Exam-style questions and sample answers have been written by the authors. In examinations, the way marks are awarded may be different.

Coursebook answers

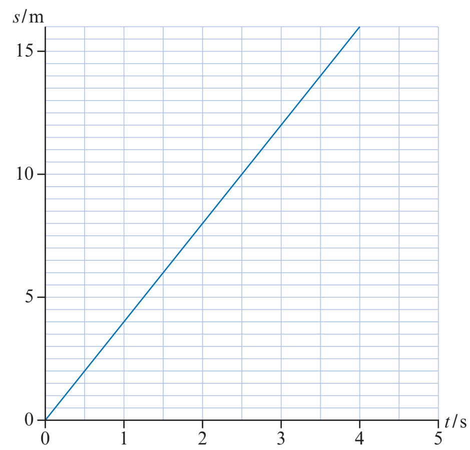
Chapter 1

Test your understanding

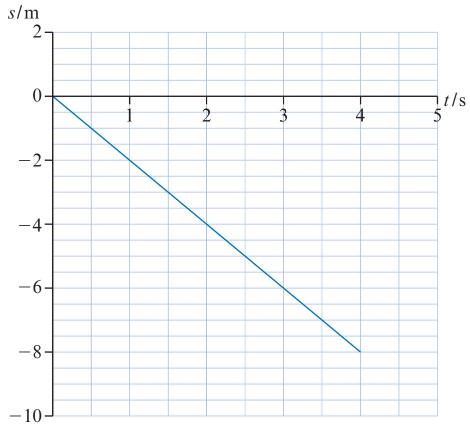
1 Distance travelled in first 1.5 h is $s = vt = 70 \times 1.5 = 105$ km. Remaining distance is 15 km and must be covered in 1.0 h so average speed must be $v = \frac{15}{1.0}\,\frac{\text{km}}{\text{h}} = 15\ \text{km h}^{-1}$.

2 We find the gradient in each case: $v_a = 3.0\ \text{m s}^{-1}$; $v_b = -4.0\ \text{m s}^{-1}$; $v_c = 8.0\ \text{m s}^{-1}$; $v_d = -4.0\ \text{m s}^{-1}$

3 **a** Displacement = 16 m



b Displacement = −8.0 m



4 The relative speed of the cyclists is $v = 35\ \text{km h}^{-1}$. They will then meet in a time of $v = \frac{70}{35} = 2.0\,\text{h}$.

a The common displacement is $s = 15 \times 2.0 = 30\ \text{km}$.

b The fly will travel a distance of $s = 30 \times 2.0 = 60\ \text{km}$.

5 Use $v = u + at$. So $8.0 = 2.0 + a \times 2.0 \Rightarrow a = \frac{8.0 - 2.0}{2.0} = 3.0\ \text{m s}^{-2}$.

6 From $s = \frac{u+v}{2}t$, we find $s = \frac{0+28}{2} \times 9 = 126 \approx 130\ \text{m}$.

7 From $v^2 = u^2 + 2as$ we find $0 = 12^2 + 2 \times a \times 45 \Rightarrow a = -\frac{12^2}{2 \times 45} = -1.6\ \text{m s}^{-2}$.

8 From $s = ut + \frac{1}{2}at^2$ we get $16 = -6.0t + \frac{1}{2} \times 2.0 \times t^2$. Solving the quadratic equation gives $t = 8.0\ \text{s}$.

9 From $s = \frac{u+v}{2}t$ we get $450 = \frac{0+v}{2} \times 15$ so $v = \frac{900}{15.0} = 60.0\ \text{m s}^{-1}$.

OR

Use $s = ut + \frac{1}{2}at^2$. So $450 = \frac{1}{2}a \times 15.0^2 \Rightarrow a = \frac{900}{225} = 4.00\ \text{m s}^{-2}$. Then from $v = u + at$, $v = 4.00 \times 15.0 = 60.0\ \text{m s}^{-1}$.

10 $v^2 = 2ad$. The speed we want, u, satisfies $u^2 = 2a\frac{d}{2} = \frac{1}{2}2ad = \frac{1}{2}v^2$. Hence $u = \frac{v}{\sqrt{2}}$.

11 a The distance travelled before the brakes are applied is $s = 40.0 \times 0.50 = 20\ \text{m}$. Once the brakes are applied the distance is given from $v^2 = u^2 + 2as$, i.e. $0 = 40^2 + 2 \times (-4.0) \times s \Rightarrow s = 200\ \text{m}$. The total distance is thus 220 m.

b 200 m as done in (a).

c $s = 220 - 200 = 20\ \text{m}$.

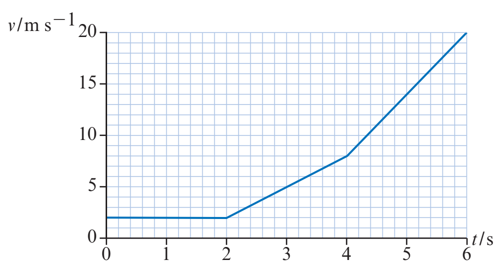
d It would be less since the speed is less.

12 a $s_1 = -\frac{1}{2} \times 10t^2 = -5t^2$ and $s_2 = -\frac{1}{2} \times 10(t-1)^2$. Two seconds after the second ball was dropped means that $t = 3.0\ \text{s}$. Then, $s_1 = -45\ \text{m}$ and $s_2 = -20\ \text{m}$. The separation is thus 25 m.

b $s_1 - s_2 = -5t^2 + \frac{1}{2} \times 10(t-1)^2 = -10t + 5$ so in magnitude this increases.

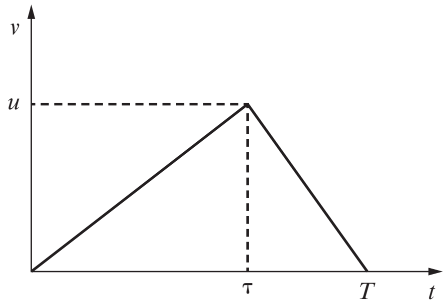
13 a The velocity at 2 s is $v_2 = 2 + 0 \times 2 = 2.00\ \text{m s}^{-1}$. The velocity at 4 s is $v_4 = 2 + 3 \times 2 = 8.00\ \text{m s}^{-1}$. The velocity at 6 s is $v_6 = 8 + 6 \times 2 = 20.0\ \text{m s}^{-1}$. Alternatively, the area under the graph is $18.0\ \text{m s}^{-1}$ and this gives the *change* in velocity. Since the initial velocity is $2.00\ \text{m s}^{-1}$, the final velocity is $20.0\ \text{m s}^{-1}$.

b



14 The acceleration is the slope of the velocity–time graph. Drawing a tangent to the curve at 2 s we find a slope of approximately $a = 2.0\ \text{m s}^{-2}$.

15 You must push the car as hard as you can but then you must also pull back on it to stop it before it crashes on the garage. The velocity–time graph must be something like:



We know that: $u = 2\tau$. During pullback, we have that the velocity is given by $v = u - 3(t - \tau) = 2\tau - 3(t - \tau) = 5\tau - 3t$. The velocity becomes zero at time T and so $0 = 5\tau - 3T$, i.e. $\tau = \frac{3T}{5}$. The area under the curve (triangle) is 15 m and is given by $\frac{1}{2}\,Tu = \frac{1}{2}T(2\tau) = \frac{1}{2}T\frac{6T}{5} = \frac{3T^2}{5}$. Hence $\frac{3T^2}{5} = 15 \Rightarrow T^2 = 25 \Rightarrow T = 5$ s.

16 **a** $v^2 = u^2 - 2gy$ so $0 = 8.0^2 - 2 \times 9.81 \times y$ and thus $y = \frac{8.0^2}{2 \times 9.81} = 3.26 \approx 3.3$ m.

b Use $s = ut + \frac{1}{2}at^2$ to get $-35 = 8 \times t - \frac{1}{2} \times 9.81 \times t^2$ and solve for time to get $t = 3.61$ s.

c $v = u + at = 8 - 9.81 \times 3.61 = -27.4$ m s^{-1}.

d $3.26 + 3.26 + 35 = 41.5$ m.

e average speed is $\frac{41.5}{3.61} = 11.5$ m s^{-1} and average velocity is $\frac{-35}{3.61} = -9.70$ m s^{-1}.

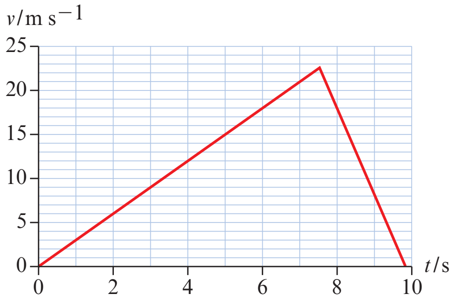
17 **a** $s = ut - \frac{1}{2}gt^2 = 20 \times 6.0 - 5 \times 6.0^2 = -60$ m. So height is 60 m.

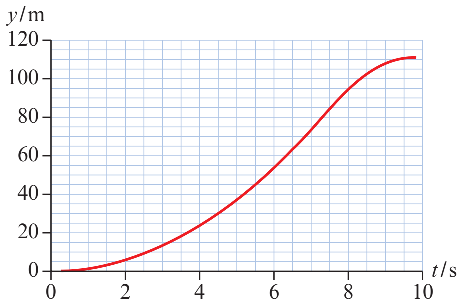
b $v = u - gt = 20 - 10 \times 6.0 = -40$ m s^{-1}, so speed is 40 m s^{-1}.

18 **a** $v^2 = u^2 + 2ay = 0 + 2 \times 3.0 \times 85$ so $v = 22.58 \approx 23$ m s^{-1}.

b $v^2 = u^2 - 2gy$ so $= 22.58^2 - 2 \times 9.81 \times y$ giving $y = 25.99 \approx 26$ m. $26 + 85 = 111$ m.

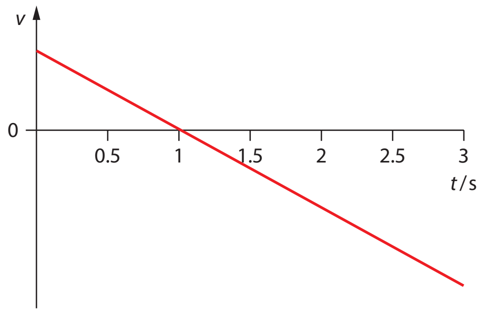
c



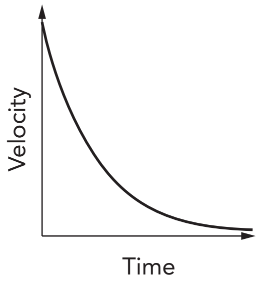


d $y = ut + \frac{1}{2}gt^2$ i.e. $111 = 0 + \frac{1}{2} \times 9.81 \times t^2$ so $t = 4.76 \approx 4.8$ s.

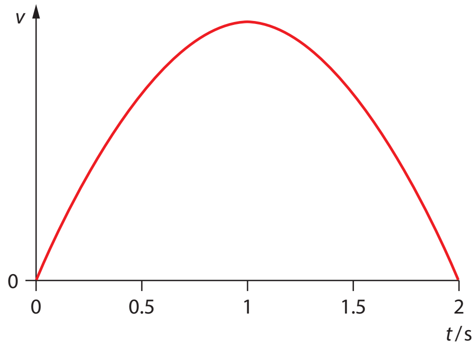
19 Velocity is the slope of the position–time graph. So we observe that the velocity is initially positive and begins to decrease. It becomes zero at 1 s and then becomes negative. The displacement graph is in fact a parabola and so the velocity is in fact a linear function. Of course we are not told that, so any shape showing the general features described above would be acceptable here.



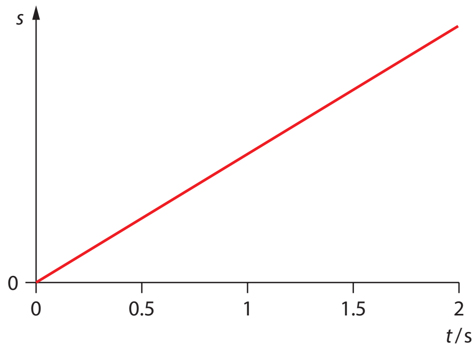
20 Velocity is the slope of the position–time graph. So we observe that the velocity is initially very large and continues to decrease all the time approaching zero. The slope and hence the velocity are always positive.



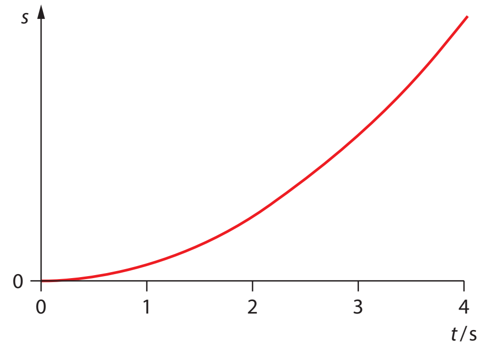
21 Velocity is the slope of the position–time graph. So we observe that the velocity is initially very small, becomes greatest at 1 s and starts decreasing thereafter becoming very small again. The slope and hence the velocity are always positive.



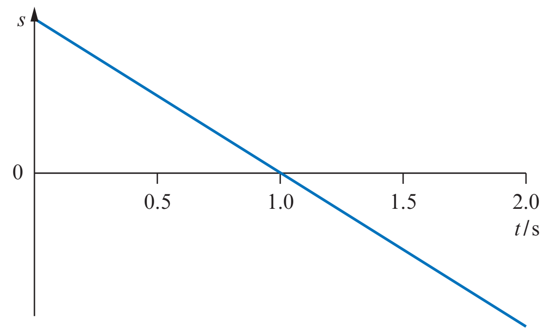
22 Velocity is constant so graph will be a straight line:



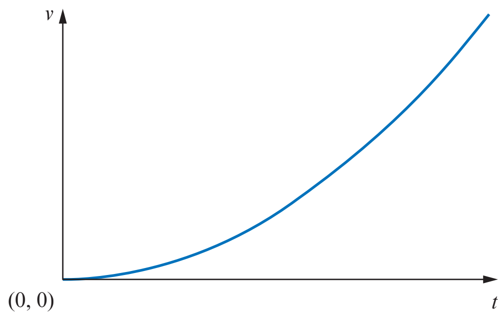
23 Acceleration is constant so the graph is a parabola.



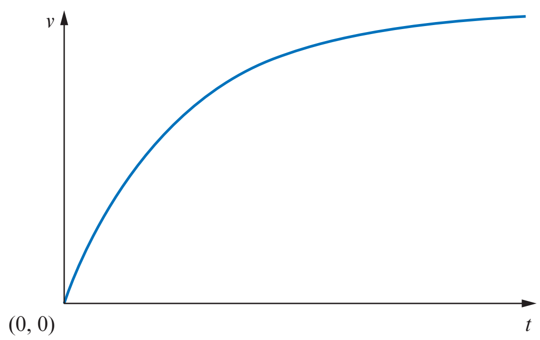
24 The gradient of the velocity graph starts large and positive, becomes zero at 1 s and then more and more negative:



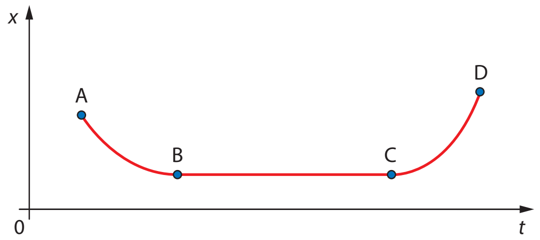
25 The gradient of the velocity graph must be increasing so:



26 Gradient must start large and positive and approach zero as time increases so:



27



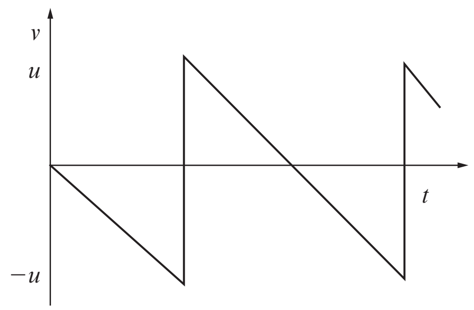
a The velocity is the slope of the position–time graph. Therefore the velocity is negative from A to B.

b Between B and C, the slope is zero and so the velocity is zero.

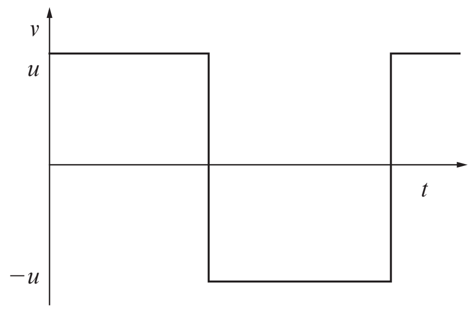
c From A to B the velocity is becoming less negative and so it is increasing. So the acceleration is positive.

d From C to D the slope is increasing meaning the velocity is increasing. Hence the acceleration is positive.

