c The speed at which the magnet is falling is increasing all the time, so the rate of change of magnetic flux is increasing and the induced emf is increasing. **[1]**

d Twice the number of turns would mean twice the magnetic flux linkage, and so twice the induced emf. So, the trace on the oscilloscope would show twice the voltage. **[1]**

12 a A larger B means a larger ε. **[1]**

b A larger N means a larger ε.

c A smaller A means a smaller ε.

d A larger ω means a larger ε **[1]** and that the frequency of ε increases. **[1]**

13 a Output voltage is less than input voltage. **[1]**

b Iron has a large relative permeability/iron core makes the magnetic field, produced by the current in the primary coil, stronger. **[1]**

c To reduce eddy currents in the iron core **[1]**

d For an emf to be induced in the secondary coil, the magnetic flux linked to it must change. **[1]** A changing current in the primary coil will produce a changing magnetic flux in the iron core that is linked to the secondary coil. **[1]**

e The transformer is ideal, so input power = output power. Power = IV, so if output V is smaller, then output I must be larger. **[1]**

14 a $N\Phi = NBA = 120 \times 4 \times 10^{-3} \times 5 \times 10^{-3}$
$= 2.4 \times 10^{-3}$ Wb **[2]**

b The perpendicular area through which the magnetic flux passes is varying sinusoidally with time. **[1]** So the sinusoidally changing magnetic flux linkage means that a sinusoidally varying induced current occurs. **[1]**

c The maximum induced current will be twice as large as it had been. **[1]** This is because the rate at which the magnetic flux linkage is changing is twice as large as it had been. **[1]**

15 a Two of the following:

- The coil on the live wire has the same number of turns as the coil on the neutral wire. **[1]**
- In normal operation, the current in each coil is the same. **[1]**
- So, the magnetic flux generated by each coil is the same magnitude. **[1]**
- But the two coils are wound oppositely around the core. **[1]**
- So, the magnetic flux from one coil cancels out the magnetic flux from the other. **[1]**

b
- If the current in the live wire is different from the current in the neutral wire, then there will be a net magnetic flux in the iron core. **[1]**
- The change from zero magnetic flux to some magnetic flux induces a current in the third coil around the iron core, due to Faraday's law. **[1]**
- The current in the relay coil magnetises the relay coil. **[1]**
- The magnetised relay coil attracts the relay switch, which moves and breaks the circuit. **[1]**

16 a Answer must have first condition. Second mark for either of the other statements.

$N_s > N_p$ **[1]**

Both coils wrapped around a laminated iron core **[1]**

Supplied with a.c. current **[1]**

b Secondary voltage must be ≥60 kV.

So: $\frac{60 \text{ kV}}{240 \text{ V}} = \frac{N_s}{120} \Rightarrow N_s = 120 \times \frac{60 \text{ kV}}{240 \text{ V}} =$ 30 000 turns **[2]**

c The sparks between the electrodes produce x-rays, which are dangerous to humans. So, the demonstration has been banned for health and safety reasons. **[1]**

17 a An induced emf will occur (in any conductor) when there is a changing magnetic flux. **[1]** The magnitude of the induced emf is proportional to the rate of change of magnetic flux. $\varepsilon \propto N\frac{d\Phi}{dt}$ **[1]**

b The changing magnetic field induces an emf in the conductor, which causes a current (an a.c. current) to flow in the vertical direction. **[1]**

c The changing electric field exerts a changing force on the electrons in the conductor, causing a changing current to flow in the vertical direction. **[1]**

d The changing magnetic field effect and the changing electric field effect produce the same result in the conductor **[1]** So, there will be an oscillating current in the conductor, which can then be used in a circuit designed to extract information from the electromagnetic wave (i.e. a radio receiver). **[1]**

Chapter 21

Exercise 21.1

1 a A positively charged mass of an atom in which negatively charged electrons were embedded.

b Geiger and Marsden fired alpha particles at gold nuclei and observed where they were scattered.

Most of the alpha particles passed through the gold foil undeflected. Some were deflected through large angles, and a few were deflected backwards. As a result of their observations, they were able to construct a new model for the atom.

c The 'size' of an atom of gold is close to 10^5 times larger than the 'size' of its nucleus. At a distance equal to the radius of an atom of gold, the Coulomb repulsive force on an alpha particle would be 10^{-10} times the force at a distance of the radius of the nucleus. A force of this size would not be sufficient to cause the large angle deflections that were observed in Geiger and Marsden's experiment.

2

Experimental observation	Conclusion
The vast majority of alpha particles passed through the gold foil undeflected.	Most of the space taken up by the atom is empty—that is it does not contain anything.
Some alpha particles were deflected through such large angles that they bounced backwards.	There was a small, positively charged and dense nucleus at the centre of the atom.

3 The distance of closest approach must be larger than the radius of a gold nucleus. So the gold nucleus must be smaller than this distance of

$$5\text{ MeV} = k\frac{Qq}{r} \Rightarrow r = k\frac{Qq}{5\text{ MeV}} =$$

$$9 \times 10^9 \times \frac{79 \times 1.6 \times 10^{-19} \times 2 \times 1.6 \times 10^{-19}}{5 \times 1.6 \times 10^{-13}} =$$

4.6×10^{-14} m.

(Actually the nucleus of a gold atom has a radius of about 7×10^{-15} m, so the alpha particle would have had a distance of closest approach that is about six-and-a-half nuclear radii.)

4 a Excited gas at a low pressure emits electromagnetic radiation as electrons of the atoms of gas fall from higher energy levels to lower energy levels, each time emitting a photon. The term *spectrum* is used to describe the different wavelengths present in these emissions. Each wavelength corresponds to a different change in energy levels.

b Place a sample of gas in a glass tube and close the tube. Excite the gas by placing a strong electric field across it. The gas will glow. A student may then observe the emission spectrum by looking at the gas through a diffraction grating.

5 Emission spectra consist of a set of discrete lines. Each line in the emission spectrum represents a wavelength. Each different wavelength is associated with a different amount of energy. These different amounts of energy are due to different energy level transitions of electrons in atoms. Because the emission lines are discrete, the energy-level transitions of the electrons must correspond to discrete amounts. This can only be true if the energy levels themselves exist in discrete energy values.

6 a The planetary model of the atom has a nucleus at the centre of the atom and electrons that orbit around the nucleus, like planets orbit around a star. The different radii of the orbits of the electrons help us to visualise the different electron energy levels.

b i The electromagnetic force keeps the electrons in orbit around the nucleus.

ii Towards the nucleus

iii Yes. The electron in its orbit is changing direction all the time, so it must be accelerating.

c A constantly accelerating electron should be constantly emitting radiation, which would cause its energy to decrease and it would spiral in towards the nucleus. Experiments show that this does not occur, so the planetary model is flawed.

7 a Since the electron is attracted to the positive charge in the nucleus, it takes energy to pull the electron away. By convention, this is given a negative value.

b The level labelled $n = 1$ is the ground state. This is the lowest energy state of the atom.

c In level $n = 1$

An atom at room temperature will have an average kinetic energy of

$E = \frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 293 = 6.1 \times 10^{-21}$ J.

This is very much smaller than the energy of the ground state, $n = 1$ ($\approx 2 \times 10^{-18}$ J).

d The electron would jump up to the next energy level, $n = 2$.

e This process is called *excitation*.

8 a In the levels $n = 2$ or higher

b Excited: the electrons are in an energy level higher than they would normally be, that is above the ground state.

c An excited electron is most likely to fall down to a lower energy level, emitting the difference in energy between the two levels as a photon.

d The process of falling to a lower energy level is called spontaneous emission.

e The energy of the atom has decreased. It has become more negative.

9 a $E_n = \frac{-13.6\text{ eV}}{n^2}$; then for $n = 2$,

$E_2 \frac{-13.6\text{ eV}}{2^2} = -3.4$ eV.

b $E_n = \frac{-13.6\text{ eV}}{n^2}$; then for $n = 3$,

$E_3 \frac{-13.6\text{ eV}}{3^2} = -1.5$ eV.

c $E_n = \frac{-13.6\text{ eV}}{n^2}$; then for $n = 4$,

$E_4 \frac{-13.6\text{ eV}}{4^2} = -0.85$ eV.

10 a A photon is a discrete particle of light energy.

b E is the photon's energy, h is Planck's constant and f is the frequency of the electromagnetic radiation.

c $E = hf$ and $f = \frac{c}{\lambda}$, so $E = h\frac{c}{\lambda}$.

d An electronvolt is the amount of kinetic energy gained by an electron that has been accelerated through a potential difference of 1 volt.

e i $E = \frac{hc}{\lambda} = \frac{6.64 \times 10^{-34} \times 3 \times 10^8}{630 \times 10^{-9}} =$

$3.16 \times 10^{-19}\text{ J} = \frac{3.16 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.98$ eV

ii $E = \frac{hc}{\lambda} = \frac{6.64 \times 10^{-34} \times 3 \times 10^8}{532 \times 10^{-9}} =$

$3.7 \times 10^{-19}\text{ J} = \frac{3.74 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.34$ eV

iii $E = \frac{hc}{\lambda} = \frac{6.64 \times 10^{-34} \times 3 \times 10^8}{430 \times 10^{-9}} =$

$4.63 \times 10^{-19}\text{ J} = \frac{4.63 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.89$ eV

11 a The four visible lines are due to transitions from $n = 6$, 5, 4 and 3 to $n = 2$.

b $E = \frac{hc}{\lambda} = \frac{6.64 \times 10^{-34} \times 3 \times 10^8}{656.3 \times 10^{-9}} = 3.04 \times 10^{-19}$ J

$= \frac{3.01 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.88\text{ eV} \approx 1.9$ eV

c $\lambda = \frac{hc}{E} = \frac{6.64 \times 10^{-34} \times 3 \times 10^8}{2.55 \times 1.6 \times 10^{-19}} = 488$ nm.

The turquoise coloured line.

12

Series name	Ending energy level	e.m. radiation emitted
Lyman	1	Ultraviolet
Balmer	2	Visible
Paschen	3	Infrared
Brackett	4	Infrared
Pfund	5	Infrared

13 The emission spectrum from a filament light bulb is continuous: it contains a wide range of wavelengths from the deep red colours to the violet end of the spectrum. The emission spectrum from an excited gas consists of a limited number of discrete lines at particular wavelengths.

14 a Taking an average wavelength of 589.3 nm,

$E = \frac{hc}{\lambda} = \frac{6.64 \times 10^{-34} \times 3 \times 10^{8}}{589.3 \times 10^{-9}} = 3.38 \times 10^{-19}$ J

$= \frac{3.38 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.11$ eV.

So, the difference in energy between these two levels must be 2.11 eV.

b The energy level $n = 2$ is actually split into two energy levels, which are about 3.4×10^{-22} J apart (or about 2 meV apart).

15 a Red

b i $E = \frac{hc}{\lambda} = \frac{6.64 \times 10^{-34} \times 3 \times 10^{8}}{632.8 \times 10^{-9}} =$ 3.15×10^{-19} J

ii $E = \frac{3.15 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.97$ eV

16 a The Fraunhofer lines are absorption lines. Radiation created in the Sun's core travels outwards and is absorbed (and re-emitted in all directions) by atoms in its outer layers at wavelengths corresponding to transitions in the atoms. This removes some of the intensity of the radiation in any particular direction, causing a dark line to occur in the emission spectrum.

b By comparing the patterns of the absorption lines with those of well-known atoms, the kind of atoms present in the outer layers can be determined. Also, the amount by which the intensity is reduced can lead to information about the relative abundance of such atoms. Together, this information can inform astrophysicists about the chemical composition of the outer regions of the Sun and other stars.

Exercise 21.2

1 a A small, dense, positively charged nucleus, around which, at some distance, electrons orbit.

b Classical physics suggests that charged particles accelerating will radiate electromagnetic energy. If electrons orbited the nucleus, as Rutherford suggested, then they would be accelerating and so should be radiating electromagnetic radiation. The loss of energy would make the electron fall into the nucleus. Since this is not observed, Bohr objected to Rutherford's model.

c Bohr proposed that there might be certain 'special' orbits in which the electron did not radiate electromagnetic energy. This compromise would allow Rutherford's model.

2 a $\text{kgm}^2\text{s}^{-1}$

b This is angular momentum, L.

c i *Quantised* means that it can take only certain discrete values and that these values will all be integer multiples of a 'unit' amount.

ii h has units of J s. J $\equiv \text{kgm}^2\text{s}^{-2}$, so J s $\equiv \text{kgm}^2\text{s}^{-1}$, which is the same as for angular momentum, L.

3 a $k\frac{e^2}{r_1^2} = \frac{mv^2}{r_1} \Rightarrow r_1 = k\frac{e^2}{mv^2}$

b $mvr_1 = \frac{h}{2\pi} \Rightarrow v^2 = \frac{h^2}{4\pi^2 m^2 r_1^2}$

So, $r_1 = k\frac{e^2}{mv^2} = k\frac{e^2}{m\frac{h^2}{4\pi^2 m^2 r_1^2}} = \frac{h^2}{4\pi^2 ke^2 m}$.

c $r_1 = \frac{h^2}{4\pi^2 ke^2 m} =$

$\frac{(6.63 \times 10^{-34})^2}{4\pi^2 \times 9 \times 10^9 \times (1.6 \times 10^{-19})^2 \times 9.1 \times 10^{-31}} =$

0.53×10^{-10} *m*

d For $n = 2$, $r_2 = 4 \times 0.53 \times 10^{-10} =$ 2.12×10^{-10} m.

For $n = 3$, $r_3 = 9 \times 0.53 \times 10^{-10} =$ 4.77×10^{-10} m.

For $n = 4$, $r_4 = 16 \times 0.53 \times 10^{-10} =$ 8.48×10^{-10} m.