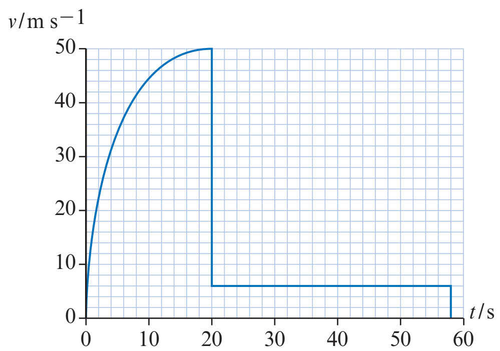
28 a



b



c

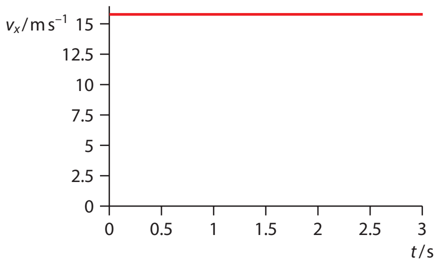


29 The time to fall to the floor is given by $y = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2 \times 1.3}{9.81}} = 0.515$ s. The horizontal distance travelled is therefore $x = v_x t = 2.0 \times 0.515 = 1.03 \approx 1.0$ m.

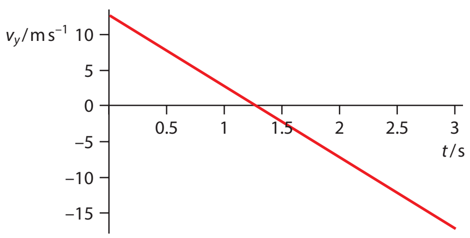
30 a The times to hit the ground are found from $y = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2y}{g}} = \sqrt{\frac{2 \times 4.0}{9.81}} = 0.903$ s and $\sqrt{\frac{2 \times 8.0}{9.81}} = 1.277$ s. The objects are thus separated by $4.0 \times (1.277 - 0.903) = 1.496 \approx 1.5$ m when they land.

b The horizontal distance travelled by the object falling from 8.0 m is $x = v_x t = 4.0 \times 1.277 = 5.108$ m. Thus the speed of the other object must be $v_x = \frac{5.108}{0.903} = 5.657 \approx 5.7$ m s^{-1}.

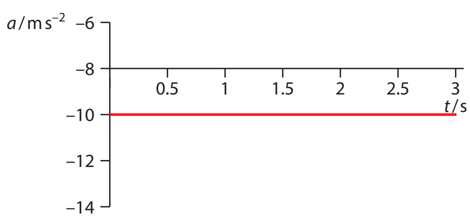
31 a $v_x = v \cos 40° = 20 \cos 40° = 15.3$ m s^{-1} and so the graph is a horizontal straight line:



b $v_y = v \sin 40° - gt = (12.9 - 10t)$ m s^{-1} so that the graph is a straight line with negative slope:

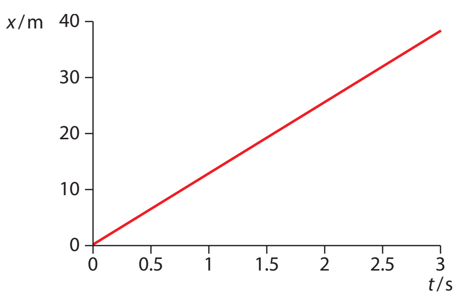


c The acceleration is constant, so the graph is a horizontal straight line.

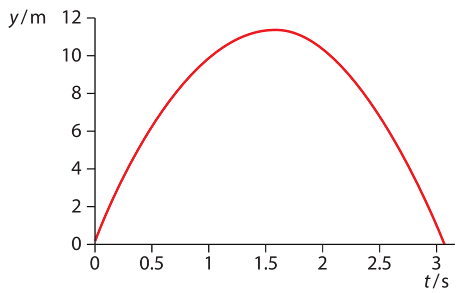


32 $u_y = u \sin 40° = 15.43$ m s^{-1}. At the highest point $0 = u_y^2 - 2gy$ so $y = \frac{u_y^2}{2g} = \frac{15.43^2}{2 \times 9.81} = 12.1 \approx 12$ m.

33 a The horizontal position is given by $x = 20t \times \cos 50° = 12.9t$ whose graph is a straight line.



b The vertical position is $y = 20t \times \sin 50° - \frac{1}{2} \times 10\,t^2 = 15.3t - 5\,t^2$ whose graph is a concave down parabola.



34 In time t the chimp will fall a vertical distance $y = \frac{1}{2}gt^2$ but so will the bullet and hence the bullet will hit the chimp.

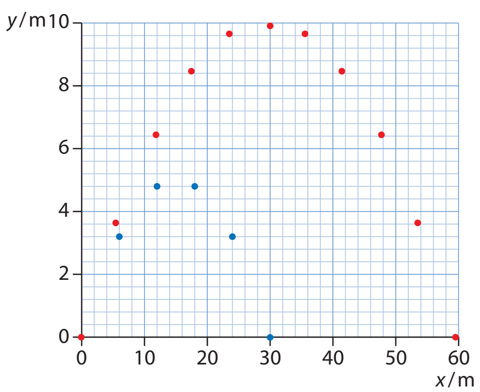
35 a i The ball covers a horizontal distance of 60 m in 2.0 s and so the horizontal velocity component is $u_x = \frac{60}{2.0} = 30\ \text{m s}^{-1}$. The ball climbs to a height of 10 m in 1.0 s and so from $y = \frac{u_y + v_y}{2}t$ we have $10 = \frac{u_y + 0}{2} \times 1.0 \Rightarrow u_y = 20\ \text{m s}^{-1}$.

ii The angle of launch is $\theta = \tan^{-1}\frac{u_y}{u_x} = \tan^{-1}\left(\frac{20}{30}\right) = 34°$.

iii The vertical component of velocity becomes zero at 1.0 s and so $v_y = u_y - gt \Rightarrow 0 = 20 - g \times 1.0 \Rightarrow g = 20\ \text{m s}^{-2}$.

b The velocity is horizontal to the right and the acceleration is vertically down.

c With $g = 40\ \text{m s}^{-2}$, the ball will stay in the air for half the time and so will have half the range. The maximum height is reached in 0.50 s and is $y = u_y t - \frac{1}{2}g\,t^2 = 20 \times 0.50 - \frac{1}{2} \times 40 \times 0.50^2 =$ 5.0 m i.e. half as great as before. This leads to:



36 a The initial velocity components are: $u_x = 20.0\cos 48° = 13.38\ \text{m s}^{-1}$ and $u_y = 20.0\sin 48° =$ $14.86\ \text{m s}^{-1}$. The ball hits the sea when the vertical position is $y = -60.0$ m. Thus $y = u_y t - \frac{1}{2}g\,t^2 \Rightarrow -60.0 = 14.86t - 4.90\,t^2$. Solving for the positive root we find $t = 5.33$ s. Hence $v_x = u_x = 13.38\ \text{m s}^{-1}$ and $v_y = u_y - gt = 14.86 - 9.81 \times 5.33 = -37.43\ \text{m s}^{-1}$. The speed at impact is thus $v = \sqrt{13.38^2 + (-37.43)^2} = 39.7 \approx 40\,\text{m s}^{-1}$ at $\theta = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\left(-\frac{37.43}{13.38}\right) = -70°$ to the horizontal.

b Some of the kinetic energy of the ball will be converted into thermal energy and so the speed at impact will be less. The horizontal component of velocity will decrease in the course of the motion and will tend to go to zero but the vertical component will never become zero (after reaching the maximum height). This means that the angle of impact will be steeper.

37 a Terminal speed is the eventual constant speed reached by a projectile as a result of an air resistance force that increases with speed.

b Initially the net force on the particle is just the weight. As the speed increases so does the resistance force. Eventually the resistance force will equal the weight and from then on the particle will move with zero acceleration, i.e. with a constant terminal speed.

38 a The height is given by $h = ut\sin 45° - \frac{1}{2}gt^2 = ut\frac{\sqrt{2}}{2} - \frac{1}{2}gt^2$

We rewrite as $gt^2 - u\sqrt{2}\,t + 2h = 0$. The roots of this quadratic equation give the times at which the projectile is at the same height h. The roots are

$$t_{1,2} = \frac{u\sqrt{2} \pm \sqrt{2u^2 - 8gh}}{2g}$$

So the horizontal distance travelled to get to the height h the first time is

$ut_1\cos 45° = u\frac{\sqrt{2}}{2}\left(\frac{u\sqrt{2} - \sqrt{2u^2 - 8gh}}{2g}\right)$ and the second time is $ut_2\cos 45° = u\frac{\sqrt{2}}{2}\left(\frac{u\sqrt{2} + \sqrt{2u^2 - 8gh}}{2g}\right)$.

The difference of these distances is what we want and this is

$$= 2 \times u\frac{\sqrt{2}}{2}\left(\frac{\sqrt{2u^2 - 8gh}}{2g}\right)$$

$$= u\sqrt{2}\frac{\sqrt{2}\sqrt{u^2 - 4gh}}{2g}$$

$$= \frac{u}{g}\sqrt{u^2 - 4gh}$$

b At the maximum height the distance in a is zero and so $h = \frac{u^2}{4g}$

39 The projectile will reach the top at $t = 3.0$ s. Hence $0 = u\sin\theta - 10 \times 3.0$ and so $u\sin\theta = 30\ \text{m s}^{-1}$. Then $0 = 30^2 - 2 \times 10 \times h$ so that $h = 45$ m.

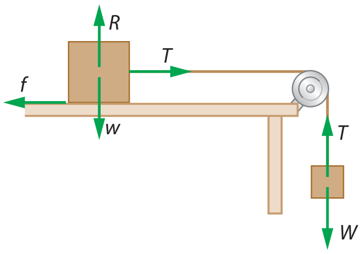
OR

$u\sin\theta \times 1 - 5 \times 1^2 = u\sin\theta \times 5 - 5 \times 5^2$ so that $4u\sin\theta = 120 \Rightarrow u\sin\theta = 30\ \text{m s}^{-1}$. The projectile will reach the top at $t = 3.0$ s and so $h = u\sin\theta \times 3 - 5 \times 3^2 = 30 \times 3 - 5 \times 3^2 = 45$ m.

Chapter 2

Test your understanding

1 The forces are shown in the diagram.



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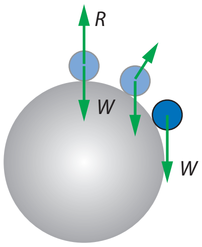
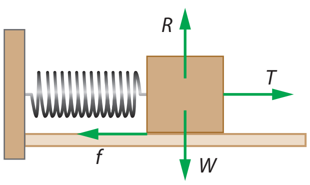


3 The tension is the same in both cases since the wall exerts a force of 50 N to the left on the string just as in the second diagram.

4



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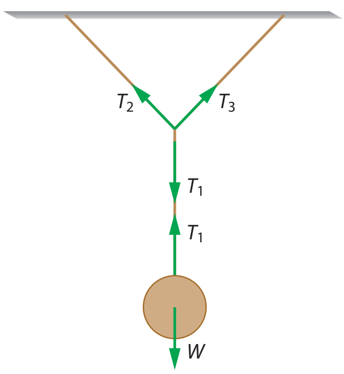


6 $F = kx$, i.e., $F \propto x$ so the new force is $F = 12 \times \frac{4}{3} = 16$ N.

7 **a** Both springs exert a force to the left. The net force is $2kx = 2 \times 220 \times 0.02 = 8.8$ N to the left.

b The right spring gives a force $kx = 220 \times 0.02 = 4.4$ N to the right. The left spring gives a force $kx = 220 \times 0.06 = 13.2$ N to the left. The net force is $13.2 - 4.4 = 8.8$ N to the left as before.

8 **a** 30 N to the right

b 6 N to the right

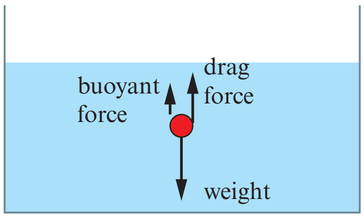
c 8 N to the left

d 15 N to the right

e 10 N down

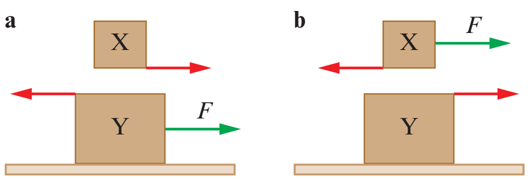
f 20 N up

9



10 There is no vertical force to balance the weight.

11



12 The largest frictional force that can develop between the mass and the table is $f_{max} = \mu_s N = 0.60 \times 2.00 \times 9.8 = 11.8$ N. This is also the tension holding the hanging mass up. Hence $mg = 11.8 \Rightarrow m = 1.2$ kg.

13 Equilibrium demands that $W = F + N \Rightarrow N = W - F = 220 - 140 = 80$ N.

14 Equilibrium demands that $W + F = N \Rightarrow N = 15 + 12 = 27$ N. This is the force that the table exerts on the block.

15 **a** Since the string is being pulled slowly, we have equilibrium until one of the strings breaks. If the lower string is being pulled with force F then the tension in the lower string will be F and the tension in the upper string will be T where $T = mg + F$. The tension in the upper string is thus greater and will reach breaking point first.

b If the lower string is pulled down abruptly, the inertia of the block will keep it momentarily motionless and so the tension in the lower string will reach a high value before the upper one does. Hence it will break.

16 Equilibrium demands that $W + F = N \Rightarrow N = 150 + 50 = 200$ N. This is the force that the table exerts on the block, and by Newton's third law this is the force the block exerts on the table.

17 The component of the weight down the plane is $Mg \sin\theta$, and for equilibrium this is also the tension in the string. To have equilibrium for the hanging mass its weight must equal the tension and so $Mg \sin\theta = mg$. Hence $\theta = \sin^{-1}\frac{m}{M} = \sin^{-1}\frac{4.0}{12} = 19°$.

18 **a** One possibility is to have the mass of the body decrease as in the case of a rocket where the fuel is being burned and ejected from the rocket.

b That happens when the mass increases as for example a cart that is being filled with water or sand while being pulled with a constant force.

19 **a** The force opposing the motion is $\mu_d mg$ and so the deceleration is $\mu_d g = 3.43\,\text{m s}^{-2}$. From $v = u - at$ we find $0 = 18 - 3.43t \Rightarrow t = 5.2$ s.

b From $v^2 = u^2 - 2as$ we find $0 = 18^2 - 2 \times 3.43 \times s \Rightarrow s = 47$ m.

c The mass has cancelled out so there would be no change.

20 **a** The net force on the block along the plane is $mg\sin\theta + \mu_d N$ directed down the plane, where N is the normal force. We know that $N = mg\cos\theta$ so the net force is $mg\sin\theta + \mu_d mg\cos\theta$. The deceleration is then $g(\sin\theta + \mu_s\cos\theta) = 9.81 \times (\sin 28° + 0.42 \times \cos 28°) = 8.24\ \text{m s}^{-2}$. So from $v = u - at$ we find $0 = 14 - 8.24t \Rightarrow t = 1.7$ s.

b From $v^2 = u^2 - 2as$ we find $0 = 14^2 - 2 \times 8.24 \times s \Rightarrow s = 12\,\text{m}$.

c When the block stops the forces along the plane are $mg\sin\theta = 4.2 \times 9.81 \times \sin 28° = 19.3$ N down the plane and the frictional force up the plane. The maximum frictional force is $\mu_s N = 0.55 \times 4.2 \times 9.81 \times \cos 28° = 20$ N and so the block will stay at rest.

d The force down the plane is greater than the frictional force so the block will not stop.

21 **a** The forces on the man are his weight, mg, and the normal force R from the floor.

i The acceleration is zero and so $R - mg = 0$, i.e., $R = mg$.

ii The acceleration is zero and so $R - mg = 0$, i.e., $R = mg$.

iii The net force is in the downward direction and equals $mg - R$. Hence, $mg - R = ma$ and so $R = mg - ma$.

iv From **iii** we have that $R = mg - mg = 0$.

b The man will be hit by the ceiling of the elevator, which is coming down faster than the man.

22 As the elevator goes up the force that must be supplied by the arm on the book upward must increase. Because you are not aware that you have to do that, the book feels heavier and so moves down a bit. As the elevator comes to a stop the force necessary to keep it up decreases and so the book feels less heavy and so moves up a bit. The same thing happens when you start going down. As the elevator comes to a stop on the way down, the force needed to keep it up is again greater than the weight so the book falls.

23 **a** The forces on the man are (i) the tension from the rope, T, on his hands upward (this is the same as the force with which he pulls down on the rope), (ii) his weight 700 N downward, and (iii) the normal force, R, from the elevator floor upward.

b On the elevator they are (i) the weight downward, (ii) the normal force, R, from the man downward, and (iii) the tension, T, in the rope upward.

c The forces on the man *and* the elevator together are $2T$ upward (one T on the elevator at the top and one T on the man from the rope). The combined weight is 1000 N. Thus

$2T - 1000 = 100a = 100 \times 0.50 \Rightarrow T = 525$ N. The net force on the man is $R + T - 700 = 70a = 35$ $\Rightarrow R = 700 - 525 + 35 \Rightarrow R = 210$ N.

d We know that

$2T - 1000 = 100a$

$R + T - 700 = 70a$

and so $2R + 2T - 1400 = 140a$

Subtracting we get $2R - 400 = 40a$ and so $a = \frac{2R - 400}{40} = \frac{400 - 400}{40} = 0\,\text{m s}^{-2}$.