4 a Electric force—the Coulomb attraction between the electron and the proton.

b $F = -k\frac{e^2}{r^2}$ (Note that since the charge on the proton is the same magnitude, but has the opposite sign to that of the electron, there is no need here to use the usual Q and q for the two charges.)

c $|F| = k\frac{e^2}{r^2} = m\frac{v^2}{r}$. $E_K = \frac{1}{2}mv^2 = \frac{r}{2}F = k\frac{e^2}{2r}$

d The electrical potential energy of the electron is $E_P = -k\frac{e^2}{r}$, so the total energy is $E_{total} = k\frac{e^2}{2r} + -k\frac{e^2}{r} = -k\frac{e^2}{2r}$.

e $E_{total} = -k\frac{e^2}{2r} = -9 \times 10^9 \times \frac{(1.6 \times 10^{-19})^2}{2 \times 0.53 \times 10^{-10}} =$ -2.17×10^{-18} J $= \frac{-2.17 \times 10^{-18}}{1.6 \times 10^{-19}} = -13.6$ eV

5 a $\frac{m_p}{m_e} = \frac{1.67 \times 10^{-27}}{9.1 \times 10^{-31}} = 1835 \approx 1840$

b Yes, because the electron's mass is only about $\frac{1}{1840} = 0.05\ \%$ of the mass of the atom.

c $\frac{\text{volume of an atom}}{\text{volume of a nucleus}} = \frac{(10^{-10})^3}{(10^{-15})^3} = 10^{15}$

d 10^{15}

e $\rho_{\text{proton}} = \frac{\text{mass}}{\text{volume}} = \frac{1.67 \times 10^{-27}}{\frac{4}{3}\pi r^3} = \frac{1.67 \times 10^{-27}}{\frac{4}{3}\pi(1.2 \times 10^{-15})^3} = 2.3 \times 10^{17}\ \text{kgm}^{-3}$

f $\frac{\rho_{\text{proton}}}{\rho_{\text{diamond}}} = \frac{2.3 \times 10^{17}}{3.51 \times 10^3} = 6.6 \times 10^{13} \sim 10^{14}$.

The answer is only a factor of ten different.

6 a $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(r_o A^{\frac{1}{3}}\right)^3 = \frac{4}{3}\pi r_o^3 A$

b $\frac{A}{V} = \frac{A}{\frac{4}{3}\pi r_o^3 A} = \frac{3}{4\pi r_o^3}$

c This is independent of A and implies that all nuclei should have the same density.

d $\frac{3m}{4\pi r_o^3} = \frac{3 \times 1.7 \times 10^{-27}}{4\pi(1.2 \times 10^{-15})^3} = 2.3 \times 10^{17}\ \text{kgm}^{-3}$

e They are the same. All nuclei have the same density.

7

Nucleus	Nucleon number	Radius ($\times 10^{-15}$ m)	Density ($\times 10^{17}$ kgm^{-3})
H	1	1.2	2.35
He	4	1.9	2.35
C	12	2.7	2.35
S	32	3.8	2.35
Sr	88	5.3	2.35
Au	197	7.0	2.35
U	238	7.4	2.35

8 a The radius of a golf ball is about 1.3 cm. So, its volume is $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times 0.013^3 = 9.2 \times 10^{-6}\ \text{m}^3$.

b $m = \rho V = 2.3 \times 10^{17} \times 9.2 \times 10^{-6} = 2.1 \times 10^{12}$ kg

c $\frac{2.1 \times 10^{12}}{46 \times 10^{-3}} = 4.6 \times 10^{13}$

Exam-style questions

Multiple-choice questions

1 B **[1]**

2 D **[1]**

3 D **[1]**

4 A **[1]**

5 C **[1]**

6 D **[1]**

7 A **[1]**

8 B **[1]**

9 D **[1]**

10 C **[1]**

Short-answer questions

11 a $E = hf = 6.63 \times 10^{-34} \times 3.08 \times 10^{15} = 20.4 \times 10^{-19}$ J **[1]**

The transition from $n = 4$ to $n = 1$ gives $E = (-1.36 - -21.8) \times 10^{-19} = 20.4 \times 10^{-19}$ J.

So, the transition responsible must be from $n = 4$ to $n = 1$. **[1]**

b Ultraviolet **[1]**

c $E_K = hf - \Delta E = (6.63 \times 10^{-34} \times 4.0 \times 10^{15}) - 21.8 \times 10^{-19}$

$= 4.72 \times 10^{-19}\ \text{J} = \frac{4.72 \times 10^{-19}\ \text{J}}{1.6 \times 10^{-19}\ \text{J eV}^{-1}} = 2.95$ eV **[2]**

12 a i This is the energy level in which the atom has the lowest amount of energy. **[1]**

ii This means that the electron is in an energy level higher than the ground state. **[1]**

iii This means that the electron has gained enough energy to escape from the atom completely—leaving behind only the proton/nucleus. **[1]**

b Electrons in energy levels above the $n = 2$ level **[1]** fall to the $n = 2$ level **[1]** emitting photons that have their energies in the visible part of the e.m. spectrum.

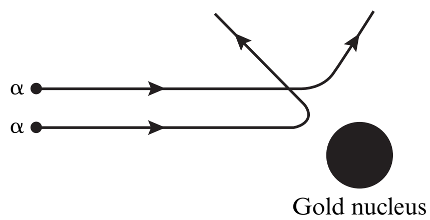
13 a $\lambda = \frac{hc}{E} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{(-2.6 - 8.8) \times 10^{-19}} =$
3.2×10^{-7} m = 320 nm **[2]**

b $n = 3 \rightarrow n = 2$ **[1]**

c $n = 4 \rightarrow n = 1$; **[1]** $n = 3 \rightarrow n = 1$ and $n = 2 \rightarrow n = 1$ **[2]**

14 a $r = r_o A^{\frac{1}{3}} = 1.2 \times 10^{-15} \times 197^{\frac{1}{3}} =$
7.0×10^{-15} m (2 s.f.) **[1]**

b



[1]

c Distance of closest approach must be $\leq 8.0 \times 10^{-15}$ m. So,
$E = k\frac{Qq}{r} = 9 \times 10^{9} \times$
$\frac{79 \times 1.6 \times 10^{-19} \times 2 \times 1.6 \times 10^{-19}}{8 \times 10^{-15}} =$
$4.55 \times 10^{-12}\ \text{J} = \frac{4.55 \times 10^{-12}\ \text{J}}{1.6 \times 10^{-19}\ \text{J}\ eV^{-1}} =$
28 MeV. **[2]**

15 a The dark lines are caused by absorption of those wavelengths that correspond to the energy-level transitions of the electrons in the atoms of the hydrogen gas.
The excited electrons then re-emit the photons but in all directions.
Since the intensity of the light in the direction being viewed has been reduced, the observer sees dark lines. **[2]**

b $E = \frac{hc}{\lambda} = \frac{6.64 \times 10^{-34} \times 3 \times 10^{8}}{434 \times 10^{-9}} =$
$4.59 \times 10{-}^{19}\ \text{J} = \frac{4.59 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.87$ eV **[2]**

c The absorption spectra from stars show discrete energy-level transitions within the atoms in the star. Scientists have catalogued the spectra of elements and compounds in laboratory experiments. The star spectra can be matched up to the catalogued spectra, giving us information about which elements and compounds are present in the star. **[1]**

16 a mass of electron, **[1]** speed of electron in its orbit, radius of electron orbit **[2]**

b $mvr = \frac{h}{2\pi}$ ($n = 1$ for ground state)
So, $v = \frac{h}{2\pi mr} = \frac{6.63 \times 10^{-34}}{2\pi \times 9.1 \times 10^{-31} \times 0.53 \times 10^{-10}}$
$= 2.2 \times 10^{6}\ \text{ms}^{-1}$ **[2]**

c It would be moving at a speed $\frac{1}{4}$ of that in the ground state.
So: $v = \frac{2.2 \times 10^{6}\ ms^{-1}}{4} = 5.5 \times 10^{5}\ \text{ms}^{1}$ **[1]**

17 a $\frac{1}{2}mv^{2} = 3.5\ \text{MeV} \Rightarrow$
$v = \sqrt{\frac{2 \times 3.5 \times 1.6 \times 10^{-13}}{6.64 \times 10^{-27}}} =$
$1.3 \times 10^{7}\ \text{ms}^{-1}$ **[2]**

b It slows down, at an increasing rate. **[2]**

c At Y, the alpha particle will have an electrical potential energy of
$E_p = k\frac{Qq}{r} = 9 \times 10^{9} \times$
$\frac{79 \times 1.6 \times 10^{-19} \times 2 \times 1.6 \times 10^{-19}}{8 \times 10^{-14}} =$
4.55×10^{-13} J. **[1]**

So, at Y, the alpha particle will have a kinetic energy of
$(3.5 \times 1.6 \times 10^{-13}) - 4.55 \times 10^{-13} =$
1.05×10^{-13} J. **[1]**

18 a For $n = 3$, $E = \frac{-13.6\ \text{eV}}{3^{2}}$.
So, the energy required for the transition $n = 1$ to $n = 3$ (or above) is $E = 13.6\ \text{eV} \times \left(\frac{1}{1^{2}} - \frac{1}{3^{2}}\right) =$
12.1 eV (3 s.f.). **[2]**

b **i** Transitions in the Balmer series will produce visible light photons. **[1]**

OR

Transitions that finish on the $n = 2$ energy level will produce visible light photons. **[1]**

ii Any two of the following:

- Atoms can transfer energy from one to another by collisions.
- At room temperature, hydrogen atoms will be in the ground state.
- Hydrogen atoms in the ground state don't have enough energy to transfer a minimum of 12.1 eV during collisions.
- (A good answer would include $E = \frac{3}{2}kT = \frac{3}{2} \times \frac{1.38 \times 10^{-23} \times 293}{1.6 \times 10^{-19}} \approx$ 0.04 eV)
- So electrons cannot get to level 3 or above in order to fall to level 2 and emit visible light photons.

Chapter 22

Exercise 22.1

1 a i $E = hf = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{2.5} = 8.0 \times 10^{-26}\text{ J} = \frac{8.0 \times 10^{-26}}{1.6 \times 10^{-19}} = 5.0 \times 10^{-7}\text{ eV}$

ii $E = hf = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{6.0 \times 10^{-6}} = 3.3 \times 10^{-20}\text{ J} = \frac{3.3 \times 10^{-20}}{1.6 \times 10^{-19}} = 0.21\text{ eV}$

iii $E = hf = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{623 \times 10^{-9}} = 3.2 \times 10^{-19}\text{ J} = \frac{3.2 \times 10^{-17}}{1.6 \times 10^{-19}} = 2.0\text{ eV}$

iv $E = hf = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.5 \times 10^{-10}} = 1.3 \times 10^{-15}\text{ J} = \frac{1.3 \times 10^{-15}}{1.6 \times 10^{-19}} = 8.1\text{ keV}$

b i $E = hf = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{630 \times 10^{-9}} = 3.2 \times 10^{-19}\text{ J}$

ii Number of photons $\text{s}^{-1} = \frac{5.0 \times 10^{-3}}{3.2 \times 10^{-19}} = 1.6 \times 10^{16}$ photons s^{-1}.

2 a Einstein was able to show that light behaved like particles.

b Until Einstein's work with the photoelectric effect, it was an accepted fact that radiation—and light in particular—was made up of waves. The behaviour of waves had been well understood for more than 100 years, so it was a very big step to consider that light could show particle-like properties, and that the accepted wave properties of light could not explain the observations of the photoelectric effect.

c The photoelectric effect is the beginning of quantum physics, which introduces the idea that radiation can behave like particles.

3 a i The photons of light do not have enough energy for them to eject an electron free of the metal surface. The metal cannot lose any of its negative charge.

ii Over time, the electrons in the metal would absorb energy from waves. Eventually, they would have enough energy to break free of the metal surface and the negatively charged plate would discharge. Since this does not happen, light cannot be behaving like a wave.

b i A photon of light with this shorter wavelength has enough energy to give to an electron so that it can break free of the metal surface, without having to wait for energy to build up.

ii This suggests that the energy needed to break an electron free of the surface must be contained in a small space over a small time—a discrete packet of energy that we call a photon. A rapid, complete discharge of the metal plate suggests that there are numerous electrons released, in turn suggesting a stream of photons.

4 a The metal plate will lose electrons at an increased rate, so the coulombmeter will show a faster decrease in charge.

b Higher intensity means that there are more photons of light per second. This allows more electrons per second to break free of the metal surface, so the charge decreases at a faster rate.

c It increases the number of photons per second. It does not change the energy that each individual photon has. So, the maximum kinetic energy of the photoelectrons would not change.

5 a $f_o = \frac{3.2 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} = 7.7 \times 10^{14}\text{ Hz}$

b $\lambda = \frac{c}{f_o} = \frac{3 \times 10^8}{7.7 \times 10^{14}} = 3.9 \times 10^{-7}\text{ m}$ (390 nm)

c Photon energy $= \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{6.5 \times 10^{-8}} = 3.06 \times 10^{-18}\text{ J} = \frac{3.06 \times 10^{-18}}{1.6 \times 10^{-19}} = 19.1\text{ eV}$

So, maximum E_K of photoelectrons = 19.1 − 3.2 = 15.9 eV.

6 a $f_o = \frac{E}{h} = \frac{4.2 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} = 1.0 \times 10^{15}\text{ Hz}$

b Ultraviolet

c Photon energy $= hf = 6.63 \times 10^{-34} \times 2.2 \times 10^{15} = 1.47 \times 10^{-18}\text{ J} = \frac{1.47 \times 10^{-18}}{1.6 \times 10^{-19}} = 9.19\text{ eV}$

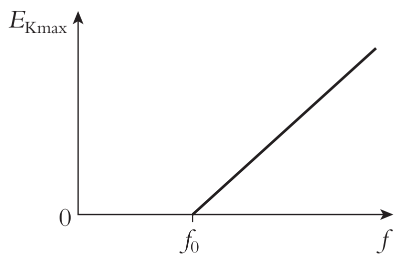
So, $E_{Kmax} = 9.19 - 4.2 = 4.99$ eV.

Therefore, $v = \sqrt{\frac{2E}{m}} =$

$\sqrt{\frac{2 \times 4.99 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} = 1.3 \times 10^6\ \text{ms}^{-1}$.

7 a E_{Kmax} is the maximum kinetic energy of the photoelectron emitted; h is Planck's constant; f is the frequency of the incident radiation; φ is the work function of the metal surface, from which the photoelectrons are emitted.

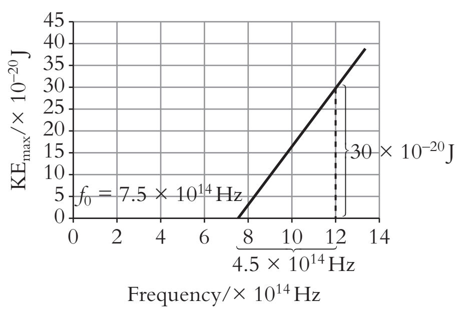
b



c Planck's constant, h

d The threshold frequency, f_o (see the diagram in **part b**), is the minimum frequency of radiation that will cause the emission of photoelectrons from the metal surface.

8 a i



The intercept on the x-axis is at 7.5×10^{14} Hz.