24 a The net force on the system of the two blocks is 24 N and the total mass is 8.0 kg so the acceleration of each block is 3.0 m s^{-2}.

b The force X exerts on Y is the net force on the 6.0 kg block and so $F = ma = 6.0 \times 3.0 = 18$ N.

25 a Treat the two masses as one body. The net force is 60.0 N and so the acceleration is $a = \frac{60.0}{40.0} = 1.50$ m s^{-2}. The net force on the back block is the tension in the string and so $T = ma = 10.0 \times 1.50 = 15.0$ N.

b The tension would now be $T = Ma = 30.0 \times 1.50 = 45.0$ N.

26 For three (planar) forces to be in equilibrium, any one force must have a magnitude that is in between the sum and the difference of the other two forces. This is the case here. Now, the resultant of the 4.0 N and the 6.0 N forces must have a magnitude of 9.0 N. Thus when the 9.0 N force is suddenly removed, the net force on the body is 9.0 N. The acceleration is therefore 3.0 m s^{-2}.

27 If m is the largest it means that the big block has a tendency to move up the plane and so friction is down the plane. Then $T = mg = Mg\sin\theta + \mu Mg\cos\theta$. So $m = M\sin\theta + \mu M\cos\theta = 15 \times \sin 25° + 0.40 \times 15 \times \cos 25° = 11.8 \approx 12$ kg. Similarly, if m is the smallest then the big block has a tendency to move down the plane and so friction is up the plane. So, $T = mg = Mg\sin\theta - \mu Mg\cos\theta$ and $m = M\sin\theta - \mu M\cos\theta = 15 \times \sin 25° - 0.40 \times 15 \times \cos 25° = 0.90$ kg.

28 $T + B = mg$. The volume of the block is $V = \frac{m}{\rho} = \frac{25}{2800} = 8.93 \times 10^{-3}$ m^3. Therefore $B = \rho_L gV = 850 \times 9.81 \times 8.93 \times 10^{-3} = 74.5$ N. Thus $T = 25 \times 9.81 - 74.5 = 170.7 \approx 170$ N.

29 a $B = mg$ so $1000 \times A \times h \times g = 750 \times A \times H \times g \Rightarrow \frac{h}{H} = 0.75$.

b The net force upward on the block is now $\rho_{water}Vg - \rho_{block}Vg = (\rho_{water} - \rho_{block})Vg$. The acceleration is then $= \frac{(\rho_{water} - \rho_{block})Vg}{M} = \frac{(\rho_{water} - \rho_{block})Vg}{\rho_{block}V} = \left(\frac{\rho_{water}}{\rho_{block}} - 1\right)g = 3.3$ m s^{-2}.

30 Let us call the mass of each person m and let there be n persons on the raft. Then $\rho_{water}Vg = \rho_{raft}Vg + nmg$ and so $n = \frac{(\rho_{water} - \rho_{raft})Vg}{mg} = \frac{(\rho_{water} - \rho_{raft})V}{m} = \frac{(1000 - 800)}{50} \times \frac{1200}{800} = 6$.

31 a At terminal speed $mg = F_{drag}$ and so $mg = 6\pi\eta rv$. The mass is given by $m = \rho_{ball}V = \rho_{ball}\frac{4\pi r^3}{3}$ so that $\rho_{ball}\frac{4\pi r^3}{3}g = 6\pi\eta rv$ and finally $v = \frac{2\rho_{ball}r^2 g}{9\eta} = \frac{2 \times 3500 \times (6.0 \times 10^{-3})^2 \times 9.81}{9 \times 0.70} = 0.39$ m s^{-1}.

b The viscosity will decrease so the terminal speed will increase.

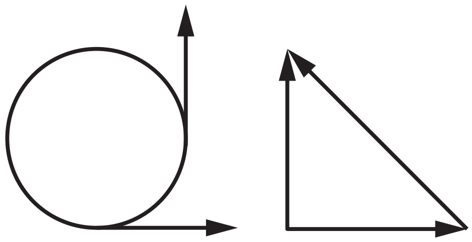
c We now have $mg = F_{drag} + B$ and so $\rho_{ball}\frac{4\pi r^3}{3}g = 6\pi\eta rv + \rho_{liquid}\frac{4\pi r^3}{3}g$ giving $v = \frac{2(\rho_{ball} - \rho_{liquid})r^2 g}{9\eta} = \frac{2 \times (3500 - 920) \times (6.0 \times 10^{-3})^2 \times 9.81}{9 \times 0.70} = 0.29$ m s^{-1}.

32 a The angular speed is just $\omega = \frac{2\pi}{T} = \frac{2\pi}{1.24} = 5.07$ rad s^{-1}. The linear speed is $v = \omega R = 5.07 \times 3.50 = 17.7$ m s^{-1}.

b The frequency is $f = \frac{1}{T} = \frac{1}{1.24} = 0.806$ s^{-1}.

33 $a = 4\pi^2 rf^2 = 4\pi^2 \times 2.45 \times (3.5)^2 = 1.2 \times 10^3$ m s^{-2}.

34 a The average acceleration is defined as $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$. The velocity vectors at A and B and the change in the velocity $\Delta\vec{v}$ are shown below.



The magnitude of the velocity vector is 4.0 m s^{-1}, and it takes a time of $\frac{2\pi \times 2.0}{4.0} = 3.14$ s to complete a full revolution.

Hence it takes a time of $\frac{3.14}{4} = 0.785$ s to complete a quarter of revolution from A to B. The magnitude of $\Delta\vec{v}$ is $\sqrt{4.0^2 + 4.0^2} = 5.66$ m s^{-1} and so the magnitude of the average acceleration is $\frac{5.66}{0.785} = 7.2$ m s^{-2}.
This is directed toward the northwest, and if this vector is made to start at the midpoint of the arc AB it is then directed toward the centre of the circle.

b The centripetal acceleration has magnitude $\frac{v^2}{r} = \frac{16.0}{2.0} = 8.0$ m s^{-2} directed towards the centre of the circle.

35 The centripetal acceleration is $a = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} = 4\pi^2 r f^2$. Hence $f = \sqrt{\frac{a}{4\pi^2 r}} = \sqrt{\frac{50}{4\pi^2 \times 10}} =$ 0.356 s^{-1} ≈ 21 min^{-1}.

36 a $\frac{\omega_P}{\omega_Q} = 1$

b $\frac{v_P}{v_Q} = \frac{\frac{\omega R}{2}}{\omega R} = \frac{1}{2}$

c $\frac{a_P}{a_Q} = \frac{\frac{\omega^2 R}{2}}{\omega^2 R} = \frac{1}{2}$

37 a The centripetal acceleration is $\frac{v^2}{r} = \frac{4.00}{0.400} = 10.0$ m s^{-2}. The tension is the force that provides the centripetal acceleration and so $T = ma = 1.00 \times 10.0 = 10.0$ N.

b From $T = ma = 20.0$ N we have $a = \frac{v^2}{r} = 20.0$ m s^{-2} and so $v = \sqrt{20 \times 0.40} = 2.83$ m s^{-1}.

c $20.0 = 1.00 \times \frac{4.00^2}{r} \Rightarrow r = \frac{16.0}{20.0} = 0.800$ m.

38 With $a = 9.8$ m s^{-2} we have that $a = \frac{4\pi^2 r}{T^2} \Rightarrow T = \sqrt{\frac{4\pi^2 \times 6.4 \times 10^6}{9.8}} = 5.08 \times 10^3$ s ≈ 85 min

39 a $a = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2 \times 50.0 \times 10^3}{(25.0 \times 10^{-3})^2} = 3.2 \times 10^9$ m s^{-2}.

b The forces on the probe are (i) its weight, mg, and (ii) the normal force N from the surface. Assuming that the probe stays on the surface, the net force would be
$mg - N = \frac{mv^2}{r} \Rightarrow N = mg - \frac{mv^2}{r} = m(g - \frac{v^2}{r}) = m(8.0 \times 10^{10} - 3.2 \times 10^9) > 0$. This is positive so the probe can stay on the surface.

40 a $v = \frac{2\pi R}{T} = \frac{2\pi \times 1.5 \times 10^{11}}{365 \times 24 \times 60 \times 60} = 2.99 \times 10^4 \approx 30$ km s^{-1}.

b $a = \frac{v^2}{r} = \frac{(2.99 \times 10^4)^2}{1.5 \times 10^{11}} = 5.96 \times 10^{-3} \approx 6.0 \times 10^{-3}$ m s^{-2}.

c $F = ma = \frac{mv^2}{r} = 6.0 \times 10^{24} \times 5.96 \times 10^{-3} \approx 3.6 \times 10^{22}$ N.

This is the force the sun exerts on the earth and by Newton's third law also the force the earth exerts on the sun.

41 The components of L are $L_x = L\sin\theta$, $L_y = L\cos\theta$.

We have that

$L\sin\theta = m\frac{v^2}{R}$

$L\cos\theta = mg$

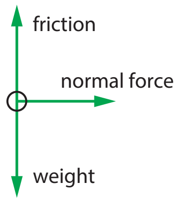
Dividing side by side,

$\frac{L\sin\theta}{L\cos\theta} = \frac{m\frac{v^2}{R}}{mg}$

$\tan\theta = \frac{v^2}{gR}$

This gives $R = \frac{v^2}{g\tan\theta} = \frac{180^2}{9.8\times\tan 35°} = 4.7\,\text{km}$.

42 a



b Let the normal force from the wall be N. Then

$N = m\frac{v^2}{r}$

$mg = f_s$ For the minimum rotation speed the frictional force must be a maximum, i.e., $f_s = \mu_s N$. That is,

$N = m\frac{v^2}{r}$

$mg = \mu_s N$

Combining gives $mg = \mu_s m\frac{v^2}{r}$ i.e. $v = \sqrt{\frac{gr}{\mu_s}} = \sqrt{\frac{9.8\times 5.0}{0.60}} = 9.04\ \text{m s}^{-1}$. From $v = 2\pi rf$ we find

$f = \frac{v}{2\pi r} = \frac{9.04}{2\pi\times 5.0} = 0.288\ \text{rev s}^{-1} \approx 17\ \text{rev min}^{-1}$.

43 The tension in the string must equal the weight of the hanging mass, i.e. $T = Mg$. The tension serves as the centripetal force on the smaller mass and so $T = m\frac{v^2}{r}$. Hence $m\frac{v^2}{r} = Mg \Rightarrow v = \sqrt{\frac{Mgr}{m}}$.

44 Let the tension be T_U in the upper string and T_L in the lower string. Both strings make an angle θ with the horizontal. We have that

$T_U\sin\theta = mg + T_L\sin\theta$

$T_U\cos\theta + T_L\cos\theta = m\frac{v^2}{r}$

We may rewrite these as

$T_U\sin\theta - T_L\sin\theta = mg$

$T_U\cos\theta + T_L\cos\theta = m\frac{v^2}{r}$

From trigonometry, $\sin\theta = \frac{0.50}{1.0} = 0.50 \Rightarrow \theta = 30°$. Furthermore, $r = \sqrt{1.0^2 - 0.50^2} = 0.866\ \text{m}$. Therefore the equations simplify to

$0.50\times(T_U - T_L) = 2.45$

$0.866\times(T_U + T_L) = 18.48$

OR

$T_U - T_L = 4.90$

$T_U + T_L = 21.34$.

Finally, $T_U = 13.1\ \text{N}$, $T_L = 8.2\ \text{N}$.

Chapter 3

Test your understanding

1 The work done is $W = Fd\cos\theta = 24 \times 5.0 \times \cos 0° = 120\,\text{J}$.

2 The work done is $W = Fd\cos\theta = 3.2 \times 2.4 \times \cos 180° = -7.7\,\text{J}$.

3 The work done is $W = Fd\cos\theta = 25 \times 15 \times \cos 20° = 352 \approx 3.5 \times 10^2\,\text{J}$.

4 The work is zero since the normal force is at right angles to the direction of motion.

5 The work is zero in both cases since the force is at right angles to the direction of motion.

6 No work is being done since the force does not move its point of application.

7 a The work done is the area under the curve and so 50 J.

b There is a force acting on the body, and so the speed increases.

8 The change in the kinetic energy is zero, and so the work done is zero.

9 The change in kinetic energy is $\Delta E_K = \frac{1}{2} \times 2.0 \times (0 - 5.4^2) = -29.16\,\text{J}$. This equals the work done by the resistive force, i.e., $f \times 4.0 \times \cos 180° = -29.16 \Rightarrow f = 7.3\ \text{N}$.

10 The work done by the force is the area under the curve, i.e., 25 J and equals the change in kinetic energy. So the kinetic energy at 5.0 m is 30 J.

11 a The work done is the area under the curve. This is a trapezoid and so $W = \frac{15 + 7.0}{2} \times 8.0 = 88\,\text{J}$.

b The work done is the change in kinetic energy and so $\frac{1}{2}mv^2 = W$, giving $v = \sqrt{\frac{2 \times 88}{2.0}} \approx 9.4\ \text{m s}^{-1}$.

12 The work done by the frictional force equals the change in kinetic energy and so $f \times 4.0 \times \cos 180° = 0 - 36$, giving $f = 9.0\,\text{N}$.

13 The work done is the same in both cases, and the change in kinetic energy is also the same. Since initially the bodies were at rest the final kinetic energies will be the same.

14 The change in kinetic energy is the work done, and this is the same for the first and second 1 m fallen. Hence the change will be 1 J.

15 a The work done by the weight is $W = mgs\cos 0° = 150 \times 12 = 1800\,\text{J}$.

b The work done by the tension is $W = Ts\cos 180° = mgs\cos 180° = -1800\,\text{J}$.

16 Along horizontal steps, the work done is zero since the force (the weight) is at right angles to the direction of motion. Along a vertical drop the work done is $mg\Delta h$. So the total work done is the sum of $mg\Delta h$ over all the vertical drops and so equals $\sum mg\Delta h = mg\sum\Delta h = mgh$. The argument is independent of the shape of the path, so the work done is also independent of the shape of the path.

17 We just saw that the work done by gravity is independent of the path so the work done by gravity is the same in both cases ($-mgh$). The work done by the person pulling on the block vertically up is $W = Th\cos 0° = mgh$ where h is the vertical height. Pulling along the plane, the work done is $W = Ts\cos 0° = (mg\sin\theta)s = mg(s\sin\theta) = mgh$, so the same.

More directly, the net work done is zero, so the work done by pulling plus the work done by gravity must be zero, so the work done by the person pulling is $+mgh$.

18 $E = \frac{1}{2}kx^2 = \frac{1}{2} \times 250 \times 0.12^2 = 1.8\,\text{J}$.

19 $E = \frac{1}{2}kx^2 = \frac{1}{2} \times 380 \times x^2 = 1.4\,\text{J}$ and so $x = \sqrt{\frac{2 \times 1.4}{380}} = 8.6 \times 10^{-2}\ \text{m}$.

20 $W = \Delta E = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2} \times 200 \times ((5.0 \times 10^{-2})^2 - (3.0 \times 10^{-2})^2) = 0.16\,\text{J}$.

21 The work done is the area under the curve and so $\frac{6.0+18}{2} \times 0.10 = 1.2\,\text{J}$.

22 In all three cases $\frac{1}{2}mv^2 = mgh$, where h is the height of the box. So the speed will be the same.

23 In both cases $\frac{1}{2}mu^2 + mgh = \frac{1}{2}mv^2$, where h is the height of the table and u is the common launch speed. Hence the landing speed will be the same.

24 **a** The minimum energy is required to just get the ball at A. Then,

$\frac{1}{2}mv^2 = mgh \Rightarrow v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 4.0} = 8.9\ \text{m s}^{-1}$.

b At position B, $\frac{1}{2} \times m \times 8.9^2 = m \times 9.8 \times 2.0 + \frac{1}{2} \times m \times v^2 \Rightarrow v = 6.3\ \text{m s}^{-1}$.

c At A: $\frac{1}{2} \times m \times 12.0^2 = \frac{1}{2} \times m \times v^2 + m \times 9.8 \times 4.0 \Rightarrow v = 8.1\,\text{m s}^{-1}$.
At B: $\frac{1}{2} \times m \times 12.0^2 = \frac{1}{2} \times m \times v^2 + m \times 9.8 \times 2.0 \Rightarrow v = 10.2\ \text{m s}^{-1}$.

25 The total energy at A is $E_A = 8.0 \times 9.8 \times 12 + \frac{1}{2} \times 8.0 \times 6.0^2 = 1085\,\text{J}$. At B it is $E_B = \frac{1}{2} \times 8.0 \times 12^2 = 576\,\text{J}$. The total energy decreased by $1085 - 576 = 509\,\text{J}$, and this represents the work done by the resistive forces. The distance travelled down the plane is 24 m and so $f \times 24 = 509 \Rightarrow f = 21\ \text{N}$.