So, the work function must be $\varphi = hf_o =$ $6.63 \times 10^{-34} \times 7.5 \times 10^{14} =$ 4.98×10^{-19} J $\left(= \frac{4.98}{1.6} = 3.1 \text{ eV}\right)$.

ii See diagram, **part a i**. h is the gradient of the graph.

So, $h = \frac{30 \times 10^{-20}}{4.5 \times 10^{14}} = 6.7 \times 10^{-34}$ J s (the accepted value for h is 6.63×10^{-34} J s).

b Any straight line of the same gradient to the right of the line on the graph.

9 a With no air molecules between the metal plate and the collector, emitted photoelectrons can move unhindered from the metal plate to the collector.

b By the process of the photoelectric effect, the illuminating light causes the emission of electrons from the metal plate. These electrons move away from the metal plate, and some will arrive at the collector, creating a photocurrent. This current is measured by the sensitive ammeter.

c i The collector is connected to the negative terminal of the variable power supply. As the collector is made more negative, more photoelectrons will be repelled from the collector. This reduces the photocurrent, and the reading on the ammeter decreases.

ii For a given photon energy from the illuminating light (and a given value for the work function of the metal surface), the maximum kinetic energy of the photoelectrons is determined. When the voltage value, in volts, on the variable power supply equals the maximum kinetic energy of the photoelectrons (in eV), then the photoelectrons will not have enough energy to overcome the repulsion from the negative collector and the photocurent will be zero. Increasing the voltage beyond this value will have no effect, since the photoelectrons already have insufficient energy to reach the collector, so the reading on the ammeter will continue to be zero.

i The value of voltage at which the photocurrent becomes zero is called the stopping potential.

ii At a smaller wavelength, the incident photons have more energy. The work function of the metal plate is the same as it had been. So, the maximum kinetic energy of the photoelectrons will be larger. This means it will require a larger stopping potential to cause the photocurrent to become zero.

iii More intense light means more photons, of the same energy as before, per second. So the original photocurrent would be greater. However, since the maximum kinetic energy of the photoelectrons is still the same, the stopping potential will also stay the same.

Exercise 22.2

1 a Diffraction and interference

b De Broglie's argument was that physics seemed to be based on symmetry and, because of that, if light could show particle properties, then particles should be able to show wave properties.

c $\lambda = \frac{h}{p}$, where λ is the wavelength of the matter wave, h is Planck's constant and p is the momentum of the particle.

2 a It showed that electrons exhibited wave properties—that is, diffraction. This supported de Broglie's hypothesis that matter should exhibit wave properties.

b Electrons were reflected from layers of atoms within the zinc crystal so that at various angles electrons showed maxima in the intensity detected by the detector. This was similar to what had been observed by Bragg with X-rays and it suggested that the wavelength of electron waves must be of the order of the spacing between the layers of atoms in the zinc crystal. Such observations lead the way for future physicists to consider matter as having wave properties.

3 a Energy gained = eV

b $p = mv$, so $p^2 = m^2v^2$ and $E_K = \frac{1}{2}mv^2$

$\frac{m^2v^2}{2m} = \frac{p^2}{2m} = E_K$.

c If $E_K = eV$, then $p = \sqrt{2m \times E_K} = \sqrt{2meV}$.

d From diffraction theory, $n\lambda = d\sin\theta$, so there will be more diffraction for larger values of λ. This means that the radii of the bright and dark rings will increase when the wavelength of the electron waves increases.

e $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$ so $\lambda \propto \frac{1}{\sqrt{V}}$; that is as V increases, λ decreases and so the radii of the bright and dark rings decreases.

4 a i $\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 2 \times 10^7} =$ 3.6×10^{-11} m

ii $140\ \text{kmhr}^{-1} = \frac{140 \times 10^3}{60 \times 60} = 38.9\ \text{ms}^{-1}$

$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{0.16 \times 38.9} = 1.1 \times 10^{-34}$ m

iii $\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{70 \times 1.0} = 9.5 \times 10^{-36}$ m

b i The electron of wavelength 3.6×10^{-11} m is going to behave like a wave because it will have the opportunity to show diffraction effects.

The wavelength is too short for the cricket ball to show diffraction effects within its surroundings.

The human's wavelength is far too short and cannot show diffraction effects within its surroundings and so cannot show wave behaviour.

ii For a human to show diffraction effects, their momentum would have to be ~10^{-35} kg ms^{-1}. Humans do not move this slowly so they do not show wave properties; they behave like particles.

5 a i $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2\ \text{meV}}} =$ $\frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 400}} =$ 6.1×10^{-11} m

ii 100 is $\frac{1}{4}$ of 400, and $\lambda \propto \frac{1}{\sqrt{V}}$.

So, new $\lambda = 1.2 \times 10^{-10}$ m.

b $\sin\theta = \frac{\lambda}{d} = \frac{10^{-10}}{3 \times 10^{-10}} = 0.33$

So, yes, there should be diffraction effects for either of these electrons.

6 a It would have to be a standing wave.

b $n\lambda = 2\pi r$ or $\lambda = \frac{2\pi r}{n}$, where n is an integer.

c $mvr_n = \frac{nh}{2\pi}$; *and* $r_n = n^2 r_1$

So, $p = mv = \frac{nh}{2\pi r_n} = \frac{nh}{2\pi n^2 r_1} = \frac{h}{2\pi n r_1}$.

d $\lambda = \frac{h}{p} = \frac{h}{\frac{nh}{2\pi r_n}} = \frac{2\pi r_n}{n}$

e i $\lambda = \frac{2\pi r_n}{n} = \frac{2\pi r_1}{1} = 2\pi \times 0.53 \times 10^{-10} =$ $3.33 \times 10^{-10}\ m$

Since $\lambda = 2\pi r_1$ and circumference of orbit $n = 1$ is also $2\pi r_1$, there is 1 wavelength present in the ground state.

ii $\lambda = \frac{2\pi r_n}{n} = \frac{2\pi \times 2^2 r_1}{2} = 2 \times 2\pi \times$ $0.53 \times 10^{-10} = 6.66 \times 10^{-10}$ m

Since $\lambda = 4\pi r_1$ and circumference of orbit $n = 2$ is $8\pi r_1$, there are 2 wavelengths present in the $n = 2$ state.

iii $\lambda = \frac{2\pi r_n}{n} = \frac{2\pi \times 3^2 r_1}{3} = 3 \times 2\pi \times 0.53 \times 10^{-10} = 9.99 \times 10^{-10}$ m

Since $\lambda = 6\pi r_1$ and circumference of orbit $n = 3$ is $18\pi r_1$, there are 3 wavelengths present in the $n = 3$ state.

f They are the same: there are n wavelengths present in the electron standing wave for quantum state n.

7 a i The interference pattern produced by the electrons is something that is considered to be a wave phenomenon. Only if electrons were exhibiting wave-like properties would they produce an interference pattern.

ii The current arriving at the fluorescent screen is $\frac{4.8\,\mu A}{2} = 2.4\,\mu A$.

So, the rate at which electrons are arriving must be $\frac{2.4 \times 10^{-6}\ Cs^{-1}}{1.6 \times 10^{-19}\ C electron^{-1}} = 1.5 \times 10^{13}\ s^{-1}$.

iii Reducing the accelerating voltage gives the electrons less kinetic energy. This means that their de Broglie wavelength is longer. Longer wavelengths will produce an interference pattern with maxima more spread out.

b i Individual flashes of light on the screen in apparently random places.

ii Over a long period of time, the usual Young slits' interference pattern will be built up.

iii Our particle model of the electron prevents us from accepting that the electron can pass through both slits at the same time. If we consider the electron to be a matter wave—as described by de Broglie—then the de-localised nature of the wave may be sufficient to allow us to consider that it passes thorugh both slits at the same time.

iv Schrödinger's argument is based on the idea of probability. He argued that the matter wave associated with the electron is a kind of probability that the electron exists in a particular place. As each electron arrives at the fluorescent screen, its probability density (related to the square of the wave function, ψ, itself) dictates the chance that it will arrive in any particular place. The bright regions of the Young slits' interference pattern are places where the probability density function of the electron waves is quite high, whereas the regions where no electrons arrive (the usual dark regions between the interference maxima) are places where the electron wave's probability density fucntion is very low. In this way, electrons will build up a pattern on the screen that depends on the probability of them arriving in any particular place—and that probability is highest where the interference maxima should be.

v Schrödinger's probability density function has a high value where Bohr's orbital radius occurs, and a lower value at other distances from the nucleus. What is most interesting, perhaps, is that this allows the electron to exist anywhere within (and outside!) the atom, with the probability that it exists at the Bohr radii being higher than anywhere else.

Exam-style questions

Multiple-choice questions

1	D	[1]
2	B	[1]
3	A	[1]
4	B	[1]
5	B	[1]
6	C	[1]
7	C	[1]
8	B	[1]
9	C	[1]

Short-answer questions

10 a Work function is the *minimum* amount of energy required by an electron to break free of the metal surface. **[1]**

b Some of the electrons will be deeper inside the metal and will, therefore, require more energy to break free. **[1]** So, the energy remaining from the photon, which becomes kinetic energy for the electron, will have a range of values. **[1]**

c $E_{\text{K max}} = \frac{hc}{\lambda} - \phi =$

$\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{200 \times 10^{-9}} - (2.2 \times 1.6 \times 10^{-19}) =$

6.4×10^{-19} J = 4.0 eV **[2]**

11 a *Wave–particle duality* means that particles can show both the behaviour of particles and of waves. **[1]**

b i Any demonstration involving diffraction or interference, especially Young's double slits.

ii The photoelectric effect: illuminate a metal surface with ultraviolet light and measure the electrons emitted from the metal surface.

c Any two of the following: **[2]**

- Electron gun, graphite crystal and fluorescent screen inside an evacuated glass flask
- Electron gun to accelerate electrons
- Variable voltage supply to electron gun
- Diffracted electrons from graphite crystal show a diffraction pattern
- Change voltage and the radius of the diffraction rings changes

12 a $8.0 \times 10^{-17}\ \text{J} = \frac{8.0 \times 10^{-17}\ \text{J}}{1.6 \times 10^{-19}\ \text{J eV}^{-1}} =$

500 eV **[1]**

So, the electrons must have been accelerated through a voltage of 500 V. **[1]**

b $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2\,meV}} =$

$\frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 8 \times 10^{-17}}} =$

5.5×10^{-11} m **[2]**

c $\theta = \sin^{-1}\left(\frac{\lambda}{s}\right) = \sin^{-1}\left(\frac{5.5 \times 10^{-11}\ \text{m}}{2 \times 10^{-10}\ \text{m}}\right) =$

16°, **[1]**

which is easily measurable/observable. **[1]**

13 a Work function: the **minimum** amount of energy that an electron requires to break free from the surface. **[1]**

b Threshold frequency: the **minimum** frequency of radiation that will cause the emission of photoelectrons. **[1]**

c Photons have energy, $E = \frac{hc}{\lambda} =$

$\frac{6.63 \times 10^{-34} \times 3 \times 10^8}{450 \times 10^{-9}} = 4.4 \times 10^{-19}$ J

$= \frac{4.4 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.75$ eV.

So, the maximum E_K is 2.75 − 1.5 = 1.25 eV (1.3 eV 2 s.f.) **[2]**

d Number of incident photons s^{-1}

$= \frac{3 \times 10^{-3}}{4.4 \times 10^{-19}} = 6.8 \times 10^{15}\ \text{s}^{-1}$

Only $\frac{1}{8}$ of these eject photoelectrons, so current $= \frac{1}{8} \times 6.8 \times 10^{15} \times 1.6 \times 10^{-19} = 0.14$ mA. **[2]**

14 a $E_{\text{Kmax}} = \frac{hc}{\lambda} - \phi =$

$\frac{6.64 \times 10^{-34} \times 3 \times 10^8}{260 \times 10^{-9}} -$

$(4.25 \times 1.6 \times 10^{-19}) = 8.6 \times 10^{-20}$ J **[2]**

b The right-hand terminal will have to be the negative terminal in order to inhibit the flow of electrons to it. **[1]**

c Minimum terminal voltage =

$\frac{8.6 \times 10^{-20}}{1.6 \times 10^{-19}} = 0.54$ V **[2]**

15 a To remove atoms that might obstruct the movement of electrons from the graphite crystal to the fluorescent screen. **[1]**

b The pattern observed on the fluorescent screen is a diffraction pattern. **[1]** Diffraction/interference is a property that we associate with wave behaviour. **[1]**

c $\lambda = s \sin\theta$ and $\theta = \tan^{-1}\left(\frac{3.5}{20.0}\right) = 9.9°$ **[1]**

So, $s = \frac{\lambda}{\sin\theta} = \frac{h}{\sin\theta \times \sqrt{2meV}} =$

$\frac{6.63 \times 10^{-34}}{\sin 9.9° \times \sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 1000}} =$

2.3×10^{-10} m. **[2]**

16 a f_o is the intercept of the x-axis, so

$f_o = 5.6 \times 10^{14}$ Hz. (**[1]** for ±0.2)

b Einstein's equation is $KE_{max} = hf - \Phi$; E_K of photoelectrons = eV, so h will be given by gradient of graph × e

$\therefore h = \frac{1.8}{4.4 \times 10^{14}} \times 1.6 \times 10^{-19} =$

6.5×10^{-34} J s. **[2]**

c Φ is the intercept on the y-axis times e. This will be

$\frac{1.8}{4.4} \times 5.6 \times 1.6 \times 10^{-19} = 3.7$ eV. **[2]**

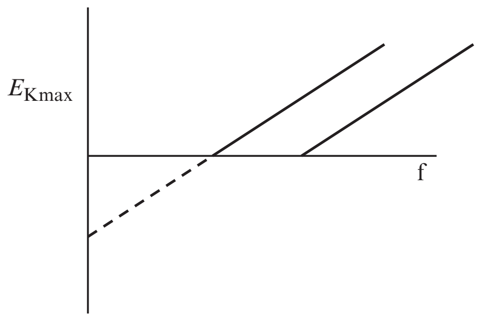
17 a Photoelectrons cannot have negative kinetic energy. The dotted line is an extrapolation backwards of the graph to show the position of the work function. **[1]**

b i f_o is the intercept on the x-axis. **[1]**

ii h is the gradient of the graph. **[1]**

iii ϕ is the intercept on the y-axis. **[1]**

c



18 a 2.4×10^{-15} m **[1]**

(Accept 10^{-15} m.)

b $p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{2.4 \times 10^{-15}} =$

2.8×10^{-19} kg ms^{-1} **[2]**

(Using $\lambda = 1 \times 10^{-15}$ m will give 6.6×10^{-19} kg ms^{-1}.) **[2]**

c Using $p = mv$:

$v = \frac{p}{m} = \frac{2.8 \times 10^{-19}}{9.1 \times 10^{-31}} = 3.1 \times 10^{11}$ ms^{-1} **[1]**

This is faster than the speed of light and cannot occur. **[1]**

So the value of m must be larger than 9.1×10^{-31} kg to make the speed of the electron below the speed of light.

19 a $\lambda = \frac{h}{mv} \Rightarrow v = \frac{h}{\lambda m} =$

$\frac{6.63 \times 10^{-34}}{2.4 \times 10^{-11} \times 1.67 \times 10^{-27}} =$

1.7×10^4 ms^{-1} **[2]**

b $\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 1.7 \times 10^4} =$

4.3×10^{-8} m **[2]**

c Any of the following: **[1]**

- Electrons are easier to obtain.
- Wavelength of electrons is likely to be larger than wavelength for protons.
- Wavelength of protons is too small to show diffraction.
- Larger wavelength for electrons means it is easier to demonstrate diffraction.

Chapter 23

Exercise 23.1

1 a A nucleon is any of the two kinds of particles in the nucleus (a neutron or a proton).

b An isotope is a form of an atom that has the same number of protons in the nucleus but a different number of neutrons.

c A nuclide is a description of a particular kind of nucleus: it expresses the nucleon number and the proton number (from which it is possible to calculate the neutron number).

2 a

Name of force	Acts on	Range	Boson responsible
Electromagnetic	charge	infinite	photon
Gravitational	mass	infinite	graviton
Weak force	nucleons	$\sim 10^{-18}$ m	W and Z boson
Strong nuclear force	nucleons	$\sim 10^{-15}$ m	gluon

b i Weak nuclear

ii Gravitational

iii Electromagnetic

iv Strong nuclear

c Three of the four fundamental forces act on the nucleons in the nucleus. Of these, the gravitational force is too weak to have any appreciable effect on holding the nucleons together. The electromagnetic force causes the positively charged protons to be repelled from each other—suggesting that they should fly apart. However, the strong nuclear force is about 100 times stronger than the electromagnetic force and is attractive between all nucleons. This overcomes the electromagnetic force and holds the nucleons together.

d The strong nuclear force acts most strongly on nucleons that are closest together. So, in a nucleus with a large number of nucleons, some of the nucleons do not 'feel' the strong force from many of the other nucleons; they only 'feel' the strong force from the nucleons immediately next to them. This allows a tightly bound alpha particle to break free of the nucleus if the nucleus is heavy enough.

e i Alpha-particles approaching the gold nuclei were repelled by the protons in the nucleus. Since the alpha particles didn't have enough energy to get close enough for the strong nuclear force to overcome the electromagnetic force of repulsion, the alpha particles were deflected.

ii If the alpha particles had had more energy so that they could approach the nucleus to within about 3 fm, the strong nuclear force would have been able to overcome the electromagnetic force and cause the alpha particle to be absorbed by the gold nucleus. Had this been the case, Rutherford would not have seen the very large angle deflections that occasionally occurred.

3 a i $\frac{1}{12}$ of the mass of a ${}^{12}_{6}C$ atom

ii The difference between the mass of the nucleons and the mass of the nucleus.

iii The minimum amount of energy required to separate all the nucleons in a nucleus.

b

Name of particle	Mass / u	Mass / kg
proton	1.007 28	1.672621×10^{-27}
neutron	1.008 67	1.674928×10^{-27}
electron	0.00055	9.11×10^{-31}

c **i** $4.0015\ \text{u} = 4.0015 \times 1.66 \times 10^{-27} = 6.64 \times 10^{-27}$ kg

ii Mass of particles = $2(1.00728 + 1.00867) = 4.0319$ u

iii The mass of the particles is greater than the mass of the alpha-particle.

iv Mass defect = $4.0319 - 4.0015 = 0.0304$ u

v Energy equivalence = 0.0304×931.5 MeV = 28.32 MeV

vi $E_B \text{ nucleon}^{-1} = \frac{28.32}{4} = 7.08 \approx 7.1 \text{ MeV nucleon}^{-1}$

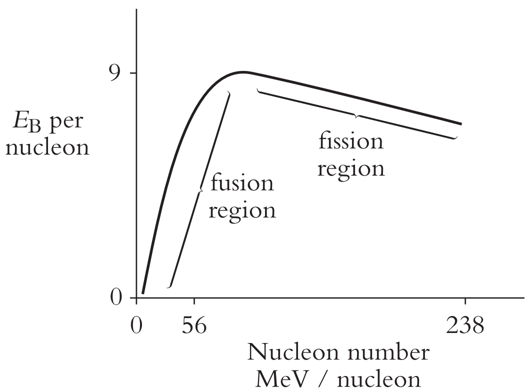
4 **a** $E_B \text{ nucleon}^{-1} = \frac{(6 \times 1.00728 + 9 \times 1.00867 - 15.0106) \times 931.5 \text{ MeV}}{15} = 6.9 \text{ MeV nucleon}^{-1}$

b $E_B \text{ nucleon}^{-1} = \frac{(11 \times 1.00728 + 13 \times 1.00867 - 23.99096) \times 931.5 \text{ MeV}}{24} = 7.8 \text{ MeV nucleon}^{-1}$

c $E_B \text{ nucleon}^{-1} = \frac{(26 \times 1.00728 + 30 \times 1.00867 - 55.93494) \times 931.5 \text{ MeV}}{56} = 8.6 \text{ MeV nucleon}^{-1}$

d $E_B \text{ nucleon}^{-1} = \frac{(28 \times 1.00728 + 34 \times 1.00867 - 61.9129) \times 931.5 \text{ MeV}}{62} = 8.8 \text{ MeV nucleon}^{-1}$

5 **a**, **c** and **e**



b The most stable nuclei are those that are held together with the most binding energy per nucleon. So, those nuclei at the peak of the curve are the most stable.

d Iron's and nickel's binding energy per nucleon are the maximum values on the curve. The implication of this is huge: iron and nickel are the heaviest elements that can be produced by nuclear fusion in stars without the extra energy from supernova events.

6 **a** In any event or process, the total energy (in all its forms) must remain constant; no energy can be lost—or gained.

b For the reaction to occur, the mass-energy of the left-hand side of the equation must be greater than the mass-energy of the right-hand side of the equation so that the reaction products can gain kinetic energy and move away from each other.

c For the first equation (which is a beta-minus decay; this will be examined in section 23.2):

Left-hand side: 46.9524 u

Right-hand side: 46.9518 + 0.00055 = 46.95235

So, mass-energy of left-hand side > mass-energy of right-hand side.

Therefore, this reaction can occur.

For the second equation (which would be a beta-plus decay—also examined in Section 23.2):

Left-hand side: 46.9524 u

Right-hand side: 46.9545 + 0.00055 = 46.95505 u

Here, the mass-energy of the left-hand side < mass-energy of the right-hand side.

So, this reaction cannot occur.

d In any event or process, the total momentum remains constant.

e **i** Energy available = (46.9524 – 46.95235) u = 0.00005 u = 46.6 keV

$|p_{Ti}| = |p_\beta| = p$

ii $\frac{E_{K\beta}}{E_{KTi}} = \frac{\frac{p^2}{2m_\beta}}{\frac{p^2}{2m_{Ti}}} = \frac{m_{Ti}}{m_\beta} = \frac{46.9518}{0.00055}$

$= 8.5367 \times 10^4$

So, the b-particle gains $\frac{85\,367}{85\,368} \times 46.6$ keV = 46.599 keV, and the Ti nucleus gains $\frac{1}{85\,368} \times 46.6$ keV $= 5 \times 10^{-4}$ keV, that is the beta particle gains about

99.999% of the available kinetic energy.

7 **a** **i** The mass-energy of A must be greater than the combined mass-energy of B + C.

ii The mass of the neutron is greater than the combined mass of the proton, the electron and the antineutrino.

iii The mass of the proton is less than the combined mass of the neutron, beta-plus particle and the neutrino.

iv The nucleus must contribute some energy to the proton so that the total mass-energy of the left-hand side of the equation becomes greater than the total mass-energy of the right-hand side of the equation.

b **i** The combined mass of A and B must be greater than the mass of C.

ii $Mass_A + Mass_B > Mass_C$

Or

$Mass_A + Mass_B + E_{KA} + E_{KB} > Mass_C + E_{KC}$

iii (2.01355 + 1.00728 – 3.01603) u = 4.47 MeV

So, this reaction can occur.

8 **a** $E = 233.03950 - (229.03163 + 4.001506)$ u $= 5.93$ MeV

b The alpha particle will gain

$\frac{\left(\frac{229.03163}{4.001506}\right)}{\left(\frac{229.03163}{4.001506} + 1\right)} \times 5.93 \text{ MeV} = 5.83 \text{ MeV}.$

c $v = \sqrt{\frac{2 \times E_K}{m}} = \sqrt{\frac{2 \times 5.83 \times 1.6 \times 10^{-13}}{4.001506 \times 1.66 \times 10^{-27}}}$

$= 1.7 \times 10^7 \text{ ms}^{-1}$

Exercise 23.2

1 **a** The nucleus

- has too many nucleons.
- has too many protons.
- has too many neutrons.
- is in an excited energy state.

b Too many nucleons: α-decay

Too many protons: β^+-decay

Too many neutrons: β^--decay

Nucleus on an excited energy state: γ-decay

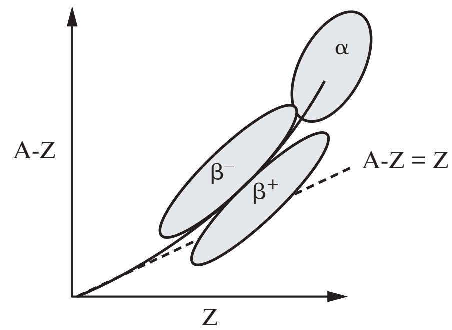
c **i** ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}\alpha$

ii ${}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + {}^{0}_{-1}\beta^- + \bar{\nu}_e$

iii ${}^{A}_{Z}X \rightarrow {}^{A}_{Z-1}Y + {}^{0}_{1}\beta^+ + \nu_e$

iv ${}^{A}_{Z}X \rightarrow {}^{A}_{Z}X + \gamma$

d



2 **a** Strong nuclear force and electrical force

b The strong nuclear force pulls the nucleons together; the electrical force pushes the protons away from each other.

c The strong nuclear force

d The strong nuclear force has a limited range (of about the size of a nucleon), so for any given nucleon, the strong nuclear force acts only on those other nucleons which are immediately adjacent.
The electrical force has an infinite range (even though it is an inverse-square law) and so can act on nucleons beyond those that are immediately adjacent. When the number of protons is very large (and this means that the number of neutrons is also very large), the electrical force can become larger than the strong nuclear force—enough to push nucleons away and cause α-decay.

e $^{241}_{95}\text{Am} \rightarrow {}^{237}_{93}\text{Np} + {}^{4}_{2}\alpha$

f **i** 241.004579 u – (236.99702 + 4.00151) u = 0.006049 u,

which is positive, so the decay can occur spontaneously.

ii 0.006049 × 931.5 = 5.6346 MeV

iii Fraction of E_K taken by α-particle depends on ratio of neptunium's mass to that of the α-particle: $\frac{237}{237 + 4} =$ 98% = 0.98 × 5.6346 = 5.52 MeV.

iv The neptunium nucleus, formed as a result of the decay process, is in an excited nuclear energy level. The nucleus then emits a gamma ray as it falls from the excited state to its ground state.

g **i** 241.004579 u − (240.00211 + 1.00728) u = −0.00481 u,

which is negative, so the decay cannot occur spontaneously.

ii 241.004579 u − (240.00305 + 1.00867) u = −0.00714 u,

which is also negative, so the decay cannot occur spontaneously.

h The alpha-particle is so tightly bound – that is has a binding energy that is so large – that its sufficiently small mass allows it to be emitted from unstable nuclei whilst obeying the conservation of energy rule. Nuclei cannot usually emit a single nucleon because a single nucleon has no binding energy, so its relatively large mass means the decay process cannot occur because it would break the conservation of energy rule.

3 **a** It will attract two electrons from nearby and become a helium nucleus.

b Rutherford and Royds were able to trap the emissions from an alpha emitter in a glass jar. After leaving the jar for some time, for the alpha particles to attract electrons from the nearby air and become helium atoms, they were able to test the gas in the jar. They found the gas to be helium and so were able to show that alpha particles were helium nuclei.

4 **a** *Ionise* means to remove one or more electrons from an atom to leave a positive ion and a free electron, called an ion pair.

b It moves relatively slowly (compared to β-particles or γ-rays).

It has a doubly positive charge.

It is relatively large (compared to β-particles or γ-rays).

c Suppose the alpha particle has 3 MeV of E_K, then it can produce $\frac{3 \times 10^6}{30} =$ 1×10^5 ion pairs.

d **i** At the start of its path, the alpha particle has lots of energy and moves at a constant speed, so its ability to ionise remains fairly constant. As the alpha particle loses a significant amount of its energy, it starts to slow down, which allows it to ionise more atoms per cm. When the alpha particle has lost almost all its energy it slows down very quickly and so doesn't travel very much farther and doesn't have enough energy left to ionise any more atoms, so the ionising events per cm fall rapidly to zero.

ii Area = total number of ion pairs. Estimate is 90 000 ion pairs. This is not significantly different from the answer to **part c**.

e **i** $\frac{8 \times 10^{-5}}{2 \times 10^{-10}} = 4 \times 10^5$ atoms thick

ii $\frac{5 \times 10^6}{30} = 1.67 \times 10^5$ ionising events

iii To pass through the paper, the alpha-particle would have to make at least 4×10^5 ionising events. With an initial kinetic energy of only 5.0 MeV, the alpha particle can make only 1.67×10^5 ionising events. So, the alpha-particle will have run out of energy before it passes through the piece of paper.

5 **a** **i** 6 protons and 6 neutrons

ii 6 protons and 8 neutrons

b It has too many neutrons.

c It will decay by β^--decay.

d ${}^{14}_{6}C \rightarrow {}^{14}_{7}C + {}^{0}_{-1}\beta^- \; (+{}^{0}_{0}\bar{\nu})$

6 **a** ${}^{3}_{2}He$

b ${}^{32}_{16}S$

c ${}^{63}_{29}Cu$

d ${}^{90}_{39}Y$

e ${}^{209}_{83}Bi$

7 **a** No particles are emitted from the nucleus (assuming we don't decribe a γ-photon as a particle!).