26 **a** $mgH = mg(2R) + \frac{1}{2}mv^2$, hence the result.

b $mg + N = \frac{mv^2}{R} \Rightarrow N = \frac{mv^2}{R} - mg$. Substituting for the speed: $N = \frac{2mg(H-2R)}{R} - mg = mg\left(\frac{2H}{R} - 5\right)$.

c We need $N \geq 0$, i.e., $mg\left(\frac{2H}{R} - 5\right) \geq 0$ so $H \geq \frac{5R}{2}$.

27 **a** From $P = Fv$, $F \times \frac{100 \times 10^3}{3600} = 90 \times 10^3 \Rightarrow F = 3240 = 3.24 \times 10^3\ \text{N} \approx 3 \times 10^3\ \text{N}$.

b The additional power will be $mg \times \sin\theta \times v = 1200 \times 9.8 \times \sin(5.0°) \times 27.8 = 28\ \text{kW}$.

28 **a** From $P = Fv$, and $F = Mg = 1.177 \times 10^4\ \text{N}$ we find $v = \frac{2.5 \times 10^3}{1.177 \times 10^4} = 0.21\ \text{m s}^{-1}$.

b Most likely some of the power produced by the motor gets dissipated in the motor itself due to frictional forces, getting converted into thermal energy, and is not used to raise the block.

29 **a** The work done is $mgh = 50 \times 9.8 \times 15 = 7350\,\text{J}$. The power is thus $\frac{7350}{125} = 59\,\text{W}$.

b $\eta = \frac{59}{80} = 0.74$.

c The work required is double and the time is therefore also double, 250 s.

30 From $P = Fv$, $F \times \frac{240 \times 10^3}{3600} = 250 \times 10^3 \Rightarrow F = 3750\ \text{N}$. When travelling uphill the engine force T is given by $T = mg\sin\theta + F = 1200 \times 9.8 \times \sin 12° + 3750 = 6195\ \text{N}$. So $6195 \times v = 250 \times 10^3$ and hence $v = \frac{250 \times 10^3}{6195} = 40.36\,\text{m s}^{-1} = 145\,\text{km hr}^{-1}$.

31 The adult since more work (mgh) is done in the same time.

32 Gravitational potential energy is transferred to kinetic energy as the bob moves to a lower position plus thermal energy. The total mechanical energy of the pendulum is being reduced. As the pendulum starts going up, kinetic energy is being transferred to gravitational potential energy and thermal energy. Eventually, all the initial mechanical energy of the pendulum will have been transferred to thermal energy.

33 Elastic potential energy is transferred to kinetic energy plus thermal energy as the mass moves from an extreme position toward the equilibrium position. The total mechanical energy of the system is being reduced. As the mass goes past the equilibrium position kinetic energy is being transferred to elastic potential energy. Eventually, all the initial mechanical energy of the system is transferred to thermal energy.

34 The vertical mass–spring system is interesting. By a simple trick we can show that we can forget about gravitational potential energy completely. We take the zero of gravitational potential energy to be at the equilibrium position. The spring is now already extended by a distance x_0 and $mg = kx_0$. Let x be the extension of the spring *past the equilibrium position*. So the actual extension is then $x + x_0$. The total energy of the system is then

$$\frac{1}{2}mv^2 + \frac{1}{2}k(x + x_0)^2 - mgx$$

We assume that the maximum extension of the spring is A past the equilibrium position. Then the total energy is also

$$\frac{1}{2}k(A + x_0)^2 - mgA$$

Therefore $\frac{1}{2}mv^2 + \frac{1}{2}k(x + x_0)^2 - mgx = \frac{1}{2}k(A + x_0)^2 - mgA$. It is left as a simple exercise to show that when we use the relation $mg = kx_0$ the last equation simplifies to $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$.
In other words, if we measure extensions from the equilibrium position we can forget the gravitational potential energy.

Then elastic potential energy gets transferred to kinetic energy as the mass moves from an extreme position toward the equilibrium position, and kinetic energy gets transferred to elastic potential as the mass moves from the equilibrium position toward an extreme position.

35 Electrical energy from the motor is converted to potential energy and thermal energy if the elevator is just pulled up. Normally a counterweight is being lowered as the elevator is being raised, which means that the net change in gravitational potential energy is zero (assuming that the counterweight is equal in weight to the elevator). In this case all the electrical energy goes into thermal energy.

36 Gravitational potential energy gets transferred into thermal energy.

37 Gravitational potential energy gets transferred into thermal energy.

Chapter 4

Test your understanding

1 $F_{ave} = \frac{\Delta p}{\Delta t} = \frac{12.0}{2.00} = 6.00 \text{ N}$

2 a $\Delta p = 2.0 \times 4.0 - (-2.0 \times 4.0) = 16 \text{ N s}$

b $\Delta p = 2.0 \times 3.0 - (-2.0 \times 4.0) = 14 \text{ N s}$

c $\Delta p = 0 - (-2.0 \times 4.0) = 8.0 \text{ N s}$.

3 a The ball is dropped from a height of h_1 so its speed right before impact will be given by (applying conservation of energy) $mgh_1 = \frac{1}{2}mv_1^2 \Rightarrow v_1 = \sqrt{2gh_1}$. The ball will leave the floor on its way up with a speed found in the same way: $\frac{1}{2}mv_2^2 = mgh_2 \Rightarrow v_2 = \sqrt{2gh_2}$. The change in momentum is therefore $mv_2 - (-mv_1) = m(\sqrt{2gh_2} + \sqrt{2gh_1})$ and hence the net average force is $\frac{\Delta p}{\Delta t} = m\frac{\sqrt{2gh_2} + \sqrt{2gh_1}}{\tau}$.

b $F = 0.250 \times \frac{\sqrt{2 \times 9.81 \times 6.0} + \sqrt{2 \times 9.81 \times 8.0}}{0.125} = 46.8 \approx 47 \text{ N}$. This is the average net force on the ball. The forces on the ball are the normal force from the floor R and its weight so $R - mg = F \Rightarrow R = F + mg = 46.8 + 0.250 \times 9.81 = 49.2 \approx 49 \text{ N}$. By Newton's third law this is also the force on the floor exerted by the ball.

4 The change in momentum of the water in 1 s is $\frac{40}{60} \times 9.0 = 6.0 \text{ N s}$ and so the force is 6.0 N.

5 Impulse is the product of the force and the time for which it acts. So a large force may produce a very small impulse if it acts for a very short time.

6 a The impulse is the area under the graph and so equals (we use the area of a trapezoid) $\frac{15 + 7}{2} \times 8.0 = 88 \text{ N s}$.

b Since the impulse is the change in momentum, $88 = mv - 0 \Rightarrow v = \frac{88}{4.0} = 22 \text{ m s}^{-1}$.

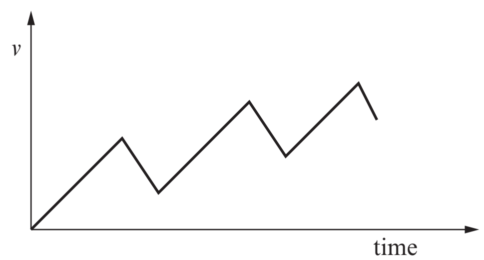
c Now, $88 = 0 - mv \Rightarrow v = -\frac{88}{4.0} = -22 \text{ m s}^{-1}$.

7 a The impulse is $0.150 \times 3.00 - (-0.150 \times 3.00) = 0.900 \text{ N s}$.

b $F_{ave} = \frac{\Delta p}{\Delta t} = \frac{0.900}{0.125} = 7.20 \text{ N}$.

8 a The impulse supplied to the system is (area under curve) $3 \times 0.5 \times 100 - 25 \times 4 = 50 \text{ N s}$. This is the change in momentum, i.e., $25 \times \Delta v = 50 \text{ N s} \Rightarrow \Delta v = 2.0 \text{ m s}^{-1}$.

b Something like (you can easily work out the numbers and slopes on the axes):



9 a From 0.5 s to 1.5 s, i.e., for 1 s.

b A rough approximation would be to treat the area as a triangle (of area $\frac{1}{2} \times 1.0 \times 120 = 60 \text{ N s}$), but this is too rough and would not be acceptable in an exam. There are roughly 120 rectangles in the area, and each has area $0.1 \times 4.0 = 0.4 \text{ N s}$ so that the total area is $0.4 \times 120 = 48 \approx 50 \text{ N s}$.

c From $F_{average} \Delta t = \Delta p$ we thus find $F_{average} = 50 \text{ N}$.

10 The impulse is the area, i.e., 100 N s. The final speed is then 25 m s^{-1}.

$$\bar{P} = \bar{F} \times \frac{u+v}{2} = 10 \times \frac{0+25}{2} = 125 \text{ W}$$

OR

kinetic energy $\frac{1}{2} \times 4.0 \times 25^2 = 1250$ J so average power is $\frac{1250}{10} = 125$ W.

11 The initial momentum is zero and so must remain zero for the rocket–fuel system. $0 = (5000 - m)v - m \times (1500 - v)$. In the first 1 second, $v = 15$ m s^{-1} and so $(5000 - m) \times 15 = m \times 1485 \Rightarrow m = \frac{75000}{1500} = 50.0$ kg in 1 second.

12 There are no external forces on the system. The forces each block exert on the other are internal forces.

13 The initial momentum is zero and will remain zero. Therefore the speed with which the 4.0 kg mass moves off is $2.0 \times 3.0 = 4.0 \times v \Rightarrow v = 1.5$ m s^{-1}

The total kinetic energy of the two bodies is $\frac{1}{2} \times 2.0 \times 3.0^2 + \frac{1}{2} \times 4.0 \times 1.5^2 = 13.5 \approx 14$ J.

14 The change in momentum of m is $-mv$ and so that of M is $+mv$.

15 The total momentum before the collision is $m \times v + 2m \times \left(-\frac{v}{2}\right) = 0$. This is also the momentum after. If u is the speed after the collision then $3m \times u = 0 \Rightarrow u = 0$.

16 The initial total momentum is $4.0 \times 24 - 12.0 \times 2.0 = +72$ N s. The final total momentum is $-4.0 \times 3.0 + 12 \times v$. Hence $-12 + 12v = 72 \Rightarrow v = +7.0$ m s^{-1}. (The ball moves to the right.)

17 The common velocity after the collision is $6.0 \times 4.0 + 0 = (6.0 + 8.0)v$, i.e., $v = \frac{24}{14} = 1.71429$ ms^{-1}. The initial kinetic energy was $\frac{1}{2} \times 6.0 \times 4.0^2 = 48$ J. The final is $\frac{1}{2} \times 14 \times 1.71429^2 \approx 21$ J, so 27 J were lost.

18 The speed of the heavy mass is v after the collision. Then $6.0 \times 14 + 0 = -6.0 \times 2.0 + 8.0v$ so $v = 12$ m s^{-1}.

The initial kinetic energy was $\frac{1}{2} \times 6.0 \times 14^2 = 588$ J. The final is $\frac{1}{2} \times 6.0 \times 2.0^2 + \frac{1}{2} \times 8.0 \times 12^2 = 588$ J so the collision is elastic.

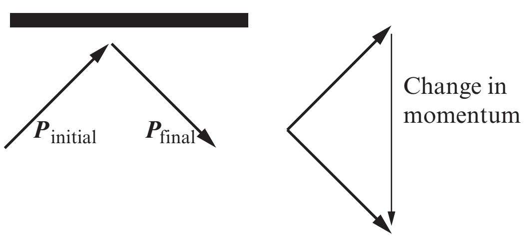
19 The two pieces have equal and opposite moment, so the ratio of the kinetic energies is

$$\frac{K_{\text{light}}}{K_{\text{heavy}}} = \frac{\frac{p^2}{2m_{\text{light}}}}{\frac{p^2}{2m_{\text{heavy}}}} = \frac{m_{\text{heavy}}}{m_{\text{light}}} = \frac{\frac{3M}{4}}{\frac{M}{4}} = 3. \text{ I.e.}$$

$K_{\text{heavy}} = \frac{1}{3}K_{\text{light}}$. Since $K_{\text{light}} + K_{\text{heavy}} = Q$

we have that $K_{\text{light}} + \frac{1}{3}K_{\text{light}} = Q \Rightarrow K_{\text{light}} = \frac{3}{4}Q$.

20 The change in momentum is given by the following vector diagram. The angle between the vectors is a right angle.



The magnitude of the initial and of the final momentum is $p = 0.250 \times 4.00 = 1.00$ N s. The direction of the change of momentum is given in the diagram. Its magnitude is $\sqrt{1.00^2 + 1.00^2} = \sqrt{2.00} = 1.41$ N s.

21 **a** $0 = mv - Mu \Rightarrow u = \frac{m}{M}v.$

b $mgh = \frac{1}{2}mv^2 + \frac{1}{2}Mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}M\left(\frac{m}{M}v\right)^2$

i.e., $2gh = v^2 + \frac{m}{M}v^2 = v^2\left(1 + \frac{m}{M}\right)$, which gives the required result.

22 **a** The rotor pushes the air downward exerting a force on the air, so by Newton's third law the air exerts an equal upward force on the rotor.

b The force on the rotor upward is the rate of change of momentum of the air and so is $1.80 \times v$. This must equal the weight and so $1.80 \times v = 1.50 \times 9.81 = 14.715$ N, giving $v = 8.18\,\text{m s}^{-1}$.

c $\bar{P} = \bar{F} \times \frac{u+v}{2} = 14.715 \times \frac{8.18}{2} = 60.2$ W OR air kinetic energy per second is $\frac{1}{2} \times 1.8 \times 8.18^2 =$ 60.2 W.

d The lift force used to be mg. Now it is 1.5 mg. The net force is 0.5 mg and so the acceleration is $0.5g = 4.90\,\text{m s}^{-2}$.

23 After the bullet gets stuck in the block the two bodies move together with speed u. Conservation of energy gives $u^2 = 2gh = 2g \times 1.5 \times (1 - \cos 34°) \Rightarrow u = 2.24\,\text{m s}^{-1}$.

The collision between the bullet and the block is inelastic, but momentum is conserved so $0.080 \times v + 0 = (0.080 + 2.0) \times 2.24 \Rightarrow v = 58\ \text{m s}^{-1}$.

24 Conservation of momentum in the x direction gives $12m + 0 = mu\cos 60° + mv\cos 30°$

$12 = u \times \frac{1}{2} + v \times \frac{\sqrt{3}}{2}$

$u + v\sqrt{3} = 24$

and in the y direction

$0 = mu\sin 60° - mv\sin 30°$

$0 = u \times \frac{\sqrt{3}}{2} - v \times \frac{1}{2}$

$v = u\sqrt{3}$

Substituting in the first set of equations,

$u + v\sqrt{3} = 24$

$u + u\sqrt{3} \times \sqrt{3} = 24$

$u = 6.0\,\text{m s}^{-1}$

Hence, $v = 6.0\sqrt{3}\,\text{m s}^{-1}$.

The initial kinetic energy was $\frac{1}{2}m \times 12^2 = 72m$.

The final is $\frac{1}{2}m \times 6.0^2 + \frac{1}{2}m \times (6.0\sqrt{3})^2 = 18m + 54m = 72m$, so we have an elastic collision.

25 **a** Conservation of momentum in the x direction gives $mw = mu\cos\theta + mv\cos(90° - \theta)$. But $\cos(90° - \theta) = \sin\theta$ and so $w = u\cos\theta + v\sin\theta$.

Conservation of momentum in the y direction gives $0 = mu\sin\theta - mv\sin(90° - \theta)$. But $\sin(90° - \theta) = \cos\theta$ and so $u\sin\theta = v\cos\theta$. From this equation $v = u\frac{\sin\theta}{\cos\theta}$ and substituting in the first equation we get

$w = u\cos\theta + u\frac{\sin\theta}{\cos\theta}\sin\theta$

$w = \frac{\cos^2\theta + \sin^2\theta}{\cos\theta}u = \frac{u}{\cos\theta}$

Thus, $u = w\cos\theta$. Similarly, $v = w\sin\theta$.

b The initial kinetic energy is $\frac{1}{2}mw^2$. The final kinetic energy is $\frac{1}{2}mu^2+\frac{1}{2}mv^2=\frac{1}{2}mw^2\cos^2\theta+\frac{1}{2}mw^2\sin^2\theta=\frac{1}{2}mw^2(\cos^2\theta+\sin^2\theta)=\frac{1}{2}mw^2$. The collision is elastic.

26 Conservation of momentum in the x direction gives $4.0w+0=4.0u\cos 64°+16\times 2.0\cos 51°$. This simplifies to $w=0.43837u+5.03456$

Conservation of momentum in the y direction gives $0=4.0u\sin 64°-16\times 2.0\sin 51°$ which becomes $u=6.92\approx 6.9\,\text{m s}^{-1}$. Hence $w=8.07\approx 8.1\,\text{m s}^{-1}$.

27 The initial momentum is zero, and so the final momentum is zero as well. Therefore the missing speed is $4.0\,\text{m s}^{-1}$.