The nucleus does not change into a different nuclide.

b ${}^{A}_{Z}X^* \rightarrow {}^{A}_{Z}X + {}^{0}_{0}\gamma$

Note: it is usual to add an asterisk to the symbol for the nuclide to show that it is in an excited state.

c Since there is no change in the kind of nucleus it is, the nucleus must go from an excited energy state to a lower energy state, emitting the gamma photon as the difference between the energies of the two states involved.

d No.

e Typical γ-rays have energies of the order of MeV. A visible light photon from an electron energy level transition in an atom would have an energy of between 1 and 2 eV. So the γ-ray energy is about a million times larger.

8 **a** Using the constant ratio rule for every 5 mm:

$\frac{400}{214} = 1.87$; $\frac{214}{115} = 1.86$; $\frac{115}{61} = 1.90$; $\frac{61}{33} = 1.85$.

Since these values are all approximately the same, the graph is exponential.

b **i** Reading from the graph, half the intensity occurs for a thickness of 5.5 ±0.2 mm.

ii 1/e = 0.37. 0.37 × 400 = 148. The thickness required for this is about 8.0 ±0.2 mm.

9 **a** $E = 2 \times 938 \text{ MeV} = 2 \times 938 \times 1.6 \times 10^{-13}$
$= 3.0 \times 10^{-10}$ J

b **i** Pair production requires the intervention of a heavy nucleus. During the production process, some of the energy of the photon is given to the nucleus. Excess energy is transformed into E_K of the two particles produced. So, the actual energy required by the photon is greater than this calculated value.

ii Most will be transformed into E_K of the proton and anti-proton, a small amount will be transformed into E_K of the nearby heavy nucleus.

10

Radiation	What is it?	Charge	Mass (amu)	Ionising ability	Stopped by?	Deflected by em field?
α	Helium nucleus	+2	4.00151	Very good	A piece of paper	Yes, weakly
β^-	Fast-moving electron	−1	0.00055	medium	Thin aluminium	Yes, strongly
β^+	Fast-moving positron	+1	0.00055	medium	Thin aluminium	Yes, strongly
γ	Electromagnetic radiation	0	0	low	Several cm of lead	No

11 a Any radiation present around us that is of natural origin.

b The Sun, cosmic rays, food, rocks, radon gas from the ground and buildings.

c Generally, the amount of radiation we are exposed to in our lives is too small for it to cause us any significant harm.

d Corrected count means that the count of radioactive decay events from a particular source has been adjusted by subtracting the background count from it.

12 a i The time it takes for half of the nuclei present to decay.

ii The number of decay events per second

b Becquerels, Bq, where 1 Bq = 1 decay per second.

c 1 Ci = 3.7×10^{10} Bq, and so 1 mCi = 3.7×10^{4} Bq.

13 a 80 g

b 40 g

c 10 g

14 **Equipment required:** GM tube and counter, radioactive sample, two stopwatches, tongs to hold the sample

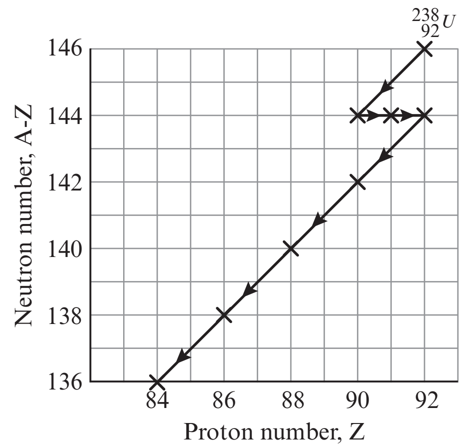
Method: Measure the background radiation for a period of 10 s several times and find the average count due to background radiation over a 10-s period. Measure the time with the stopwatch. 2: Place the sample against the GM tube. 3: Start one of the stopwatches and use the other stopwatch and GM tube and counter to measure the count over a 10-s period three times. 4: Find the average count over the 10-s period. 5: Subtract the background count to get a corrected count. 6: Repeat step 2 every minute and record all results in a table.

Plot the graph of the average count for 10 s against time for about half an hour.

Use the graph to find the half-life in the usual way.

15 a A series of decays that lead eventually to a stable nucleus.

b



c $^{238}_{92}\text{U} \rightarrow ^{234}_{90}\text{Th} + ^{4}_{2}\alpha$

$^{234}_{90}\text{Th} \rightarrow ^{234}_{91}\text{Pa} + ^{0}_{-1}\beta^-$

$^{234}_{91}\text{Pa} \rightarrow ^{234}_{92}\text{U} + ^{0}_{-1}\beta^-$

$^{234}_{92}\text{U} \rightarrow ^{230}_{90}\text{Th} + ^{4}_{2}\alpha$

$^{230}_{90}\text{Th} \rightarrow ^{226}_{88}\text{Ra} + ^{4}_{2}\alpha$

$^{226}_{88}\text{Ra} \rightarrow ^{222}_{86}\text{Rn} + ^{4}_{2}\alpha$

$^{222}_{86}\text{Rn} \rightarrow ^{218}_{84}\text{Po} + ^{4}_{2}\alpha$

$^{218}_{84}\text{Po} \rightarrow ^{214}_{82}\text{Pb} + ^{4}_{2}\alpha$

Exercise 23.3

1 a Scattering thorugh small angles still occurred, but the back-scattering and the scattering through very large angles that Lord Rutherford had observed no longer occurred.

b The strong nuclear force

c The much-higher-energy α-particles were now able to approach the gold nuclei to within about 10^{-15} m. At this distance, the strong nuclear force is dominant over the Coulomb repulsive force, and the α-particles were absorbed within the gold nuclei, thus making the large-angle scattering absent from observations. This provided suitable evidence for the existence of the strong nuclear force.

2 a They have too many neutrons.

b i The β^- spectrum is continuous; the α-spectrum is discrete, consisting on one or more vertical lines extending upwards from the kinetic energy axis.

ii Pauli suggested that if there were another particle emitted, along with the β^--particle, then the two particles could share the available energy—allowing the energy spectrum of the β^--particle to be continuous.
The other particle turned out to be the electron antineutrino.

c ${}^{13}_{6}\mathrm{C} \rightarrow {}^{13}_{7}\mathrm{N} + {}^{0}_{-1}\beta + {}^{0}_{0}\bar{\nu}$

d i $\Delta m = 13.99994\ u - (13.99922 + 0.00055)\ u = 0.00017\ u$

$\therefore E = 0.00017 \times 931.5 = 158$ keV

ii Max. fraction of available energy = $\frac{13.99922}{0.00055 + 13.99922} = 99.996\%$—very nearly all of it!

3 a ${}^{A}_{Z}\mathrm{X}^{*} \rightarrow {}^{A}_{Z}\mathrm{X} + {}^{0}_{0}\gamma$

b i $9 + 4.4 = 13.4$, so the two release the same total energy.

ii The mass deficit in the beta-decay equation defines the total amount of energy that is released—by whatever means. So, since both modes of decay are examples of the same β-decay process (involving the same nuclei), both modes must release the same total energy.

iii If the daughter nucleus produced is in an excited state—that is, one that is above its ground state—then the daughter nucleus will emit a gamma ray to lower its energy down to an energy level lower than its current level, and eventually to its ground state. This may involve more than one γ-emission and so suggests that more than one nuclear energy state may exist.

4 a 5.443 MeV + 0.102 MeV = 5.545 MeV

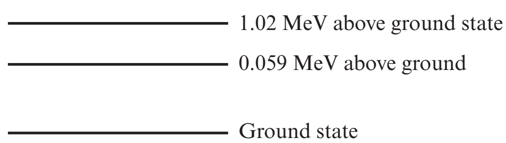
5.443 MeV + 0.043 MeV + 0.059 MeV = 5.545 MeV

5.486 MeV + 0.059 MeV = 5.545 MeV

So, all modes release the same total energy.

b The mass deficit in the α-decay equation defines the total amount of energy that is released—by whatever means. So, since all three modes of decay are examples of the same α-decay process (involving the same nuclei), all three modes must release the same total energy.

c



d They are very much larger. In this example, the excited nuclear energy levels are 59 keV and 102 keV; excited energy levels for electrons in atoms are of the order of about 10 eV, so the nuclear energy levels are of the order of 10 000 times larger.

e Electromagnetic radiations from the electron energy level transitions in atoms have energies that are of the order 10^{-4} of the energies of the gamma rays emitted by excited nuclei.

5 a $N = N_o e^{-\lambda t}$, so after $t = t_{\frac{1}{2}}$, $N = \frac{N_o}{2}$ and so, $\frac{N_o}{2} = N_o e^{-\lambda t_{\frac{1}{2}}}$

$\therefore -\ln 2 = -\lambda t_{\frac{1}{2}} \Rightarrow \lambda = \frac{\ln 2}{t_{\frac{1}{2}}}$.

This is, in fact, the usual way in which the link between half life, $t_{\frac{1}{2}}$, and the decay constant, λ, is derived.

b i $\lambda = \frac{\ln(2)}{t_{\frac{1}{2}}} = \frac{0.693}{1.25 \times 10^9 \times 3.15 \times 10^7}$

$= 1.8 \times 10^{-17}\ \text{s}^{-1}$

ii $\lambda = \frac{\ln(2)}{t_{\frac{1}{2}}} = \frac{0.693}{9.96 \times 60} = 1.2 \times 10^{-3}\ \text{s}^{-1}$

iii $\lambda = \frac{\ln(2)}{t_{\frac{1}{2}}} = \frac{0.693}{5.27 \times 3.15 \times 10^7}$

$= 4.2 \times 10^{-9}\ \text{s}^{-1}$

6 a $t_{\frac{1}{2}} = \frac{\ln(2)}{\lambda} = \frac{0.693}{1.28 \times 10^{-5}} = 5.4 \times 10^4$ s

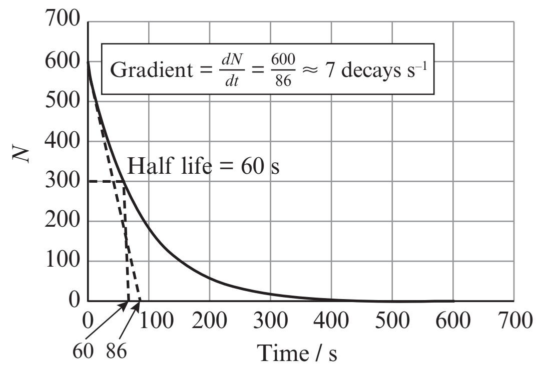
b $5.4 \times 10^4\ \text{s} = \frac{5.4 \times 10^4}{60 \times 60} = 15.0$ hours

i After 15 hours, the activity of a sample of $^{24}_{11}$Na will be $\frac{1}{2}$.

ii After 30 hours, the activity of a sample of $^{24}_{11}$Na will be $\frac{1}{4}$.

7 a $t_{\frac{1}{2}} = \frac{0.693}{\lambda} = \frac{0.693}{0.01155} = 60$ s

b and **c**



d $\left.\frac{-dN}{dt}\right|_{t=0} = \lambda N_0 = 0.01155 \times 600 = 6.93 \approx 7$

8 a The radioactive isotope $^{14}_{6}$C decays by β^- emission. In living tissue, the proportion of $^{14}_{6}$C to $^{12}_{6}$C remains constant, but in dead tissue, the proportion decreases as the radioactive isotope decays. So, if a known quantity (say, 5 g) of fossilised organic material is measured for its count rate and then compared to the corrected count rate for a sample of 5 g of the same kind of living organic material, an approximate age of the fossilised material can be found, using half-life.

b i $\sqrt{144} = 12$

ii $18 = \frac{144}{2^3}$, so the material will be three half-lives old = 3 × 5730 years = 17 190 years.

c $A = A_o e^{-\lambda t} \Rightarrow t = \frac{\ln\left(\frac{A}{A_o}\right)}{-\lambda} = t_{\frac{1}{2}} \times \frac{\ln\left(\frac{A}{A_o}\right)}{-0.693} =$

$5730 \times \frac{\ln(0.92)}{-0.693} = 690$ years.

9 Measure the mass of the sample accurately. Use the mass of the sample and the relative atomic mass of the nuclide to calculate the number of nuclei present. Measure the corrected count rate with a GM tube placed 1 cm away from the sample. Assuming that the sample emits its decay products in all directions, find the fraction of the area of the GM tube window to the area of a sphere of radius 1 cm. Divide the corrected count rate by this fraction to get the activity of the sample.

Find the decay constant using $\lambda = \frac{A}{N}$.

Determine the half-life by using $t_{\frac{1}{2}} = \frac{0.693}{\lambda}$.

Chapter 24

Exercise 24.1

1 a The splitting of a nucleus of large mass into two smaller nuclei of smaller mass (usually accompanied by one or more neutrons).

b The fission process occurs without the need for an absorbed neutron.

c The fission process requires a neutron to be absorbed.

2 a Two smaller-mass nuclei have binding energy per nucleon that is larger than a heavy mass nucleus, such as uranium or plutonium. So the fission process is energetically possible since the difference between the total binding energy of the fission products and the original, heavier, nucleus is the energy released by the fission process.

b Small-mass nuclei, such as deuterium and tritium, have binding energy per nucleon that is smaller than heavier-mass nuclei, such as helium. So when two small-mass nuclei are fused together to make a heavier-mass nucleus, the total binding energy becomes greater and energy is thus released. Another way of looking at this is that the combined mass of the two small-mass nuclei is greater than the mass of the heavier nucleus. The difference in mass between the two becomes the energy released in the fusion process.

c The boundary between the two processes occurs where the binding energy per nucleon is greatest. This happens for the elements iron and nickel.

3 a Nuclear fission

b Neutron

c Kinetic energy of the fission fragments

4 a $235 + 1 = 137 + 95 + 4$. So, $n = 4$.

b The mass deficit is $(234.994 + 1.0087) - (136.877 + 94.886 + 4 \times 1.0087) = 0.2049$ u.

So, the energy released is $0.2049 \times 931.5 = 191$ MeV.

c The induced fission of ${}^{235}_{92}$U can produce a range of pairs of nuclei, not just Cs and Rb. This means that the energy released will be different for each particular example of the fission process.
The average energy released by all possible fission reactions is 215 MeV.

5 a ${}^{235}_{92}$U absorbs a slow neutron and undergoes fission quite easily. The nucleus ${}^{238}_{92}$U does not readily undergo fission; it absorbs neutrons without further nuclear processes.

b Specific energy of natural uranium = $0.6\% \times 8.0 \times 10^{13} = 4.8 \times 10^{11}$ J kg^{-1}

c $200 \text{ MeV} = 200 \times 1.6 \times 10^{-13} = 3.2 \times 10^{-11}$ J from the fission of one nucleus.