The initial kinetic energy was $\frac{1}{2}\times 4.0\times 12^2+\frac{1}{2}\times 6.0\times 8.0^2=480\,\text{J}$. The final is $\frac{1}{2}\times 4.0\times 6.0^2+\frac{1}{2}\times 6.0\times 4.0^2=120\,\text{J}$, so the loss in KE is 360 J.

28 Conservation of momentum in the x direction gives $mv\cos\frac{\theta}{2}+mv\cos\frac{\theta}{2}=2m\times\frac{v}{2}$, i.e., $\cos\frac{\theta}{2}=\frac{1}{2}$ and so $\frac{\theta}{2}=60°$. Hence $\theta=120°$.

Chapter 5

Test your understanding

1 Use $\Delta\theta = \frac{(\omega + \omega_0)t}{2}$ to get $\Delta\theta = \frac{(15 + 3.5) \times 5.0}{2} = 46.25 \approx 46$ rad.

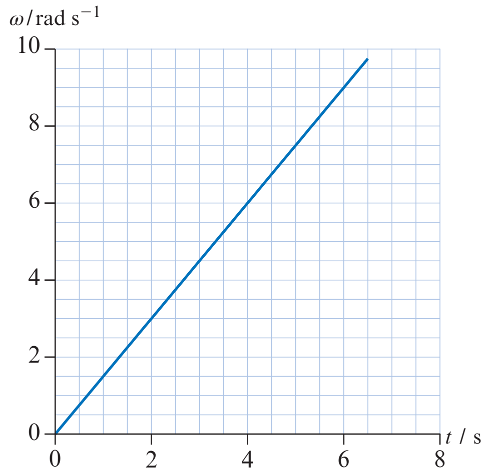
2 Use $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$ to get $\omega^2 = 5.0^2 + 2 \times 2.5 \times 54$ and so $\omega = 17.18 \approx 17$ rad s^{-1}.

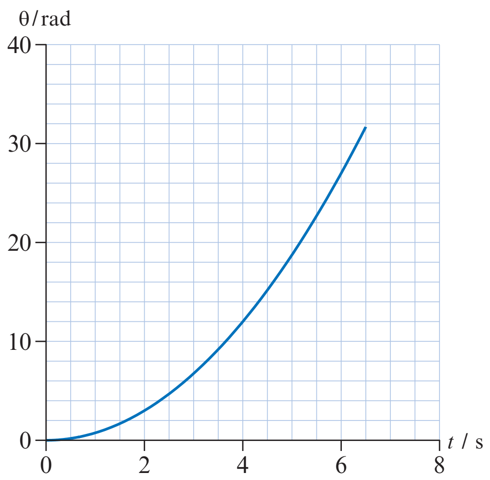
3 Use $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$ to get $12.4^2 = 3.2^2 + 2 \times \alpha \times 20 \times 2\pi$. Hence $\alpha = 0.571 \approx 0.57$ rad s^{-2}.

4 Use $\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$ to get $35 = 4.0 \times 6.0 + \frac{1}{2}\alpha \times 6.0^2$. Hence $\alpha = 0.611 \approx 0.61$ rad s^{-2}.

5 **a** Use $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$ to get $\omega^2 = 0 + 2 \times 1.5 \times 5 \times 2\pi$ and so $\omega = 9.71 \approx 9.7$ rad s^{-1}.

b The time taken is 6.5 s.





6 $T_1 + T_2 = 450 + 120 = 570$ N

Taking torques about the right support gives

$T_1 \times 5.0 = 120 \times 3.0 + 450 \times 2.5 \Rightarrow T_1 = 297$ N. Hence $T_2 = 273$ N.

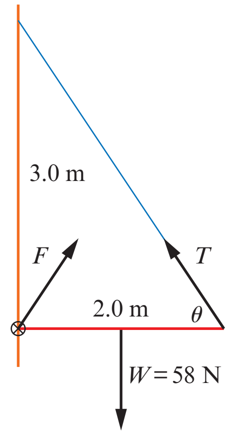
7 Let the forces at each support be L (for left) and R (for right). These are vertically upward.

Then Translational equilibrium: $L + R = 700\,\text{N}$

When the rod is about to tip, $L = 0$. Hence $R = 700$ N.

Rotational equilibrium: Taking torques about the right support: $300 \times 0.80 = 400 \times x$ giving $x = 0.60$ m.

8 **a** The forces are as shown in the following diagram:



Translational equilibrium demands that

$T_x = F_x$

$T_y + F_y = 58$

Rotational equilibrium demands that (we take torques about an axis through the point of support at the wall)

$58 \times 1.0 = T_y \times 2.0 \Rightarrow T_y = 29\,\text{N}$.

Now $T_y = T\sin\theta$ and from the diagram, $\tan\theta = \frac{3.0}{2.0} = 1.5$. Hence $\theta = 56.31°$. Thus

$T = \frac{T_y}{\sin\theta} = \frac{29}{\sin 56.31°} = 34.854 \approx 35\,\text{N}$.

b Since $T_y = 29\,\text{N}$, $F_y = 58 - 29 = 29\,\text{N}$ also. Finally, $F_x = T_x = 34.854 \times \cos 56.31° = 19.33\,\text{N}$. Hence the wall force is $F = \sqrt{F_x^2 + F_y^2} = \sqrt{19.33^2 + 29^2} = 34.854 \approx 35\,\text{N}$. The angle it makes with the horizontal is $\tan^{-1}\frac{29}{19.33} = 56.31° \approx 56°$.

9 The angular acceleration is 4.0 rad s^{-2}. Hence the torque is $\tau = I\alpha = 0.12 \times 4.0 = 0.48$ Nm.

10 **a** The angular acceleration is $\alpha = \frac{\tau}{I} = \frac{28}{3.2} = 8.75$ rad s^{-2}. Hence the angular velocity is $\omega = \alpha t = 8.75 \times 4.0 = 35$ rad s^{-1}.

b $E_K = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 3.2 \times 35^2 = 1960\,\text{J}$ OR $\Delta E_K = \tau\Delta\theta$. The angular displacement is found from $35^2 = 0^2 + 2 \times 8.75 \times \Delta\theta \Rightarrow \Delta\theta = 70$ rad. So $\Delta E_K = \tau\Delta\theta = 28 \times 70 = 1960\,\text{J}$.

11 The mass of the full big disc is $M\frac{R^2}{R^2 - r^2}$. We can think of the hole as a disc of negative mass $M\frac{r^2}{R^2 - r^2}$. Then

$$I = \frac{1}{2}M\frac{R^2}{R^2 - r^2} \times R^2 - \frac{1}{2}M\frac{r^2}{R^2 - r^2} \times r^2$$
$$= \frac{1}{2}M\frac{R^4 - r^4}{R^2 - r^2}$$
$$= \frac{1}{2}M\frac{(R^2 - r^2)(R^2 + r^2)}{R^2 - r^2}$$
$$= \frac{1}{2}M(R^2 + r^2)$$

12 The torque provided by the force about the axis of rotation is $\tau = FR$. We know that $\tau = I\alpha$. The moment of inertia of the cylinder is $I = \frac{1}{2}MR^2$ and so $\alpha = \frac{\tau}{I} = \frac{2F}{MR} = \frac{2 \times 6.5}{5.0 \times 0.20} = 13$ rad s^{-2}. Therefore $\omega = \omega_0 + \alpha t = 0 + 13 \times 5.0 = 65$ rad s^{-1}.

13 Let h be the height from which the bodies are released. The point particle will have $Mgh = \frac{1}{2}Mv^2 \Rightarrow v = \sqrt{2gh}$.

For the others,

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\frac{v^2}{R^2}$$

$$2gh = v^2\left(1 + \frac{I}{MR^2}\right)$$

$$v = \sqrt{\frac{2gh}{1 + \frac{I}{MR^2}}}$$

So

Sphere: $v = \sqrt{\frac{2gh}{1 + \frac{2}{5}\frac{MR^2}{MR^2}}} = \sqrt{\frac{2gh}{\frac{7}{5}}} = \sqrt{2gh} \times \sqrt{\frac{5}{7}}$

Cylinder: $v = \sqrt{\frac{2gh}{1 + \frac{1}{2}\frac{MR^2}{MR^2}}} = \sqrt{\frac{2gh}{\frac{3}{2}}} = \sqrt{2gh} \times \sqrt{\frac{2}{3}}$

Ring: $v = \sqrt{\frac{2gh}{1 + \frac{MR^2}{MR^2}}} = \sqrt{\frac{2gh}{2}} = \sqrt{2gh} \times \sqrt{\frac{1}{2}}$.

Hence $v_{\text{ring}} < v_{\text{cylinder}} < v_{\text{sphere}} < v_{\text{point}}$.

14 a One revolution corresponds to 2π radians and so the disc is making $\omega = \frac{45}{2\pi}\frac{\text{rev}}{\text{s}} = \frac{45}{2\pi}\frac{\text{rev}}{\frac{1}{60}\text{min}} = \frac{45 \times 60}{2\pi} = 429.7 \approx 430$ rpm.

b The angular acceleration has to be $\alpha = \frac{45}{4.0} = 11.25$ rad s^{-2}. From $\tau = I\alpha$

$FR = \frac{1}{2}MR^2\alpha$

we deduce that $F = \frac{1}{2}MR\alpha = \frac{1}{2} \times 12 \times 0.35 \times 11.25 = 23.6 \approx 24$ N.

c The work done is the change in kinetic energy, i.e., $W = \Delta E_K = \frac{1}{2}I\omega^2 = \frac{1}{2} \times (\frac{1}{2} \times 12 \times 0.35^2) \times 45^2 = 744.1875 \approx 740$ J

OR the angle turned is found from $0 = 45^2 + 2 \times (-11.25)\Delta\theta$ and so $\Delta\theta = 90$ rad. The work done is $W = \tau\Delta\theta = 23.6 \times 0.35 \times 90 = 743.4 \approx 740$ J.

d The kinetic energy divided by time, i.e., $\bar{P} = \frac{744.1875}{4} = 186$ W.

e From $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$ we find $0 = 45^2 + 2 \times (-11.25)\Delta\theta$ and so $\Delta\theta = 90$ rad. This corresponds to $\frac{90}{2\pi} = 14.3 \approx 14$ revolutions.

15 a $\tau = I\alpha \Rightarrow Mg\frac{L}{2} = \frac{1}{3}ML^2\alpha$. Thus

$\alpha = \frac{3g}{2L} = \frac{3}{2} \times \frac{9.81}{1.20} = 12.26 \approx 12.3$ rad s^{-2}.

b It is not: the torque of the weight about the axis is decreasing because the perpendicular distance between the weight and the axis is decreasing.

c Conservation of energy: $Mg\frac{L}{2} = \frac{1}{2}I\omega^2$. Hence $MgL = \frac{1}{3}ML^2\omega^2$ and $\omega = \sqrt{\frac{3g}{L}} = \sqrt{\frac{3 \times 9.81}{1.20}} = 4.95$ rad s^{-1}

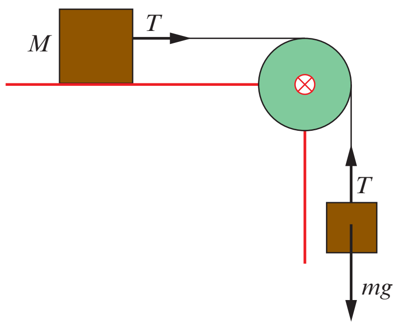
16 From Newton's second law: $F = Ma$ and $Fd = I\alpha = \frac{1}{2}MR^2\alpha$. Combing these two equations gives

$Mad = \frac{1}{2}MR^2\alpha$

$M\alpha Rd = \frac{1}{2}MR^2\alpha$

$d = \frac{R}{2}$

17 a The relevant forces are as shown.



Therefore

$mg - T = ma$

$T = Ma$

Hence, adding side by side, $mg = Ma + ma$ and so $a = \frac{mg}{M+m}$.

b Using the hint we now have

$mg - T_1 = ma$

$T_2 = Ma$ and $(T_1 - T_2)R = I\alpha$

Assuming no slipping at the pulley, $\alpha = \frac{a}{R}$ and so $(T_1 - T_2) = \frac{Ia}{R^2}$. Adding the first two equations gives $mg - (T_1 - T_2) = (m + M)a$ OR $mg - \frac{Ia}{R^2} = (m + M)a$, i.e., $mg - \frac{1}{2}Ma = (m + M)a$ so that finally $a = \frac{mg}{m + \frac{3}{2}M}$.

18 a $E_K = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$, $v = \omega R$ so $E_K = \frac{1}{2}M\omega^2R^2 + \frac{1}{2}(\frac{1}{2}MR^2)\omega^2 = \frac{3}{4}M\omega^2R^2$.

b Conservation of energy: $\frac{3}{4}M\omega_0^2R^2 + Mgh = \frac{3}{4}M\omega^2R^2$ so $\omega^2 = \omega_0^2 + \frac{4}{3}\frac{gh}{R^2}$. Hence

$\omega^2 = 5.0^2 + \frac{4}{3}\frac{9.81 \times 0.80}{0.20^2} = 286.6 \Rightarrow \omega = 16.9 \approx 17\ \text{rad s}^{-1}$

c You can get the angular acceleration by repeating what we did in the textbook for the sphere rolling down an incline to get $\alpha = \frac{2}{3}\frac{g}{R}\sin\theta = \frac{g}{3R}$ (since $\sin\theta = \sin 30° = \frac{1}{2}$) OR you can argue as follows: $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$. The distance travelled on the incline is $\frac{h}{\sin\theta} = \frac{h}{\sin 30°} = 2h$. The number of revolutions is then $\frac{2h}{2\pi R}$. So the angle by which the cylinder turned is $\Delta\theta = \frac{2h}{2\pi R} \times 2\pi = \frac{2h}{R}$. Hence $\omega^2 = \omega_0^2 + \frac{4\alpha h}{R}$. Comparing with $\omega^2 = \omega_0^2 + \frac{4}{3}\frac{gh}{R^2}$ we see that $\frac{4\alpha h}{R} = \frac{4}{3}\frac{gh}{R^2}$ and so $\alpha = \frac{g}{3R} = \frac{9.81}{3 \times 0.20} = 16.35 \approx 16\ \text{rad s}^{-2}$.

d Net torque is $I\alpha = \frac{1}{2}MR^2\alpha = \frac{1}{2} \times 4.0 \times 0.20^2 \times 16.35 = 1.3\ \text{N m}$

19 $\Delta L = \tau\Delta t$ so $\Delta L = 2.8 \times 3.2 = 9.0\ \text{J s}$

20 $\tau = \Delta L/\Delta t = 24/4.0 = 6.0\ \text{Nm}$

21 X: $E_K = \frac{1}{2}I\omega^2 = \frac{1}{2} \times \frac{1}{12}ML^2\omega^2 = \frac{1}{24} \times 2.40 \times 1.20^2 \times 4.50^2 = 2.916 \approx 2.92\,\text{J}$;

$L = I\omega = \frac{1}{12}ML^2\omega = \frac{1}{12} \times 2.40 \times 1.20^2 \times 4.50 = 1.296 \approx 1.30\,\text{J s}$.

Y: $E_K = \frac{1}{2}I\omega^2 = \frac{1}{2} \times \frac{1}{3}ML^2\omega^2 = 11.7\,\text{J}$; $L = I\omega = \frac{1}{3}ML^2\omega = 5.18\,\text{J s}$.

22 There are no external torques so angular momentum is conserved: $I_1\omega_1 = I_2\omega_2$, i.e., $\frac{2}{5}MR^2\omega_1 = \frac{2}{5}\left(\frac{M}{10}\right)\left(\frac{R}{50}\right)^2\omega_2$, leading to $\frac{\omega_2}{\omega_1} = 2.5 \times 10^4$.

23 $L = I\omega$ and $E_K = \frac{1}{2}I\omega^2$. Combining we get $E_K = \frac{L^2}{2I}$. Hence $I = \frac{L^2}{2E_K} = \frac{240^2}{2 \times 920} = 31.3 \approx 31\ \text{kg m}^2$.

24 **a** There are no external torques so angular momentum is conserved: $I_1\omega_1 = I_2\omega_2$, i.e.,

$I_1\omega_1 = (I_1 + I_2)\omega_2 \Rightarrow \omega_2 = \omega_1\frac{I_1}{I_1 + I_2} = 1.2 \times \frac{2.4}{2.4 + 1.8} = 0.69\ \text{rad s}^{-1}$.

b Initial kinetic energy is $\frac{1}{2} \times 2.4 \times 1.2^2 = 1.73$ J. Final kinetic energy is $\frac{1}{2} \times 4.2 \times 0.69^2 = 1.0$ J. So energy lost is 0.73 J.

25 The angular momentum before and after the ring begins to move must be the same. Before it moves, the angular momentum is zero. After it begins to move it is $L = I\omega + mvR$. Hence $I\omega + mvR = 0$, i.e., $\omega = -\frac{mvR}{I} = -\frac{0.18 \times 0.80 \times 0.50}{0.20} = -0.36\ \text{rad s}^{-1}$. (The minus sign indicates that the ring rotates in the opposite direction to that of the car.)