In 1 kg there are $\frac{1000}{235} \times 6.023 \times 10^{23} = 2.6 \times 10^{24}$ nuclei.

So, the specific energy $= 3.2 \times 10^{-11} \times 2.6 \times 10^{24} = 8.3 \times 10^{13}$ J kg^{-1}, which is about 8×10^{13} J kg^{-1}.

d This value is about 10^4 greater than the specific energy for fossil fuels.

e Enriched nuclear fuel is uranium that has had its ${}^{235}_{92}$U content increased. A greater percentage of the uranium fuel rod can undergo fission, increasing the specific energy of the fuel. This improves the efficiency of the nuclear power station.

6 a i The energy released by the fission process is in the form of kinetic energy of the fast neutrons. It is this kinetic energy that is destined to become the electrical energy output from the power station.

ii The uranium nuclei will not readily absorb fast-moving neutrons; they will just ‘bounce off’. (In technical terms, physicists say that the cross section for absorption—that is, the probability—is too small for fast-moving neutrons and is very much larger for slow, or thermal, neutrons). So, the fast-moving neutrons have to be slowed down to a speed that is acceptable to the uranium nuclei. Only then will the neutrons be absorbed.

b The fast-moving neutrons collide with the atoms of the moderator. Each collision makes the neutrons lose a small amount of their energy (about 30% or so). So, after many collisions, the neutrons will have lost sufficient energy to slow them down to thermal speeds, at which they can then be absorbed by other uranium nuclei.

In addition to this, the energy lost by the neutrons is gained by the atoms of the moderator, making the moderator become very hot. It is this thermal energy that is then transferred, by heat exchangers, and will be used to heat water to make steam to drive turbines and so on.

c If one of the neutrons emitted during the fission process can be used to initiate another fission process, then a chain of reactions can occur: a chain reaction. This requires two things: first, the neutrons are moderated, and second, some of the neutrons need to be removed so that only one neutron will become available to induce another fission reaction.

d Control rods. The control rods are made from materials whose atoms will readily absorb neutrons without becoming themselves unstable. In this way, inserting the control rods into the nuclear reactor allows some of the neutrons to be absorbed.

7 A small mass of uranium will have a relatively large value for the surface area-to-mass ratio. This means that if fission events occur within the mass, then it will be fairly easy for some of the neutrons emitted to escape from the surface of the fuel rod. If this occurs then there will be insufficient neutrons to keep a sustained chain reaction going. A mass large enough to have a surface area to mass ratio that is small enough to keep most of the neutrons produced inside the fuel rod is called the critical mass.

8 **a** Nuclear binding energy in the nucleus of uranium → kinetic energy of fission fragments → thermal energy of moderator → thermal energy in heat exchanger → thermal energy of steam → rotational energy of turbines → electrical energy produced by generator.

b **i** Fast-moving neutrons (from the fission process) collide with atoms/molecules of the moderator, slowing down the neutrons so they are more likely to be absorbed by uranium-235 nuclei. This transfer of energy heats the moderator. This thermal energy is transferred via a heat exchanger to a more conventional system that produces steam. Typically, a moderator can be water or graphite.

ii Control rods absorb neutrons. This reduces the number of neutrons that are able to collide with, and be absorbed by, uranium-235 nuclei. This allows a controlled chain reaction to occur, that is a sequence of fission processes that does not increase in number but keeps a steady output of energy. Inserting (or withdrawing) the control rods reduces (or increases) the number of fission reactions occurring, thus controlling the amount of energy produced. Control rods are usually made from boron.

iii The heat exchanger takes the thermal energy from the moderator and uses it to produce steam for the turbines. The heat exchanger is a closed system so that if it is contaminated in any way by radioactive material, it will not affect the surrounding power station. Common heat-exchanger materials include pressurised water and carbon dioxide gas.

c **i** Removing the moderator would reduce or stop the output of the nuclear power station.

It would not slow down the fast neutrons, so fewer fission reactions occur.

There would be no facility to transfer the energy from the kinetic energy of the fast neutrons to the heat exchanger in order to produce steam for the turbines.

ii Without the control rods, it would not be possible to sustain a chain reaction. Too many neutrons will be available to produce further fission reactions, which go on to produce even more neutrons. The nuclear power station would probably overheat, melt down and explode.

d The nuclear waste from a fission reactor is highly radioactive, with long half-lives and chemically reactive. It must be disposed of somewhere where it will not be a risk to living things for thousands of years. It is an expensive technological challenge.

Exam-style questions

Multiple-choice questions

1 B **[1]**

2 A **[1]**

3 B **[1]**

4 A **[1]**

5 B **[1]**

6 C **[1]**

7 D **[1]**

8 C **[1]**

9 C **[1]**

10 C **[1]**

11 B **[1]**

Short-answer questions

12 **a** Two protons; one neutron **[1]**

b Mass deficit = (2 × 1.007276 + 1.008665 − 3.01603) u = 0.0072 u **[1]**

So, binding energy = 0.0072 × 931.5 MeV = 6.72 MeV **[1]**

Therefore, the binding energy per nucleon $\frac{6.72}{3}$ = 2.24 MeV nucleon^{-1}. **[1]**

c ${}^{2}_{1}D + {}^{2}_{1}D \rightarrow {}^{3}_{2}He + {}^{1}_{0}n$ **[1]**

13 **a** ${}^{233}_{92}U \rightarrow {}^{229}_{90}Th + {}^{4}_{2}\alpha$ **[2]**

b E_K available = (4.00787 − 4.001506) = 0.00636 u

= 0.00636 × 931.5 MeV = 5.92 MeV **[2]**

c The conservation of momentum states that the total momentum before the interaction must equal the total momentum after the interaction.
If the alpha particle moves off in one direction, then the daughter nucleus left behind must move off in the opposite direction. This will require some of the energy available from the alpha decay process. What remains is the actual E_K of the alpha particle. **[1]**

14 **a** Mass available = (220.01140 − 216.00192 − 4.001506) u = 0.00797 u **[1]**

So, E_K available is 0.00797 × 931.5 MeV = 7.43 MeV.

b The Po nucleus is 54 times more massive than the α-particle, so its velocity will be $\frac{1}{54}$ of that of the α-particle. So, its E_K will be $\frac{1}{54}$ of that of the α-particle. **[2]**

c $E_{K\alpha} = \frac{54}{55} \times 7.43$ MeV = 7.29 MeV **[2]**

15 **a** ${}^{14}_{6}C \rightarrow {}^{14}_{7}N + {}^{0}_{-1}\beta^{-} + \bar{\nu}_e$ **[2]**

b Mass available for conversion into energy = 14.003241 − 13.999231 − 0.00055 = 0.00346 u

This energy is 0.00346 × 931.5 MeV = 3.22 MeV.

(Note that in this calculation, the mass of the electron antineutrino has been ignored. Although it is, in fact, non-zero, the mass of the antineutrino is likely to be too small to have an effect here.)

So, the E_K of the electron must be less than 3.22 MeV. **[2]**

c Some of this E_K is required for the mass of the electron antineutrino. Some of this energy is also required for the E_K of the electron antineutrino and for the E_K of the daughter nucleus. So, the maximum E_K of the electron is never observed to be as high as 3.22 MeV. **[1]**

16 a $\lambda = \frac{\ln(2)}{t_{\frac{1}{2}}} = \frac{0.693}{433 \times 3.15 \times 10^7}$

$= 5.1 \times 10^{-11}\ s^{-1}$ **[1]**

b $A = \lambda N \Rightarrow N = \frac{A}{\lambda} = \frac{5 \times 10^{-6} \times 3.7 \times 10^{10}}{5.1 \times 10^{-11}}$

$= 3.6 \times 10^{15}$ nuclei **[2]**

c In one year, the activity of the sample won't drop appreciably because its half-life is so long; the advertised activity will be very close to the actual activity. **[1]**

17 a ${}^{90}_{38}Sr \rightarrow {}^{90}_{39}Y + {}^{0}_{-1}\beta^- + \bar{\nu}_e$ **[2]**

b The total energy available is equal to the mass–energy difference between the parent nucleus and the three product particles. This is a constant.This energy is shared between the α-particle and the $\bar{\nu}_e$, so the electron has a spectrum of energies between zero and the total energy available. **[2]**

c The nucleus left behind $\left({}^{90}_{39}Y\right)$ has a mass that is about 160 000 times that of the β^--particle, so its kinetic energy will be 1/160 000th of the β^--particle's. This is too small for it to be significant. **[1]**

18 a $\lambda = \frac{\ln 2}{t_{\frac{1}{2}}} = \frac{0.693}{1.42 \times 10^{17}} = 4.88 \times 10^{-18}\ s^{-1}$ **[1]**

b $N = \frac{1 \times 10^{-3} \times 6.023 \times 10^{23}}{238} = 2.53 \times 10^{18}$ **[2]**

c $\frac{dN}{dt} = -\lambda N = 4.88 \times 10^{-18} \times 2.53 \times 10^{18}$

$= 12.3$ decays s^{-1} **[2]**

19 a The count rate that the GM tube has measured has the value of the background radiation count rate subtracted from it so that the corrected count rate is due only to the sample. **[1]**

b $\frac{CCR}{CCR_o} = e^{-\lambda t} \Rightarrow \lambda = \frac{-\ln\left(\frac{CCR}{CCR_o}\right)}{t} = \frac{-\ln\left(\frac{380}{470}\right)}{20 \times 60}$

$= 1.8 \times 10^{-4}\ s^{-1}$ **[2]**

c $t_{\frac{1}{2}} = \frac{0.693}{1.8 \times 10^{-4}} = 3.85 \times 10^3\ s =$

1.07 hours **[1]**

d This method has two major problems with a half-life as long as this:

The activity of a sample is likely to be very small. The activity may not be significantly different from (or smaller than) the background activity.
So, trying to make a corrected count rate would be meaningless. **[1]**

There will not be any appreciable change in the activity of the sample over a period of time in which the measurements might be made. So, a calculation of this kind would produce a ln (1) = 0, meaning that $t_{\frac{1}{2}}$ cannot be calculated **[1]**

20 a 1 + 13 = 13 + 1, so the particle must have 1 nucleon.

And 1 + 6 = 7 + 0, so the particle has no charge.

Therefore, the particle must be a neutron. **[1]**

b Mass deficit = (1.007276 + 13.000055) u – (13.001889 + 1.008665) u = −0.003223 u **[2]**

So, the proton needs a minimum E_K of 0.003223 × 931.5 MeV = 3.002 MeV. **[1]**

c Conservation of momentum. (statement alone = **[1]**) Since before the reaction the proton had momentum, after the reaction the nitrogen nucleus and the neutron will have to have momentum too. This will require extra energy. **[1]**

21 a Nuclear fission **[1]**

b Mass defect for energy production

= ((236.0526 − (143.92292 + 88.91781 + (3 × 1.008665))) u **[1]**

= 0.185875 u **[1]**

So, energy available = 0.185875 × 931.5 MeV = 173.14 MeV. **[1]**

c Energy is released as E_K of the fission fragments. **[1]**

22 a 200 MeV = $200 \times 1.6 \times 10^{-13} = 3.2 \times 10^{-11}$ J **[1]**

Then, using the equation $E_K = \frac{3}{2}kT$,

$T = \frac{2 \times E_K}{3 \times k} = \frac{2 \times 3.2 \times 10^{-11}}{3 \times 1.38 \times 10^{-23}}$

$= 1.5 \times 10^{12}$ K. **[1]**

b $\frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times (20 + 273)$

$= 6.1 \times 10^{-21}\text{ J} = \frac{6.1 \times 10^{-21}}{1.6 \times 10^{-19}}$

$= 3.8 \times 10^{-2}$ eV **[1]**

c $0.7^n \times 3.2 \times 10^{-11} = 6.1 \times 10^{-21}$

$\Rightarrow 0.7^n = \frac{6.1 \times 10^{-21}}{3.2 \times 10^{-11}} = 1.9 \times 10^{-10}$

So, $n = \frac{\log(1.9 \times 10^{-10})}{\log(0.7)} = 63.$

So, the neutron will need to make about 60 collisions for its energy to be reduced to that of a thermal neutron. **[2]**

Chapter 25

Exercise 25.1

1 **a** The joining together of two light nuclei to form a heavier nucleus.

b If the combined masses of the two nuclei (or nucleus and another particle) are greater than the mass of the heavier nucleus (and any other particles produced), then energy is released.

c Kinetic energy of the particles and any gamma rays produced.

2 Mass deficit = (1.007276 + 1.008665 − 2.013553) u = 0.002388 u

So, energy released = 0.002388 × 931.5 MeV = 2.22 MeV

3 **a** **i** Mass deficit = (2 × 1.007276) − (2.013553 + 0.00055) u = 0.000449 u

So, energy available = 0.000449 × 931.5 MeV = 0.418 MeV

ii $\frac{1}{1.007276 \times 1.66 \times 10^{-27}} = 5.98 \times 10^{26}$ protons kg^{-1}

An alternative method might be
1 gramme of hydrogen is 1 mole.
So 1 kg is 1000 moles =
6×10^{26} protons kg^{-1}.

iii $5.98 \times 10^{26} \times 0.418 \times 1.6 \times 10^{-13} = 4.0 \times 10^{13}\ \text{Jkg}^{-1}$

iv The ${}^{0}_{1}\beta^{+}$ particle will quickly annihilate with one of the electrons from the hydrogen. This will produce two gamma rays, which have energy.

v The combined mass of the ${}^{0}_{1}\beta^{+}$ particle and the electron is 2 × 0.00055 u = 0.0011 u. This is equivalent to 0.0011 × 931.5 = 1.02 MeV.

vi So, the total energy available from each fusion reaction is 0.418 + 1.02 = 1.438 MeV

Therefore, the energy from 1 kg of hydrogen will be
$5.98 \times 10^{26} \times 1.438 \times 1.6 \times 10^{-13} = 1.38 \times 10^{14}\ \text{Jkg}^{-1}$.

vii It is about 5 million times more.

b **i** They will need to overcome the electrostatic repulsion of the Coulomb force if they are to join together. This will require transferring kinetic energy into electrical potential energy so that the two protons get close enough to each other for the strong nuclear force to overcome the Coulomb force. If one of the two protons were stationary, then as the other approaches, the stationary proton would be deflected backwards (away from the approaching proton), which may mean that the two protons could not get close enough for the strong nuclear force to overcome the electrostatic repulsion.

ii Combined $E_K = E_p$ at a separation of 2.4×10^{-15} m

So,

$$E_K \text{ of each proton} = \frac{1}{2} \times k\frac{q^2}{r}$$
$$= \frac{1}{2} \times 9 \times 10^{9} \times \frac{(1.6 \times 10^{-19})^2}{2.4 \times 10^{-15}}$$
$$= 4.8 \times 10^{-14}\ \text{J}$$
$$= \frac{4.8 \times 10^{-14}\ \text{J}}{1.6 \times 10^{-13}\ \text{J MeV}^{-1}}$$
$$= 0.3\ \text{MeV.}$$

iii $T = \frac{2 \times 0.3 \times 1.6 \times 10^{-13}}{3 \times 1.38 \times 10^{-23}} = 2.3 \times 10^{9}\ \text{K}$

iv If the temperature in the Sun's core is of the order of 10^7 K, then, because of the Maxwellian distribution of energies that the protons will have, some protons, although not a large proportion of them, will have energies significantly larger than the average (i.e. the small number of protons in the largest energy tail of the distribution). This means that it is possible for this reaction to occur, although it does so with a low probability.

v This reaction process—the production of deuterium nuclei from protons—is the start of a chain of other fusion processes, all of which contribute to the energy production of a star like the Sun. So, if this fusion reaction happens with only a low probability, then a relatively low number of deuterium nuclei will be available for the other fusion reactions. This limits the energy that a star like the Sun can produce.

4 a i The gravitational collapse of a cloud of (mostly hydrogen) gas, following the Rayleigh–Jeans criterion, transfers gravitational potential energy into thermal energy. This heats up the core of the star to temperatures that eventually allow thermonuclear fusion to occur.

ii The large gravitational forces acting on all parts of the star, cause the constituent particles within it to be forced very close together, increasing its density. This is particularly the case at the core of the star. So, the core becomes very dense.

b i Large number of protons in close proximity partly negates the low probability of them fusing.
The overall effect is to allow sufficient numbers of fusion reactions to occur, producing the deuterium nuclei that will go on to produce more energy by other fusion processes.

ii Despite the large kinetic energies that the protons have at these high temperatures, the large density of the core—and the large gravitational force acting against thermal expansion—keeps the density high enough to keep the protons in the core region available for the fusion reactions.

iii The transfer of gravitational potential energy into thermal energy causes both the high density of the core and the high temperatures in the core. So, the high density has to be accompanied by a high temperature.

c The technological difficulties are based on the three conditions that the star's core satisfies: temperature, density and confinement.

Temperature: It is possible to produce protons with effective temperatures—or such high energies—using particle accelerators. This isn't seen, as yet, as a viable method of producing high-energy protons.

Density: The biggest difficulty of this is that, at the current time, it is only possible to produce protons of these high energies in small amounts, amounts that are insufficient to sustain the fusion reactions necessary to produce usable fusion power. This means that the high densities of protons required are not yet easily achievable.

Confinement: Keeping the hot, energetic protons in a small space is not an easy thing to do, despite technological advances in strong magnetic and electric fields.

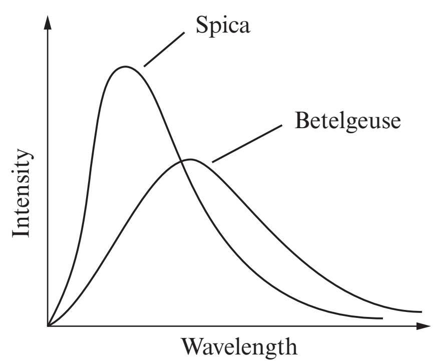
5 a $(1.007276 + 7.016)\text{ u} - (2 \times 4.0015)\text{ u} = 0.0203\text{ u}$

This is $0.0203 \times 931.5 = 18.9$ MeV.

b $18.9\text{ MeV} = 18.9 \times 1.6 \times 10^{-13} = 3.02 \times 10^{-12}$ J. This would have been split evenly between the two alpha particles, so each alpha particle would get 1.5×10^{-12} J (9.38 MeV) of kinetic energy.

Exercise 25.2

1 a



b **i** The hotter star will have its peak intensity at a smaller wavelength.

ii Wien's displacement law. $\lambda_{peak} = \frac{2.9 \times 10^{-3}}{T}$, where λ_{peak} is the wavelength at which maximum intensity occurs and T is the absolute temperature.

iii It doesn't! There is no emissivity term in Wien's law.

c $T = \frac{2.9 \times 10^{-3}}{650 \times 10^{-9}} = 4462 = 4500$ K (2 s.f.)

d **i** $\lambda_{peak} = \frac{2.9 \times 10^{-3}}{4300} = 670$ nm (2 s.f.)

ii With the peak in the spectrum at this wavelength (almost right at the far red end of the visible spectrum), there will be a large amount of energy at wavelengths that are too long to be visible. These infrared wavelengths will add to the visible part of the emission to make the luminosity (the total emitted power) larger than one would expect by considering the visible wavelengths only.

e $T = \frac{2.9 \times 10^{-3}}{1.063 \times 10^{-3}} = 2.7$ K

2 **a** *Luminosity* is the total power radiated by the star.

b $L = 4\pi\sigma R^2T^4$

c **i** $L = \sigma AT^4 = 5.67 \times 10^{-8} \times 4\pi \times (7 \times 10^8)^2 \times 5700^4 = 3.7 \times 10^{26}$ W

ii $L = \sigma AT^4 = 5.67 \times 10^{-8} \times 4\pi \times (8.2 \times 10^{11})^2 \times 3500^4 = 7.2 \times 10^{31}$ W

iii $L = \sigma AT^4 = 5.67 \times 10^{-8} \times 4\pi \times (4.9 \times 10^{10})^2 \times 11\,200^4 = 2.7 \times 10^{31}$ W

d **i** $\frac{L}{L_\odot} = \frac{3.7 \times 10^{26}}{3.8 \times 10^{26}} = 0.97 \approx 1$

ii $\frac{L}{L_\odot} = \frac{7.2 \times 10^{31}}{3.8 \times 10^{26}} = 1.9 \times 10^5$

iii $\frac{L}{L_\odot} = \frac{2.7 \times 10^{31}}{3.8 \times 10^{26}} = 7.1 \times 10^4$

3 **a** $\frac{L_{Procyon}}{L_\odot} = \frac{\sigma A_{Procyon} T^4_{Procyon}}{\sigma A_\odot T^4_\odot} = \frac{4 \times 6530^4}{5700^4} = 6.9$

b $\frac{L_{Sirius\ A}}{L_\odot} = 25.4 = \frac{\sigma A_{Sirius\ A} T^4_{Sirius\ A}}{\sigma A_\odot T^4_\odot}$

$= \frac{\sigma 4\pi (r_{Sirius\ A})^2\, T^4_{Sirius\ A}}{\sigma 4\pi (r_\odot)^2 T^4_\odot}$

So, $\frac{r_{Sirius\ A}}{r_\odot} = \sqrt{25.4 \times \frac{T^4_\odot}{T^4_{Sirius\ A}}}$

$= \sqrt{25.4 \times \frac{5700^4}{9940^4}} = 1.7$

4 **a** $\frac{L_X}{L_Y} = \frac{\sigma 4\pi r_X^2 T_X^4}{\sigma 4\pi r_Y^2 T_Y^4} = 500$ and $\frac{T_X}{T_Y} = 20$

So, $\frac{r_X}{r_Y} = \sqrt{\frac{500}{20^4}} = 0.06$

b $\frac{L_{Betelgeuse}}{L_{Rigel}} = \frac{\sigma 4\pi (r_{Betelgeuse})^2\, T^4_{Betelgeuse}}{\sigma 4\pi (r_{Rigel})^2\, T^4_{Rigel}} =$

$\frac{(1100 \times r_\odot)^2\,(0.6 \times T_\odot)^4}{(70 \times r_\odot)^2\,(2 \times T_\odot)^4} = \frac{1100^2 \times 0.6^4}{70^2 \times 2^4} = 2.0$

5 **a** Apparent brightness: the amount of energy received at the Earth per second per unit area—or the received power per unit area at the Earth.

b **i** $b = \frac{L}{4\pi d^2} = \frac{3.83 \times 10^{26}}{4\pi(1.5 \times 10^{11})^2} =$

1.4×10^3 Wm^{-2}

ii This is usually known as the solar constant, S.

c $b = \frac{L}{4\pi d^2} = \frac{5.0 \times 10^{28}}{4\pi(4 \times 9.46 \times 10^{15})^2} =$

2.8×10^{-6} Wm^{-2}

6 $\frac{b_\alpha}{b_{Sun}} = \frac{\frac{L_\alpha}{4\pi(d_\alpha)^2}}{\frac{L_\odot}{4\pi(d_\odot)^2}} = \frac{L_\alpha (d_\odot)^2}{L_\odot (d_\alpha)^2} =$

$1.52 \times \left(\frac{8.33}{4.3 \times 365 \times 24 \times 60}\right)^2 = 2.1 \times 10^{-11}$

7 **a** Using Wien's displacement law,

$T = \frac{2.9 \times 10^{-3}}{400 \times 10^{-9}} = 7250 = 7300$ K (2 s.f.)

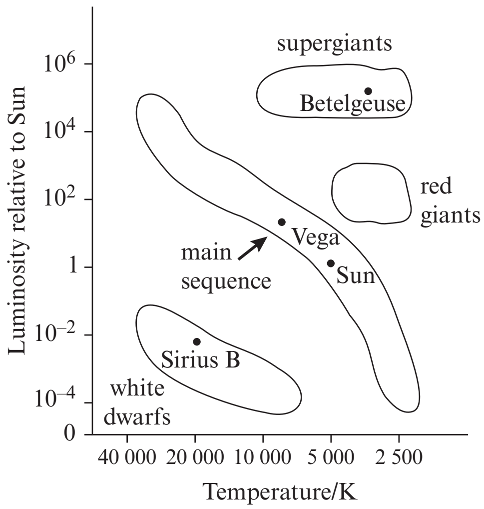
b $d = \sqrt{\frac{L}{4\pi b}} = \sqrt{\frac{7.2 \times 10^{27}}{4\pi \times 2.8 \times 10^{-10}}} =$

1.43×10^{18} m

$= \frac{1.43 \times 10^{18}}{9.46 \times 10^{15}} = 151$ ly

8 **a** A Hertzsprung–Russell diagram is a graph showing the relationship between the luminosity of stars and their surface temperatures.

b, **c** and **d**



9 **a** Main sequence stars have a range of temperatures and luminosities that, on the HR diagram, form a band from top left to bottom right.

b Red giant stars have quite small surface temperatures and large luminosities, putting them in a clump above and to the right of the main sequence.

c Supergiant stars have extremely large luminosities and a mid-range of temperatures, putting them in a clump above the red giants on the HR diagram.

d White dwarf stars have high surface temperatures and small luminosities, putting them in a region below and to the left of the main sequence on the HR diagram.

10 **a** A main sequence star's luminosity is proportional to its mass to the power 3.5 ($L \propto M^{3.5}$)

b $(10M_\odot)^{3.5} = 10^{3.5} \times M_\odot{}^{3.5} = 3162\ M_\odot{}^{3.5}$, which is about 3200 $L_\odot$.

c $5^{3.5} \approx 280$. So, the luminosity of the star will be $280 \times 3.8 \times 10^{26} = 1.1 \times 10^{29}$ W.

d **i** $E = \alpha Mc^2$

ii $t = \dfrac{\alpha Mc^2}{L}$

iii $t \propto \dfrac{\alpha Mc^2}{M^{3.5}} \Rightarrow t \propto M^{-2.5}$

iv $10^{-2.5} \times 10^{10} = 3.2 \times 10^7$ years = 3.2 million years

11 **a** Gravitational potential energy is transformed into thermal energy as the cloud of gas contracts and heats up.

b **i** Thermonuclear fusion means that small nuclei fuse to form heavier particles.

ii The end products are helium nuclei and energy.

12 **a** When the core of the star has converted 12% of the star's hydrogen into helium the amount of helium in the core of the star starts to inhibit the further production of helium by the proton–proton chain. The helium nuclei simply 'get in the way'. This causes instabilities to occur in the star, because the rate of energy production has become less. The gravitational—inward directed—forces then start to overcome the thermal expansion forces in the core.

b After moving off the main sequence, the star will become a red giant star. After that, it will become a planetary nebula and leave behind a small core. If the mass of this core is less than 1.4 solar masses, it will become a white dwarf. Since the white dwarf cannot continue to produce energy, it will gradually cool and fade until it becomes a brown dwarf, and eventually, it will become too cold to radiate in the visible part of the electromagnetic spectrum; it will be a black dwarf.

c In a white dwarf star, the gravitational forces are balanced by the electron degeneracy pressure. This is a consequence of the Pauli exclusion principle.

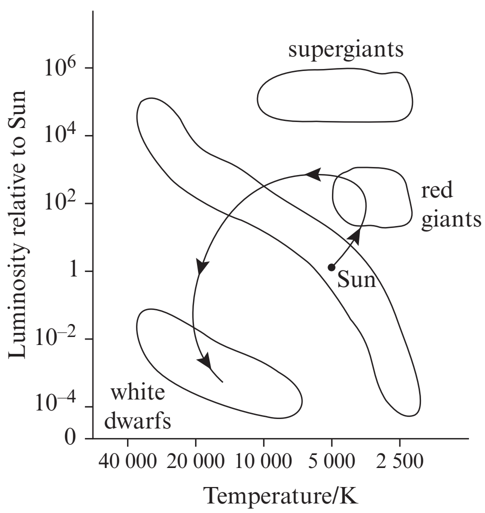
13 **a** It will become a supergiant star once it leaves the main sequence. After a period of continued fusion of heavier elements, it undergoes a supernova explosion, leaving behind a dense core. If the mass of this core is less than 1.4 solar masses (this is called the Chandrasekhar limit) then the core will become a white dwarf. If the mass of the core is 1.4 to 3.0 solar masses (called the Oppenheimer–Volkoff limit), it will become a neutron star. If the mass of the core is greater than three solar masses, it will become a black hole.

b The Chandrasekhar limit is the maximum mass that a white dwarf star can have. For a mass below this limit, electron degeneracy pressure can oppose gravitational forces and allow the star to be a stable white dwarf. For a mass larger than this, electron degeneracy pressure is no longer sufficient to oppose the gravitational forces and the star will contract further into a neutron star.

c neutron degeneracy pressure.

d The Oppenheimer–Volkoff limit states that for masses greater than three solar masses, the core of the star will not be able to provide sufficient neutron degeneracy pressure to keep the star in hydrostatic equilibrium. The core will collapse into a black hole.