26 **a** Motion of block: $mg - T = ma$.

Motion of cylinder: $TR = I\alpha$, i.e. $TR = \frac{1}{2}MR^2\frac{a}{R}$, i.e. $T = \frac{1}{2}Ma$. Hence

$$mg - \frac{1}{2}Ma = ma$$

$$a = \frac{mg}{m + \frac{M}{2}}$$

Putting in numbers, $a = 5.0\,\text{m s}^{-2}$. Hence $\alpha = 25\ \text{rad s}^{-2}$.

b $T = \frac{1}{2}Ma = \frac{1}{2} \times 4.0 \times 5.0 = 10\,\text{N}$.

c $v = at = 5.0 \times 2.0 = 10\,\text{m s}^{-1}$; $\omega = \alpha t = 25 \times 2 = 50\ \text{rad s}^{-1}$.

d $\Delta L = I\Delta\omega = \frac{1}{2}MR^2\Delta\omega = \frac{1}{2} \times 4.0 \times 0.20^2 \times 50 = 4.0\,\text{J s}$; $\Delta L = mvR = 2.0 \times 10 \times 0.20 = 4.0\,\text{J s}$.

e Tension is an internal force if we consider the system to be the cylinder plus the block. The only external force is the weight of the block, and so the *total torque on the system* is $mgR = 2.0 \times 10 \times 0.2 = 4.0\,\text{N m}$.

f The total change in angular momentum, i.e., $4.0 + 4.0 = 8.0$ J s OR $\tau\,\Delta t = 4.0 \times 2.0 = 8.0$ J s.

Chapter 6

Test your understanding

1 Because the ground is part of the earth and the earth is accelerating because it spins about its axis and rotates around the sun. But the accelerations associated with these motions are very small and can be ignored. The acceleration due to the rotation on its axis is $a = 0.034\,\text{m s}^{-2}$ and that due to the rotation around the sun is $a = 5.95 \times 10^{-3}\,\text{m s}^{-2}$.

2 You can think of many such experiments. One is to let a ball drop from rest. The ball will fall vertically down (as far as you are concerned) in exactly the same way as if the train were at rest.

3 You can hang a pendulum from the ceiling. If the train accelerates, the string will not be vertical. If the string is displaced in a given direction, the direction of acceleration will be opposite to that direction.

4 **a** $x' = x - vt = 20 - 15 \times 5.0 = -55\,\text{m}$.

b $u = u' + v = 5.0 + 15 = 20\,\text{m s}^{-1}$.

5 **a** $x = x' + vt = 24 + (-25) \times 5.0 = -101\,\text{m}$.

b $u' = u - v = 15 - (-25) = 40\,\text{m s}^{-1}$.

6 $1.80c$.

In the following questions the frames S and S' have their usual meaning, i.e. S' moves past S with velocity v and when the origins coincide, clocks are set to zero. You are expected to use relativistic equations for these questions.

7 **a** $\gamma = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \dfrac{1}{\sqrt{1-0.850^2}} = 1.90$ and $\gamma = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \dfrac{1}{\sqrt{1-(-0.850)^2}} = 1.90$.

b $\gamma = 4 = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ so $1 - \dfrac{v^2}{c^2} = \dfrac{1}{16}$ and $\dfrac{v^2}{c^2} = \dfrac{15}{16}$ so that finally $v = 0.968c$.

c No, because by definition $\gamma \geq 1$. Attempting to solve $\gamma = \dfrac{1}{2} = \dfrac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ gives a meaningless answer.

8 The gamma factor is $\gamma = \dfrac{1}{\sqrt{1-0.75^2}} = 1.51$. Then $x' = \gamma(x - vt) = 1.51 \times (600 - 0.75c \times 2.0 \times 10^{-6}) =$ $226\,\text{m}$, $t' = \gamma(t - \frac{v}{c^2}x) = 1.51 \times (2.0 \times 10^{-6} - \dfrac{0.75c}{c^2} \times 600) = 1.51 \times (2.0 \times 10^{-6} - \dfrac{0.75}{c} \times 600) = 0.76\,\mu\text{s}$.

9 The gamma factor is $\gamma = \dfrac{1}{\sqrt{1-0.60^2}} = 1.25$. Then $x = \gamma(x' + vt) = 1.25 \times (520 + 0.60c \times 4.0 \times 10^{-6}) =$ $1550\,\text{m}$, $t = \gamma(t' + \frac{v}{c^2}x') = 1.25 \times (4.0 \times 10^{-6} + \dfrac{0.60c}{c^2} \times 520) = 1.25 \times (4.0 \times 10^{-6} + \dfrac{0.60}{c} \times 520) = 6.3\,\mu\text{s}$.

10 The explosion event has coordinates $x' = 0$ and $t' = 6.0\,\mu\text{s}$ in S'. The gamma factor is $\gamma = \dfrac{1}{\sqrt{1-0.98^2}} = 5.0$. So $x = \gamma(x' + vt) = 5.0 \times (0 + 0.98c \times 6.0 \times 10^{-6}) = 8.8 \times 10^3\,\text{m}$

$t = \gamma(t' + \frac{v}{c^2}x') = 5.0 \times (6.0 \times 10^{-6} + 0) = 30\,\mu\text{s}$.

11 The origin of frame S' will get to $x = 120\,\text{m}$ in a time of $t = \dfrac{120}{0.60c} = 6.67 \times 10^{-7}\,\text{s}$. Hence

$t' = \gamma(t - \frac{v}{c^2}x) = 1.25 \times (6.67 \times 10^{-7} - \dfrac{0.60c}{c^2} \times 120) = 5.3 \times 10^{-7}\,\text{s}$

12 a $\Delta x' = \gamma(\Delta x - v\Delta t) = 1.25 \times (1200 - 0.600 \times 3 \times 10^8 \times 6.00 \times 10^{-6}) = 150\,\text{m}$ and

$\Delta t' = \gamma(\Delta t - \frac{v}{c^2}\Delta x) = 1.25 \times (6.00 \times 10^{-6} - \frac{0.600}{3 \times 10^8} \times 1200) = 4.5 \times 10^{-6}\,\text{s}$.

b If $\Delta t'' = 0$, then $6.00 \times 10^{-6} - \frac{v}{c^2} \times 1200 = 0$, i.e. $v = \frac{6.00 \times 10^{-6} c^2}{1200} = 1.5c$ which is impossible.

13 The rocket gets to the planet in $\Delta t = \frac{24\text{ ly}}{0.80c} = 30$ y. So, $\Delta t' = \gamma(\Delta t - \frac{v}{c^2}\Delta x) = \frac{5}{3} \times (30 - \frac{0.80}{c} \times 24) = \frac{5}{3} \times (30 - 19.2) = 18$ y.

(After reading the next section you may also say: the time to get to the planet is 30 y by earth clocks. The time interval for travelling to the planet is a proper time interval for the rocket so time for the rocket is $\frac{30}{\gamma} = 18$ y.

You may also say: the distance between earth and the other planet is Lorentz contracted to $\frac{24}{\gamma} = 14.4$ ly for the rocket and so 'planet arrives at rocket' in $\frac{14.4}{0.80c} = 18$ y.)

14 a The event 'signal is emitted' has coordinates $x' = 0$ and $t' = 4.0$ y in the rocket frame. For earth: $x = \gamma(x' + vt) = 1.25 \times (0 + 0.60c \times 4.0) = 1.25 \times (0 + 2.4) = 3.0$ ly. Hence the signal will arrive after 3.0 years according to earth. When the signal is emitted, clocks on earth show

$t = \gamma(t' + \frac{v}{c^2}x') = 1.25 \times (4.0 + \frac{0.60c}{c^2} \times 0) = 5.0$ y, so the signal is received by earth 8.0 years after rocket left earth.

b This can be done in many ways.

Method 1: Consider the events 'rocket leaves earth' and 'signal arrives at earth'. These happen at the same point for earth so the interval of 8.0 years is proper for earth. So the rocket will measure a time of $\gamma \times$ proper $= 1.25 \times 8.0 = 10$ y. (The rocket emitted the signal 4.0 years after leaving earth so the signal took 6.0 years to arrive).

Method 2: The rocket sends the signal after 4.0 years. In this time the earth 'moved away' a distance of $0.60c \times 4.0 = 2.4$ ly. The signal takes time T to get to earth (according to the rocket) and will travel a distance cT getting there. In the meantime the earth will travel an additional distance of $0.60cT$ and so $cT = 2.4 + 0.60cT$. This gives $T = 6.0$ years. So signal arrives when rocket clocks show 10 years.

Method 3: The signal arrives on earth at $x = 0$ and $t = 8.0$ y. Hence $t' = \gamma(t - \frac{v}{c^2}x) = 1.25 \times (8.0 - \frac{0.60}{c} \times 0) = 10$ y.

15 a If E is to cause L, some form of signal must travel from E to L. The speed of this signal will be $\frac{\Delta x}{\Delta t}$. Since no signal can travel faster than light $\frac{\Delta x}{\Delta t} \le c$.

b If E causes L there cannot be such a frame. Suppose there was. Then in this frame $\Delta t' = t'_L - t'_E < 0$. But $\Delta t' = \gamma(\Delta t - \frac{v\Delta x}{c^2}) = \gamma\Delta t(1 - \frac{v}{c^2}\frac{\Delta x}{\Delta t}) < 0$. This means that $\frac{v}{c^2}\frac{\Delta x}{\Delta t} > 1$, i.e. $v > \frac{c^2}{\frac{\Delta x}{\Delta t}} > c$ since $\frac{\Delta x}{\Delta t} \le c$.

16 a The gamma factor is $\gamma = \frac{1}{\sqrt{1 - 0.90^2}} = 2.294$. The time interval of 5.0 min is a proper time interval for the earth observer, and so for the Zenga invader the time interval is $\gamma \times 5.0 = 2.294 \times 5.0 = 11.47 \approx 11\,\text{min}$.

b 11 minutes, by exactly the same argument as in (a).

17 The length of the cube in the direction of motion is contracted and so the volume of the cube decreases. The density therefore increases. The density will be $\gamma\rho$.

18 The gamma factor is $\gamma = \frac{1}{\sqrt{1-0.95^2}} = 3.20$.

The train observers measure the proper time interval. So the ground observers measure $\gamma \times 1.0 = 3.20 \times 1.0 = 3.2\,\text{s}$.

19 The gamma factor is $\gamma = \frac{1}{\sqrt{1-0.95^2}} = 3.20$. The length of 100 m is the contracted length, and so the length at rest (the proper length) is $100\gamma = 3.20 \times 100 = 320\,\text{m}$.

20 a The gamma factor is $\gamma = \frac{30}{28} = 1.07$. The speed is then $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{\gamma^2} \Rightarrow \frac{v}{c} = \sqrt{1-\frac{1}{\gamma^2}} = \sqrt{1-\frac{1}{1.07^2}} = 0.36$.

b Since the trains are identical the proper length of train A is 30 m, which B will measure to be length contracted to 28 m.

c Obviously, 30 m.

21 a The interval of $5.0 \times 10^{-8}\,\text{s}$ is a proper time interval. The gamma factor is $\gamma = \frac{1}{\sqrt{1-0.95^2}} = 3.20$ so the time interval for the observer in the lab is $\gamma \times 5.0 \times 10^{-8} = 3.20 \times 5.0 \times 10^{-8} = 1.6 \times 10^{-7}\,\text{s}$.

b The distance travelled is $vt = 0.95 \times 3.0 \times 10^8 \times 1.6 \times 10^{-7} = 45.6 \approx 46\,\text{m}$.

22 a The time is $\frac{x}{v} = \frac{50\,\text{ly}}{0.995c} \approx 50.3$ yr.

b The time taken according to the spacecraft clocks will be the proper time, and this is $\frac{50.3}{\gamma}$ yr. The gamma factor is $\gamma = \frac{1}{\sqrt{1-0.995^2}} \approx 10$. Hence the time is $\frac{50.3}{10} = 5.03$ yr. The students are just over 23 years old when they get to Vega.

23 a According to the ground the light signal will take time T. In this time the rocket will move a distance vT closer to the mirror. Hence, $cT = D + (D - vT) \Rightarrow T = \frac{2D}{c+v} = \frac{4.8 \times 10^{12}}{1.90 \times 3.0 \times 10^8} = 8.42 \times 10^3 \approx 8.4 \times 10^3\,\text{s}$.

b The time for the rocket is the proper time interval since the signal is emitted and received at the same place. The gamma factor is $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-0.90^2}} = 2.29$. Hence $T' = \frac{8.42 \times 10^3}{2.29} = 3.7 \times 10^3\,\text{s}$.

24 a $u' = \frac{u-v}{1-\frac{uv}{c^2}}$, $v = 0.6c$, $u = 0.8c$. Then $u' = \frac{0.2c}{1-0.8 \times 0.6} = 0.385c$.

b The answer is obviously $-0.385c$, but we can verify this from $u' = \frac{u-v}{1-\frac{uv}{c^2}}$ where now $v = 0.8c$, $u = 0.6c$ so that $u' = \frac{-0.2c}{1-0.8 \times 0.6} = -0.385c$.

25 a $u' = \frac{u-v}{1-\frac{uv}{c^2}}$, $v = -0.6c$, $u = 0.8c$. Then $u' = \frac{1.40c}{1-0.8 \times (-0.6)} = 0.946c$.

b The answer is obviously $-0.946c$, but we can verify this from $u' = \frac{u-v}{1-\frac{uv}{c^2}}$ where now $v = 0.8c$, $u = -0.6c$ so that $u' = \frac{-1.40c}{1-(-0.6) \times 0.8} = -0.946c$.

26 Here we need to use $u = \frac{u'+v}{1+\frac{u'v}{c^2}}$ with $v = 0.60c$ and $u' = 0.70c$. This gives $u = \frac{0.70c + 0.60c}{1 + 0.70 \times 0.60} = 0.915c$.

27 Here we need to use $u = \frac{u'+v}{1+\frac{u'v}{c^2}}$ with $v = -0.60c$ and $u' = 0.70c$. This gives $u = \frac{0.70c + (-0.60c)}{1 + 0.70 \times (-0.60)} = 0.172c$.

28 a The lifetime is $t = \frac{x}{v} = \frac{2.00 \times 10^3}{0.95 \times 3.0 \times 10^8} = 7.02 \times 10^{-6}$ s.

b This observer measures the proper time interval between the events 'muon created' and 'muon decays', and so $\tau = \frac{t}{\gamma} = \frac{7.02 \times 10^{-6}}{\frac{1}{\sqrt{1-0.95^2}}} = 2.19 \times 10^{-6}$ s.

29 The lifetime of the pion according to the lab is $t = \frac{20}{v}$ and also $t = \tau\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \times 2.6 \times 10^{-8}$.

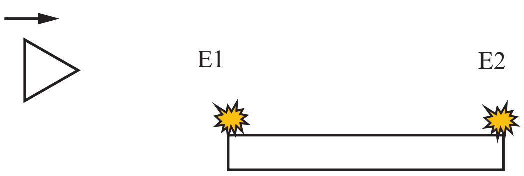
Hence, $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \times 2.6 \times 10^{-8} = \frac{20}{v}$. This is best solved on the **solver** of your graphic display calculator.

Otherwise, $v^2 \times (2.6 \times 10^{-8})^2 = 20^2\left(1 - \frac{v^2}{c^2}\right)$. This means $\frac{v^2}{c^2} \times 60.84 = 400\left(1 - \frac{v^2}{c^2}\right) \Rightarrow \frac{v^2}{c^2} = \frac{400}{460.84} \Rightarrow$ $\frac{v}{c} = 0.931$ so that finally $v = 2.8 \times 10^8$ m s^{-1}.

30 a The interval of 2.0 μs is proper and so the gamma factor is 4.0. $4 = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ so $v = \frac{\sqrt{15}}{4}c = 0.968c$.

b $\Delta x' = \gamma(\Delta x - v\Delta t) = 4.0 \times (0 - 0.968 \times 3 \times 10^8 \times 2.0 \times 10^{-6}) = -2.3$ km.

31



$\Delta t' = t_2' - t_1' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right) = \gamma \times \left(0 - \frac{v\Delta x}{c^2}\right) < 0$ and so event 2 occurs first.

OR

To an observer at rest in the middle of the space station the arrival of the light signals is simultaneous. Since these arrivals occur at the same point in space, they are simultaneous for the rocket as well. But as far the rocket is concerned the space station observer is moving away from the light at E2. So for the lights to arrive at the same time, E2 happened first.