14 a



b As the star moves off the main sequence it expands. This causes its luminosity to increase and its surface temperature to decrease. After a period of time in the red giant region of the HR diagram, the star undergoes a planetary nebula event, and the core left behind will move from the red giant region into the white dwarf region because it is now smaller and hotter. The effect of the reduced surface area is greater than the effect of the increased temperature, so the luminosity of the star decreases.

15 At some distance away from the black hole, the escape velocity becomes smaller than the speed of light, allowing radiation to escape the huge gravitational field. This is often called the *event horizon*. Matter (e.g. from a binary companion star) that has been attracted by the huge gravitational field of the black hole emits X-rays as it speeds up on its path towards the black hole. Astronomers can observe this X-ray emission from the region outside the event horizon and infer the existence of a black hole.

16 a The mass deficit is (1.007276 + 2.013553) u − 3.01493 u = 0.0059 u.

So, energy released is E = 0.0059 × 931.5 = 5.49 MeV.

b The mass deficit is (2 × 3.01493) u − (4.001506 + 2 × 1.007276) u = 0.01381 u.

So, energy released is $E = 0.01381 \times 931.5 =$ 12.86 MeV.

c $4\,{}^1_1\text{H} \rightarrow {}^4_2\text{He} + 2\,{}^0_1\beta^+ + 2\,{}^0_0\nu$ (Only the particles are shown.)

d $2 \times 1.44 + 2 \times 5.49 + 12.86 = 26.7\ MeV$

e Number of reactions $\text{s}^{-1} = \frac{3.8 \times 10^{26}}{26.7 \times 1.6 \times 10^{-13}}$ $= 8.9 \times 10^{37}\ \text{s}^{-1}$

f Mass lost $\text{s}^{-1} = 8.9 \times 10^{37} \times 4 \times 1.007276 \times 1.66 \times 10^{-27} = 5.9 \times 10^{11}\ \text{kgs}^{-1}$

g $0.75 \times 2 \times 10^{30} = 1.5 \times 10^{30}$ kg

h time $= \frac{0.12 \times 1.5 \times 10^{30}}{5.9 \times 10^{11}} = 3.1 \times 10^{17}$ s $=$ $\frac{3.1 \times 10^{17}\text{s}}{3.15 \times 10^7\ \text{s year} -1} = 9.8 \times 10^9$ years

i 9.8 − 4.6 = 5.2 billion years.

Exercise 25.3

1 a i ${}^1_1\text{p} + {}^{12}_6\text{C} \rightarrow {}^{13}_7\text{N} + \gamma$

ii ${}^{13}_7\text{N} \rightarrow {}^{13}_6\text{C} + {}^0_1\beta^+ + {}^0_0\nu$

iii ${}^1_1\text{p} + {}^{13}_6\text{C} \rightarrow {}^{14}_7\text{N} + \gamma$

iv ${}^1_1\text{p} + {}^{14}_7\text{N} \rightarrow {}^{15}_8\text{O} + \gamma$

v ${}^{15}_8\text{O} \rightarrow {}^{15}_7\text{N} + {}^0_1\beta^+ + {}^0_0\nu$

vi ${}^1_1\text{p} + {}^{15}_7\text{N} \rightarrow {}^{12}_6\text{C} + {}^4_2\text{He}$

b **i** No.

ii No.

iii No.

c It uses 4 protons and produces a helium nucleus, 2 beta-plus-particles and 2 neutrinos, and some gamma radiation.

d For a proton to fuse with a carbon nucleus (or a nitrogen nucleus), it will need to have more kinetic energy than it needs to fuse with an other proton, as in the proton–proton chain. This means that the temperature of the core of the star will have to be higher. A larger-mass star will have converted more gravitational potential energy into thermal energy in collapsing from a nebula into a protostar and then into a main sequence star. The hotter core will then provide protons with enough kinetic energy to approach the heavier nuclei close enough for the strong nuclear force to overcome the Coulomb repulsion and allow the two particles to fuse.

2 **a** $(1.007276 + 11.99945 - 13.00189) \times 931.5 = 4.5047$ MeV

b $(13.00189 - 13.00006 - 0.00055) \times 931.5 + 1.02$ MeV $= 2.2123$ MeV

c $(1.007276 + 13.00006 - 13.99922) \times 931.5 = 7.5601$ MeV

d $(1.007276 + 13.99922 - 14.99867) \times 931.5 = 7.2899$ MeV

e $(14.99867 - 14.99626 - 0.00055) \times 931.5 + 1.02$ MeV $= 2.7526$ MeV

f $(1.007276 + 14.99626 - 11.99945 - 4.001506) \times 931.5 = 2.4033$ MeV

g Total energy released $= 4.5047 + 2.2123 + 7.5601 + 7.2899 + 2.7526 + 2.4033 = 26.7$ MeV

h $4\,{}^{1}_{1}\mathrm{p} \rightarrow {}^{4}_{2}\mathrm{He} + 2\,{}^{0}_{1}\beta^{+} + 2\,{}^{0}_{0}\nu$ (Only the particles are shown.) This is exactly the same as the overall effect of the proton–proton chain.

i The energy production of the CNO cycle is the same as the energy production of the proton–proton chain.

3 **a** The two nuclear reactions are

$${}^{4}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{8}_{4}\mathrm{Be} + {}^{0}_{0}\gamma$$

quickly followed by

$${}^{8}_{4}\mathrm{Be} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{12}_{6}\mathrm{C} + {}^{0}_{0}\gamma.$$

b The ${}^{8}_{4}\mathrm{Be}$ nucleus is highly unstable and will decay, by alpha decay, into two alpha particles (the half-life for this decay is of the order of 10^{-16} s). So, the second of the triple-alpha processes has to occur very quickly indeed.

c Mass deficit $= (3 \times 4.001506 - 11.99945)$ u $= 0.005068$ u

So, the energy released $= 0.005068 \times 931.5$ MeV $= 4.72$ MeV.

d The surface temperature of the star increases.

e **i** It will end up as a white dwarf.

ii The very central part of the core will contain carbon; outside this will be helium and outside that hydrogen.

iii No.

4 **a** The star needs to have enough mass for its core to have a high enough temperature so that the helium nuclei have enough kinetic energy to approach and be absorbed by nuclei.

b Adding helium nuclei to other nuclei can happen up to iron and nickel. After that, the binding energy per nucleon starts to decrease with increasing mass, so further fusion is not possible spontaneously.

5

Initial mass	Eventual fate of star
$M < 0.25M_{\odot}$	White dwarf with helium core
$0.25M_{\odot} < M < 8M_{\odot}$	White dwarf with carbon core
$9M_{\odot} < M < 12M_{\odot}$	White dwarf with oxygen, neon or magnesium core
$12M_{\odot} < M < 40M_{\odot}$	Neutron star
$M > 40M_{\odot}$	Black hole

6 a A sizeable flux of neutrons is required. Neutrons can build up in stars over the long period during which the stars have been producing heavier and heavier nuclei. Also, in the very heaviest of stars, the extremely high core temperatures can cause nuclei to break apart during collisions. Some of the collisions will split helium nuclei into protons and neutrons, adding to the neutron flux available. In supernovae, the large amounts of energy available from the explosion do the same thing, thus increasing the flux of neutrons in the star.

b The equation gives a positive mass deficit. (Actually, the mass deficit is 0.027 u.) This means that the reaction can occur spontaneously.

c $^{59}_{26}\text{Fe} \rightarrow ^{59}_{27}\text{Co} + ^{0}_{-1}\beta^{-} + ^{0}_{0}\overline{\nu}_{e}$

d Since neutrons are unstable (*per se*), nuclei that absorb a neutron will decay by beta-minus decay, producing a nucleus with a higher atomic number. Providing there is a sufficient neutron flux—and the supernova explosion will satisfy this—further neutron absorption followed by further beta-minus decay will produce higher and higher atomic number nuclides.

7 The s-process requires a small flux of neutrons (these are by-products of other processes involving carbon, oxygen and silicon), which allows nuclei to absorb a neutron, which then decays by β^{-}-decay. Because the process happens slowly, there is enough time for the neutron to decay. This produces nuclei of higher atomic number.

8 In the r-process, a large neutron flux means that the capture of neutrons by nuclei happens relatively easily in very short times. Neutrons do not have time to decay (by the usual β^{-}-decay), so the nucleus formed will be a heavier isotope of the same element.

9 A type 1a supernova occurs from a binary star system, one star of which is a white dwarf with a large gravitational field. Material from the companion star is attracted to the white dwarf, which increases the mass of the white dwarf. When the mass of the white dwarf becomes larger than the Chandrasekhar limit (1.4 solar masses), the white dwarf will collapse (because the electron degeneracy pressure is insufficient to balance the gravitational force). Further fusion of carbon and oxygen produces such a large amount of radiation pressure that the star explodes into a supernova.

Since the supernova occurs because the mass of the white dwarf has become 1.4 solar masses, the resulting luminosity of the star will be a constant value—hence the idea of it being a standard candle. This constant value can then be used to find the distance of the supernova from the Earth.

10 A type 2 supernova occurs after several stages of fusion of successively heavier nuclei in stars heavier than the Sun. Nuclei fuse in the core of a star until the supply of nuclei runs out. This makes the star collapse (because the hydrostatic equilibrium has been lost and gravitational force is dominant). The result of the collapse is a rapid heating of the core, which enables heavier nuclei to fuse together until the supply of these nuclei runs out. The process repeats itself, producing shells of successively heavier nuclei around the core until the innermost shell is made from iron. Once iron is reached, no more fusion can occur and hydrostatic equilibrium is lost for the last time. The star collapses under its gravitational force, which is too strong for electron degeneracy pressure, and neutrons are produced. The resulting neutron degeneracy pressure is extremely large, and the star explodes, producing elements heavier than iron.

Type 1a supernovae have a luminosity that is higher than type 2 supernovae (about 10 times higher) and this luminosity decreases at a decreasing rate over the next year or so.

Type 2 supernovae have a luminosity that decreases sharply for a few days and then levels out for a month or so before decreasing sharply again. After about three months, the luminosity decreases gradually for about a year.

Exam-style questions

Multiple-choice questions

1 C [1]

2 D [1]

3 A [1]

4 A [1]

5 D [1]

6 A [1]

7 B [1]

8 B [1]

Short-answer questions

9 a Nuclear fusion [1]

b $\Delta m = ((2.014102 + 3.01605) - (4.002604 + 1.008665))\text{ u} = 0.01888\text{ u}$ [1]

So, $E = 0.01888 \times 931.5 = 17.59\text{ MeV}$ ($= 17.59 \times 1.6 \times 10^{-13} = 2.81 \times 10^{-12}\text{ J}$). [2]

(Accept answer in MeV.)

c Energy is released in the form of E_K of the alpha particle and the neutron. [1]

10 a Supergiant region [1]

b More massive [1]

c 1.4 solar masses [1]

d Eventually, the material ejected from the star will form a new nebula and allow the formation of new stars. [1]

11 a $\frac{L}{L_\odot} = \left(\frac{M}{M_\odot}\right)^{3.5} \Rightarrow M = \left(\frac{L}{L_S} \times M_\odot{}^{3.5}\right)^{\frac{1}{3.5}} = \left(\frac{L}{L_S}\right)^{\frac{1}{3.5}} \times M_\odot =$ [1]

$3000^{\frac{1}{3.5}} \times M_\odot = 9.85\,M_\odot \approx 10\,M_\odot$ [1]

b The star will expand to become a red supergiant. [1]

After that, it will undergo a supernova event, [1]

leaving behind a neutron star or a black hole. [1]

12 a Fusion of protons into helium [1]

By the proton–proton chain or by the CNO cycle [1]

b i Stars A and B have the same luminosity, but A is hotter **[1]**, so, since $L = \sigma A T^4$, star A must have a smaller surface area and, therefore, be smaller. [1]

ii Stars B and C have the same temperature, but star B has a larger luminosity **[1]** so, since $L = \sigma A T^4$, star C must have a smaller surface area and, therefore, be smaller. [1]

13 a Main sequence → red supergiant → supernova → neutron star [2]

b The maximum mass of a neutron star is about 3.0 solar masses. [1]

Above this mass and the star will collapse into a black hole. [1]

c It does not have enough mass. [1]

After the supernova event, the remnants of the star (which will be less than three solar masses, because most of the star's mass will be lost n the supernova event) will be kept in equilibrium by neutron degeneracy pressure, preventing gravitational forces from collapsing the star into a black hole. [1]

14 a i The total emitted power of the star [1]

ii The power received at the Earth per unit area from a star [1]

b $r = \sqrt{\frac{L}{4\pi b}} = \sqrt{\frac{3.8 \times 10^{26}}{4\pi \times 1.3 \times 10^{3}}} =$
$1.5 \times 10^{11}\text{ m}$ [2]

c $A_{star}T^4_{star} = A_\odot T^4_\odot \Rightarrow \frac{4\pi r^2_\odot}{4\pi r^2_{star}} = \frac{T^4_{star}}{T^4_\odot} \Rightarrow$
$\frac{r_\odot}{r_{star}} = \sqrt{\frac{T^4_{star}}{T^4_\odot}} = \sqrt{0.5^4} = 0.25$ [2]

15 a $\lambda_{max} = \frac{2.9 \times 10^{-3}}{T} = \frac{2.9 \times 10^{-3}}{5200} =$
558 ≈ 560 nm **[2]**

b $L = 4\pi\sigma R^2 T^4 = 4\pi \times 5.67 \times 10^{-8} \times (5 \times 10^8)^2 \times 5200^4 = 1.3 \times 10^{26}$ W **[2]**

c It will eventually become a white dwarf. **[1]**

16 a $\rho = \frac{\text{mass}}{\text{volume}} \Rightarrow R = \sqrt[3]{\frac{3M}{4\pi\rho}} =$
$\sqrt[3]{\frac{3 \times 3 \times 10^{30}}{4\pi \times 1.8 \times 10^{17}}} = 15.8$ km ≈ 16 km **[2]**

b 6×10^{30} kg $= 3M_\odot$ **[1]**

The Oppenheimer–Volkoff limit states that a neutron star with a mass larger than $3M_\odot$ will collapse into a black hole. **[1]**

c neutron degeneracy pressure. **[1]**

17 a $L = 4\pi\sigma R^2 T^4 = 4\pi \times 5.67 \times 10^{-8} \times (6.96 \times 10^8)^2 \times 5778^4 = 3.8 \times 10^{26}$ W **[2]**

b Fusion of protons into helium (by the proton–proton chain) **[1]**

c Main sequence **[1]**

Red giant **[1]**

(Planetary nebula)

White dwarf **[1]**