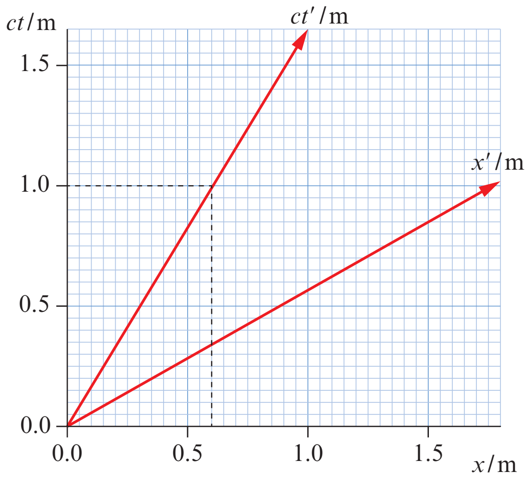
32 The gamma factor is $\gamma = \frac{1}{\sqrt{1-0.80^2}} = 1.67$, $\Delta t' = t_2' - t_1' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right) = 1.67 \times \left(0 - \frac{0.800 \times 3 \times 10^8 \times 1200}{(3 \times 10^8)^2}\right)$ $= -5.3 \times 10^{-6}$ s and so event 2 occurs first.

33 Let T be the time according to earth clocks when the signal will be emitted. Mandy's birthday occurs when $\Delta x' = 0$ and $\Delta t' = 1.0$ y. On earth we therefore have $\Delta t = \gamma\left(\Delta t' + \frac{v}{c^2}\Delta x'\right) = \gamma$ y. So the signal will travel for a time of $\gamma - T$ to get to the spacecraft. The spacecraft is a distance of $v\gamma$ from earth when the signal must be received. Hence $c(\gamma - T) = v\gamma$. Solving for T: $T = \gamma\left(1 - \frac{v}{c}\right)$. This simplifies to

$$T = \gamma\left(1 - \frac{v}{c}\right)$$
$$= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\left(1 - \frac{v}{c}\right)$$
$$= \frac{1}{\sqrt{\left(1-\frac{v}{c}\right)\left(1+\frac{v}{c}\right)}}\left(1 - \frac{v}{c}\right)$$
$$= \sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}$$

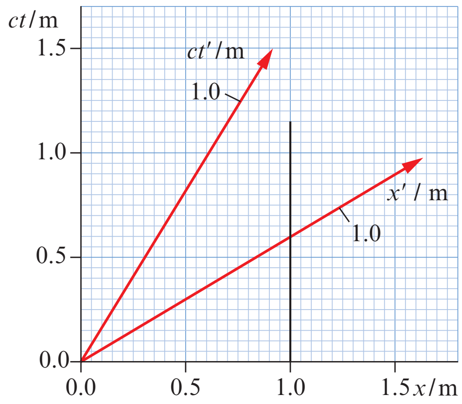
Putting in numbers: $T = \sqrt{\frac{0.4c}{1.6c}} = 0.5$ y.

34

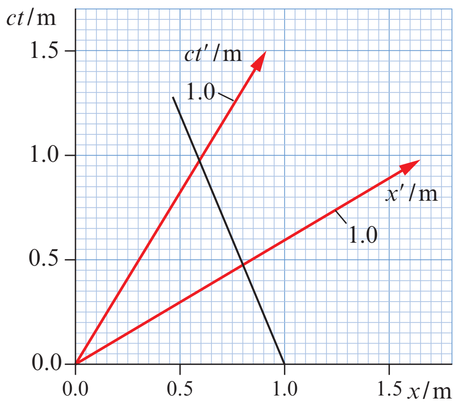


Using the dashed line and $v = \frac{x}{t} = \frac{x}{ct}c = \frac{0.6}{1.0}c = 0.6c$.

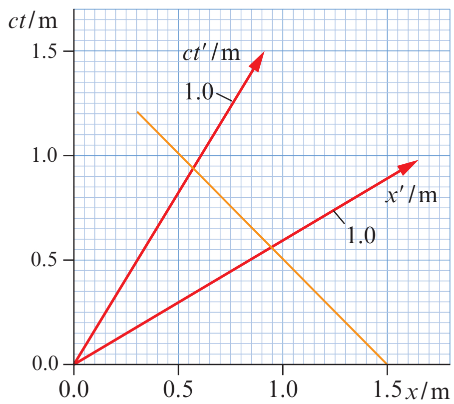
35 a i



ii

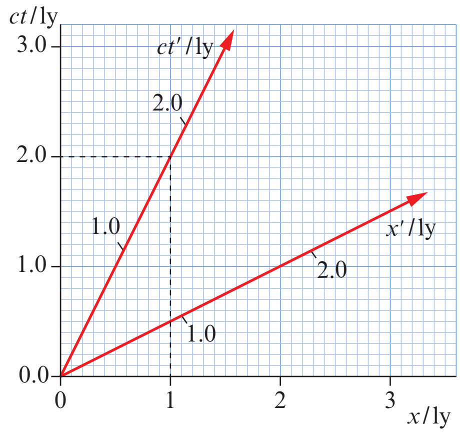


b **i** and **ii**



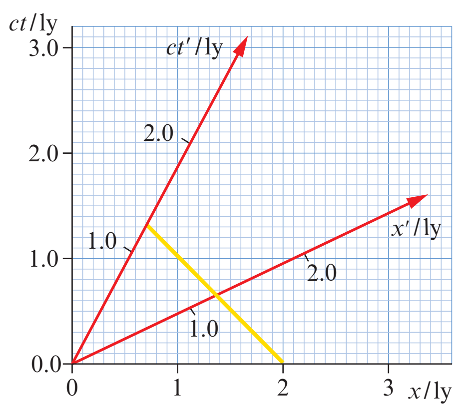
About 0.95 m for S and 0.75 m for S'.

36 a



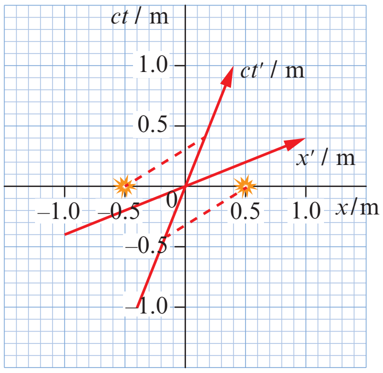
Using the dashed line and $v = \frac{x}{t} = \frac{x}{ct}c = \frac{1.0}{2.0}c = 0.50c$.

b

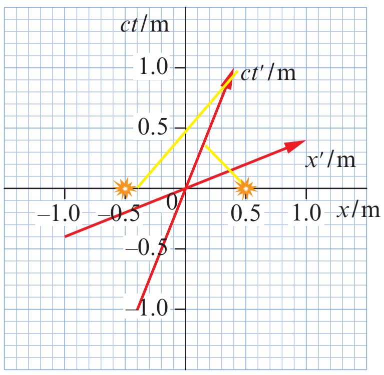


About 1.3 ly for S and 1.1 ly for S'.

37 **a** The right one turns on first:



b



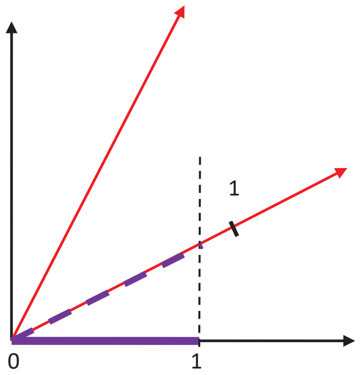
c Light from the right lamp.

38 **a** The speed of the primed frame is $0.5c$. The gamma factor is then $\gamma = \frac{1}{\sqrt{1 - 0.50^2}} = 1.1547 \approx 1.2$. Event P has the same x' co-ordinate as event Q. The co-ordinates of Q in S are $x = 1$, $ct = 0$. Hence $x' = \gamma(x - vt) = 1.2 \times (1 - 0) = 1.2\,\text{m}$. The time co-ordinate of P is clearly zero. Knowing that the space co-ordinate of P is about 1.2 m we can measure along the primed x axis to estimate the position of the point with $x' = 1\,\text{m}$ is approximately at point R.

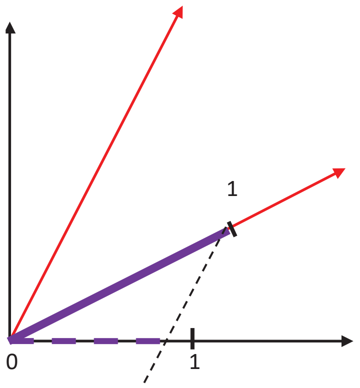
b For the general case again consider event Q that has co-ordinates in S of $x = 1\,\text{m}$ and $t = 0$. Then $x' = \gamma(x - vt) = \gamma(1 - 0) = \gamma$. $(x', ct') = (\gamma, 0)$

39 The speed is $0.6c$. The gamma factor is then 1.25. Locate 1.25 on the black axes and draw vertical and horizontal lines. These intersect the red axes at (1.0) and (0, 1).

40 a



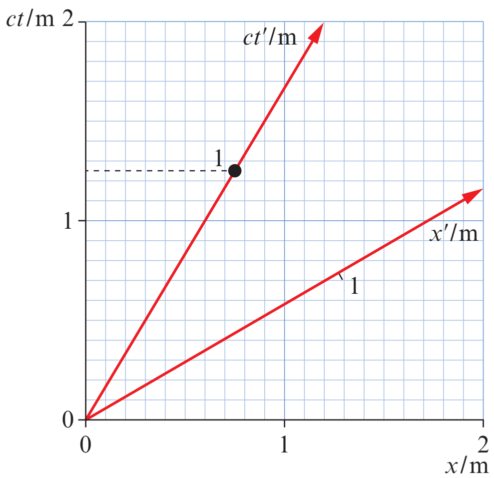
b



i Purple line.

ii Dashed line intersects the x axis at a point less than 1 m.

41 a



b About 1.25 m.

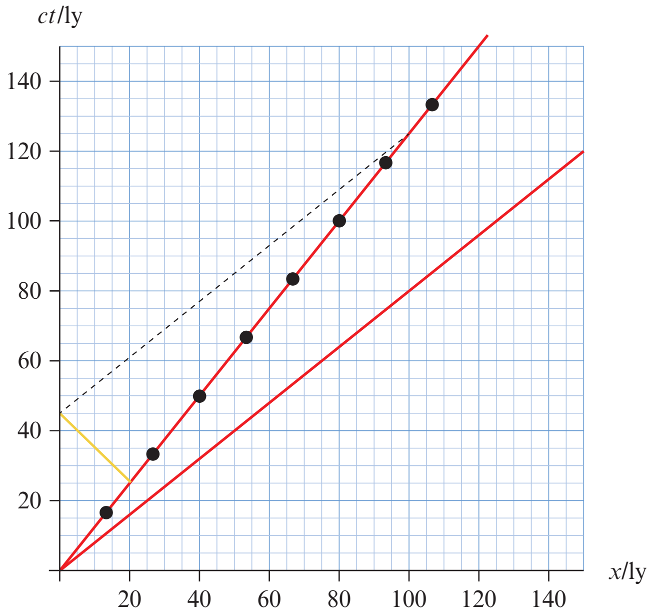
42 a i The rocket gets to the planet in $\frac{20\text{ ly}}{0.80c} = 25\text{ y}$ according to earth. The signal will take 20 years to return to earth, i.e. 45 years after the rocket left earth.

ii According to the rocket, the rocket meets the planet $\frac{25\text{ y}}{\gamma} = \frac{25\text{ y}}{\frac{5}{3}} = 15\text{ y}$ after leaving earth.

According to the rocket the distance separating earth and the planet is $\frac{20\text{ ly}}{\gamma} = \frac{20\text{ ly}}{\frac{5}{3}} = 12\text{ ly}$.

The signal will take a time T to get to earth. In this time the earth will move away a distance $0.80cT$. Therefore, $12 + 0.80cT = cT$ giving $T = \frac{12}{0.2c} = 60$ yr. Hence rocket clocks show $15 + 60 = 75$ yr.

b The dots on the time axis of the rocket are separated by 10 years (by rocket clocks). The signal is emitted (yellow line) when earth clocks show 25 yr from a point 20 ly away. The signal arrives at earth after 25 years i.e. when earth clocks show 45 years. The dotted line allows us to find the rocket time when the signal arrives at earth. The time is 75 years.

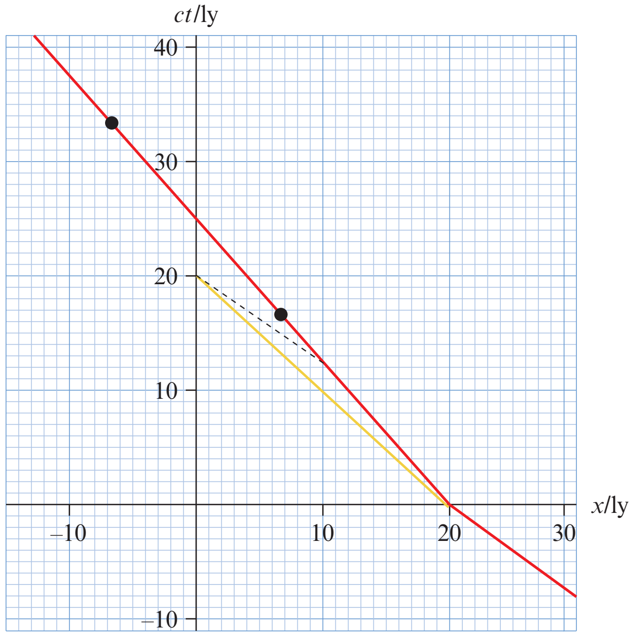


c i The signal will take 20 years to return to earth.

ii According to the rocket the distance separating earth and the planet is $\frac{20\text{ ly}}{\gamma} = \frac{20\text{ ly}}{\frac{5}{3}} = 12\text{ ly}$.

The signal will take a time T to get to earth. In this time the earth will move closer a distance $0.80cT$. Therefore, $12 - 0.80cT = cT$ giving $T = \frac{12}{1.8c} = 6.7$ yr.

d The dots on the time axis of the rocket are separated by 10 years (by rocket clocks). The dotted line is parallel to the rocket space axis. The origins do not coincide at $t = 0$.



Chapter 7

Test your understanding

1 The average kinetic energy is proportional to the absolute temperature. At $T = 0$, the average kinetic energy must then be zero. Since kinetic energy cannot be negative, zero average kinetic energy means zero kinetic energy:

$\bar{E}_K = \frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 300 = 6.2 \times 10^{-21}$ J

2 The kinetic energies are the same since the temperature is the same. Helium molecules are lighter than neon molecules so they move faster on average.

3 Internal energy is the total random kinetic energy of the molecules plus their intermolecular potential energy. Heat is energy that is being transferred from one body to another as a result of a temperature difference.

4 We cannot know which body has the higher internal energy without knowing the masses of the bodies.

5 Molecules vibrate about their equilibrium positions. When heat Q is supplied, raising the temperature, the molecules vibrate with a greater amplitude. Because the shape of the potential is not symmetric, the equilibrium position at higher temperature shifts to higher values, i.e. the solid expands.

6 No and no.

7 One cubic metre of copper has mass 8.9×10^3 kg and so contains $\frac{8.9 \times 10^3 \times 10^3}{64} = 1.39 \times 10^5$ mol OR $1.39 \times 10^5 \times 6.02 \times 10^{23} = 8.37 \times 10^{28}$ molecules. The volume corresponding to each molecule is then $\frac{1}{8.37 \times 10^{28}} = 1.19 \times 10^{-29}$ m^3. Assuming this volume to be a cube for convenience, the side of the cube must be $\sqrt[3]{1.19 \times 10^{-29}} = 2.3 \times 10^{-10}$ m. Assuming that each copper atom sits at the centre of a cube, their separation must be 2.3×10^{-10} m.

8 **a** Yes, by conservation of energy the heat gained by one body must equal the heat lost by the other.

b No, the temperature changes depend also on mass and specific heat capacity.

9 **a** From the definition, $Q = mc\Delta\theta \Rightarrow c = \frac{Q}{m\Delta\theta} = \frac{385}{0.150 \times 5.00} = 513$ J kg^{-1} K^{-1}.

b It is the same.

10 The energy provided is $20 \times 3.0 \times 60 = 3600$ J. Hence $0.090 \times 420 \times 4.0 + 0.310 \times c \times 4.0 = 3600 \Rightarrow$ $c = 2.8 \times 10^3$ J kg^{-1} K^{-1}.

11 The energy provided is $40 \times 4.0 \times 60 = 9600$ J. Hence $25 \times 15.8 + 0.140 \times c \times 15.8 = 9600 \Rightarrow$ $c = 4.2 \times 10^3$ J kg^{-1} K^{-1}. The obvious assumptions are that the liquid and the calorimeter are heated uniformly and that none of the energy supplied gets lost to the surroundings.

12 The loss of potential energy is $mgh = 1360 \times 10 \times 86 = 1.17 \times 10^6$ J. Then, $C\Delta\theta = 1.17 \times 10^6 \Rightarrow$ $\Delta\theta = \frac{1.17 \times 10^6}{16 \times 10^3} = 73$ K.

13 **a** $C = m_1c_1 + m_2c_2 = 45.0 \times 450 + 23.0 \times 4200 = 1.17 \times 10^5$ J K^{-1}.

b $\Delta Q = C\Delta\theta \Rightarrow \frac{\Delta Q}{\Delta t} = C\frac{\Delta\theta}{\Delta t}$. Hence $450 = 1.17 \times 10^5 \times \frac{\Delta\theta}{\Delta t} \Rightarrow \frac{\Delta\theta}{\Delta t} = 3.9 \times 10^{-3}$ K s^{-1}. For a change of temperature of 20.0 K we then require a time of $\frac{20}{3.9 \times 10^{-3}} = 5.2 \times 10^3$ s = 87 min.

14 The energy transferred from the water and the aluminium container is $Q = 0.300 \times 4200 \times 10 + 0.150 \times 900 \times 10 = 13\,950$ J. This is used to raise the temperature of ice to the melting point of 0 °C, melt the ice at 0 °C and raise the temperature of the melted ice (which is now water) to the final temperature of 0 °C. Thus $13\,950 = m \times 2100 \times 10 + m \times 334 \times 10^3 + m \times 4200 \times 10$. Hence $m = 0.039$ kg.

15 The mass of ice is $m = 20 \times 0.06 \times 900 = 1080$ kg. So we need $Q = 1080 \times 2100 \times 5 + 1080 \times 334 \times 10^3 = 3.7 \times 10^8$ J.

16 **a** Let the surface area (in square meters) of the pond be A. Then in time t the energy falling on the surface will be $Q = 600 \times A \times t$. The volume of ice is $V = A \times 0.01$ and so its mass is $m = (A \times 0.01) \times 900$. Then $600 \times A \times t = (A \times 0.01) \times 900 \times 334 \times 10^3$. We see that the unknown surface area cancels out and is not required. Then, $t = \frac{0.01 \times 900 \times 334 \times 10^3}{600} = 5010 \text{ s} \approx 84$ min.

b This assumes that none of the incident radiation is reflected from the ice and that all the ice is uniformly heated.

17 **a** $Q_1 = 1.0 \times 2100 \times 10 = 2.1 \times 10^4$ J.

b $Q_2 = 1.0 \times 334 \times 10^3 = 3.34 \times 10^5$ J.

c $Q_3 = 1.0 \times 4200 \times 10 = 4.2 \times 10^4$ J.

18 The water will lose an amount of thermal energy $1.00 \times 4200 \times 10 = 42\,000$ J. This energy is used to melt the ice and then raise the temperature of the melted ice to 10 °C. Thus $m \times 334 \times 10^3 + m \times 4200 \times 10 = 42000 \Rightarrow m = 0.11$ kg.

19 $0.150 \times 4200 \times (T - 30) + 0.100 \times 334 \times 10^3 + 0.100 \times 4200 \times T = 0.050 \times 2260 \times 10^3 + 0.050 \times 4200 \times (100 - T)$

This long equation can be solved for T (preferably using the Solver of your calculator) to give $T = 95$ °C.

20 $0.200 \times 334 \times 10^3 + 0.200 \times 4200 \times 50 = m \times 2260 \times 10^3 + m \times 4200 \times (100 - 50)$ giving $m = 44$ g.

21 Water received more heat from the red body and so $mc_R(90 - 40) > mc_B(90 - 20)$ implying $c_R > c_B$.

22 See textbook.

23 Marble is a better conductor of heat.

24 Water is a better conductor of heat than air.

25 Conduction transfers heat to the water through the bottom of the pan. Convection and conduction transfer the heat to the top surface of the water.

26 Vapour collects above the coffee surface from evaporation. Blowing this vapour away helps continue the evaporation process and hence cools the hot coffee.

27 The bridge loses heat from both sides, above and below, whereas the road over ground loses heat from one side only. In addition, there is a lot of metal in the bridge that conducts heat away.

28 Warm air rises so a fan slowly pushing it down can help warm a room better.

29 The air over land is warmer than the air over the water. Therefore it rises and cooler air from over the water takes its place.

30 Radiation.

31 **a** The rate of energy transfer is the same by energy conservation.

b $k_X A \frac{\Delta T_X}{L} = k_Y A \frac{\Delta T_Y}{L}$ hence $k_X \Delta T_X = k_Y \Delta T_Y$. Since the constants are different the temperature differences are different.

32 $kA\frac{(80-T)}{L} = 2kA\frac{(T-20)}{L}$, i.e. $80 - T = 2T - 40 \Rightarrow T = 40\ °\text{C}$

33 The room is losing heat at a rate (power) given by $kA\frac{(T_{\text{room}} - T_{\text{out}})}{L} = 0.15 \times 12 \times \frac{(24-(-5.0))}{0.20} = 261\ \text{W}$.
For the temperature in the room to remain constant, this must also be the power of the heater.

34 We require 334 J to melt the ice. The power at which energy is being transferred is

$kA\frac{(T_{\text{hot}} - T_{\text{cold}})}{L} = 350 \times \pi \times (0.01)^2 \times \frac{(100-0)}{0.25} = 43.98\ \text{W}$.

$Pt = Q \Rightarrow t = \frac{Q}{P} = \frac{334}{43.98} = 7.6\ \text{s}$

35 $\frac{\sigma \times 900^4}{\sigma \times 300^4} = 3^4 = 81$

36 **a** See textbook.

b A piece of charcoal.

c By $\frac{(273+100)^4}{(273+50)^4} = 1.78$

37 $\lambda T = 2.9 \times 10^{-3} \Rightarrow T = \frac{2.9 \times 10^{-3}}{410 \times 10^{-9}} = 7073 \approx 7100\ \text{K}$

38 From $\lambda T = 2.9 \times 10^{-3}$, star X has the higher temperature.

39 **a** $P = \sigma A T^4$ so increases by $2 \times 2^4 = 32$

b $I = \sigma T^4$ so increases by $2^4 = 16$

40 The sphere, because it has a larger surface area.

41 **a** $\lambda T = 2.9 \times 10^{-3} \Rightarrow T = \frac{2.9 \times 10^{-3}}{5.0 \times 10^{-6}} = 580\ \text{K}$

b $\lambda \times 2 \times 580 = 2.9 \times 10^{-3} \Rightarrow \lambda = 2.5\ \mu\text{m}$

c It stays the same. The area does not enter the formula for this graph at all.

42 **a** The peak wavelength is about 495 nm and so $\lambda T = 2.9 \times 10^{-3} \Rightarrow T = \frac{2.9 \times 10^{-3}}{495 \times 10^{-9}} = 5860\ \text{K}$.

b Yes, because there is so little variation of the spectral intensity over visible wavelengths.

43 We use $b = \frac{L}{4\pi d^2}$ so that $b = \frac{4.5 \times 10^{28}}{4\pi \times (8.3 \times 10^{17})^2} = 5.2 \times 10^{-9}\ \text{W m}^{-2}$.

44 $b = \frac{3.9 \times 10^{26}}{4\pi \times (1.5 \times 10^{11})^2} = 1379.3 \approx 1400\ \text{W m}^{-2}$

45 $L = 4\pi d^2 b = 4\pi \times (6.6 \times 10^{17})^2 \times 3.0 \times 10^{-8} = 1.6 \times 10^{29}\ \text{W}$

46 $d = \sqrt{\frac{L}{4\pi b}} = \sqrt{\frac{6.2 \times 10^{32}}{4\pi \times 8.4 \times 10^{-10}}} = 2.4 \times 10^{20}\ \text{m}$

47 $\frac{b_A}{b_B} = \frac{\frac{L_A}{4\pi d^2}}{\frac{L_B}{4\pi d^2}} \Rightarrow \frac{9.0 \times 10^{-12}}{3.0 \times 10^{-13}} = \frac{L_A}{L_B}$, i.e. $\frac{L_A}{L_B} = 30$.

48 **a** $\frac{L_{\text{A}}}{L_{\text{B}}} = \frac{\sigma A (4T)^4}{\sigma A T^4} = 256$.

b $\frac{b_{\text{A}}}{b_{\text{B}}} = \frac{\frac{L_A}{4\pi d_{\text{A}}^2}}{\frac{L_B}{4\pi d_{\text{B}}^2}}$; hence $1 = 256 \times \frac{d_{\text{B}}^2}{d_{\text{A}}^2} \Rightarrow \frac{d_{\text{B}}}{d_{\text{A}}} = \frac{1}{16}$.

49 a Since $L = \sigma A T^4 = \sigma 4\pi R^2 T^4$: $R_A^2(5000)^4 = R_B^2(10\,000)^4 \Rightarrow \frac{R_A}{R_B} = \sqrt{\frac{(10\,000)^4}{(5000)^4}} = 4$.

b $\frac{b_A}{b_B} = \frac{\frac{L_A}{4\pi d_A^2}}{\frac{L_B}{4\pi d_B^2}}$ and so $2 = \frac{d_B^2}{d_A^2} \Rightarrow \frac{d_A}{d_B} = 0.71$.

50 We have that $b = \frac{L}{4\pi d^2}$ and $L = \sigma A T^4$. Combining, $b = \frac{\sigma A T^4}{4\pi d^2}$. Hence, $\frac{b_A}{b_B} = \frac{\frac{\sigma A T_A^4}{4\pi d_A^2}}{\frac{\sigma A T_B^4}{4\pi d_B^2}} = \frac{\frac{T_A^4}{d_A^2}}{\frac{T_B^4}{d_B^2}} = \frac{T_A^4 d_B^2}{T_B^4 d_A^2} \Rightarrow$

$\frac{T_A}{T_B} = \sqrt[4]{\frac{b_A d_A^2}{b_B d_B^2}}$. Since this is a ratio we do not have to change units (light years to meters.) Hence,

$\frac{T_A}{T_B} = \sqrt[4]{\frac{8.0 \times 10^{-13} \times 120^2}{2.0 \times 10^{-15} \times 150^2}} = 4$.

51 Since $L = \sigma A T^4 = \sigma 4\pi R^2 T^4$:

a $\frac{5.2 \times 10^{28}}{3.9 \times 10^{26}} = \frac{R^2(4000)^4}{R_\odot^2(6000)^4} \Rightarrow \frac{R}{R_\odot} = \sqrt{\frac{5.2 \times 10^{28}}{3.9 \times 10^{26}} \times \frac{(6000)^4}{(4000)^4}} = 25.9 \approx 26$.

b $\frac{4.7 \times 10^{27}}{3.9 \times 10^{26}} = \frac{R^2(9250)^4}{R_\odot^2(6000)^4} \Rightarrow \frac{R}{R_\odot} = \sqrt{\frac{4.7 \times 10^{27}}{3.9 \times 10^{26}} \times \frac{(6000)^4}{(9250)^4}} = 1.46 \approx 1.5$.

52 $\frac{b_X}{b_Y} = 1 = \frac{\frac{\sigma 4\pi R_X^2 T^4}{d_X^2}}{\frac{\sigma 4\pi R_Y^2 T^4}{d_Y^2}} \Rightarrow \frac{R_X}{R_Y} = \frac{d_X}{d_Y} = 2$

53 a The color of the star corresponds to a particular wavelength. This is the peak wavelength in the spectrum, which in turn is related to surface temperature through Wien's law, $\lambda T = 2.9 \times 10^{-3}$ K m.

b The blue star since it has a lower peak wavelength.

54 We know that $L = \sigma A T^4$ and so $\frac{L_A}{L_B} = \frac{\sigma A_A T_A^4}{\sigma A_B T_B^4}$. Since the radius of A is double that of B, $\frac{L_A}{L_B} = 4 \times \frac{T_A^4}{T_B^4}$.

From Wien's law, $\lambda T = const$ and so $650 \times T_A = 480 \times T_B \Rightarrow \frac{T_A}{T_B} = \frac{480}{650}$. Hence, $\frac{L_A}{L_B} = 4 \times \left(\frac{480}{650}\right)^4 = 1.2$.

Chapter 8

Test your understanding

1 **a** The wavelength at the peak of the graph is determined by temperature, and since the wavelength is the same so is the temperature.

b The ratio of the intensities at the peak is about $\frac{1.1}{1.9} \approx 0.6$.

2 We have that $e\sigma A T^4 = P \Rightarrow T = \sqrt[4]{\frac{P}{e\sigma A}}$, i.e. $T = \sqrt[4]{\frac{1.35 \times 10^9}{0.800 \times 5.67 \times 10^{-8} \times 5.00 \times 10^6}} = 278$ K.

3 $P_{emit} = e\sigma A T^4 = e\sigma 4\pi R^2 T^4 = 0.80 \times 5.67 \times 10^{-8} \times 4\pi \times (0.50)^2 \times 300^4 = 1.2$ kW

$P_{absorb} = e\sigma A T^4 = e\sigma 4\pi R^2 T^4 = 0.80 \times 5.67 \times 10^{-8} \times 4\pi \times (0.50)^2 \times 330^4 = 1.7$ kW

4 **a** $I_{emit} = e\sigma T^4 = 0.70 \times 5.67 \times 10^{-8} \times 300^4 = 321.49 \approx 320$ W m^{-2}

b $P = 321.49 \times 4\pi r^2 = 321.49 \times 4\pi \times (0.60)^2 = 1454$ W. Hence $I = \frac{P}{4\pi d^2} = \frac{1454}{4\pi (2.2)^2} = 24$ W m^{-2}.

5 **a** Intensity is the power received or emitted per unit area from a source of radiation.

b $P_{net} = 0.90 \times 5.67 \times 10^{-8} \times 1.60 \times ((273 + 37)^4 - (273 - 15)^4) = 61.52 \approx 62$ W m^{-2}.

6 The incident intensity must be 150 W m^{-2} and so the albedo is 0.20.

7 **a** $\frac{75}{340} = 0.22$

b $\frac{30}{160} = 0.19$

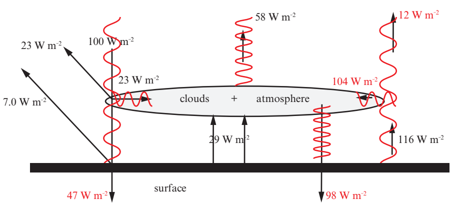
c $\frac{75 + 30}{340} = 0.31$

d $340 - 75 - 160 = 105$ W m^{-2}

e $340 - 75 - 30 = 235$ W m^{-2}

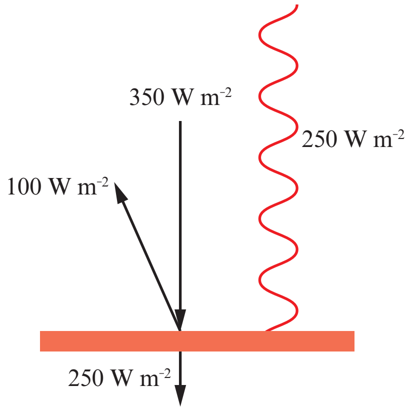
8 $320 + 180 - 85 = 415$ W m^{-2}

9 Required intensities asked in **a–d** are shown in red in the following energy flow diagram.



e Solar intensity incident on surface is $100 - 23 - 23 = 54$ W m^{-2}, so surface albedo is $\frac{7}{54} = 0.13$.

10 a The energy flow diagram is shown below.



b The reflected intensity is $350 - 250 = 100$ W m^{-2} and so the albedo is $\frac{100}{350} = 0.29$.

c It has to be equal to that absorbed, i.e. 250 W m^{-2}.

d Use $e\sigma T^4 = I \Rightarrow T = \sqrt[4]{\frac{I}{e\sigma}}$; $T = \sqrt[4]{\frac{250}{(1-0.29)\times 5.67\times 10^{-8}}} = 281$ K. You must be careful with these calculations in the exam. You must be sure as to whether the question wants you to assume a black body or not. Strictly speaking, in a model without an atmosphere the planet's surface cannot be taken to be a black body—if it were no radiation would be reflected!

11 The equilibrium temperature of the earth is determined by equating the radiated intensity to the absorbed intensity. An increase in greenhouse gas concentrations means more IR radiation is trapped and more radiation is sent back down to earth. Since the radiated intensity is proportional to T^4 the temperature will rise.

12 a We must have that $\sigma A T^4 \propto \frac{1}{d^2} \Rightarrow T \propto \frac{1}{\sqrt{d}}$.

b Using our knowledge of propagation of uncertainties, we deduce that $\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta d}{d}$ and so $\frac{\Delta T}{T} = \frac{1}{2}\times 1.0\% = 0.005$. Hence, $\Delta T = 0.005T = 0.005 \times 288 = 1.4$ K.

c The sun is losing mass in the fusion reactions taking place in the core of the sun. So the gravitational force it exerts on earth is decreasing.

d Assuming the rate remains constant, 1% of the present distance is 1.5×10^9 m. So the time will be $\frac{1.5\times 10^9}{0.015} = 10^{11}$ years! The sun will have engulfed the earth when it becomes a red giant star much earlier than this time.

13 a $S = 1360 \times \left(\frac{1.5}{1.08}\right)^2 = 2.62 \times 10^3$ W m^{-2}.

b The average intensity is $\frac{S}{4} = \frac{2620}{4} \approx 655$ W m^{-2}. Of this, 0.75 is reflected so the absorbed intensity is $0.25 \times 655 = 164$ W m^{-2}. Therefore $\sigma T^4 = 164$ and $T = \sqrt[4]{\frac{164}{5.67\times 10^{-8}}} = 232$ K. If we argue that the planet is not a black body and has emissivity 0.25, the temperature rises to 329 K.

c Either way, the calculated temperature neglecting the greenhouse effect gives a temperature very much lower than the actual temperature. This means the greenhouse effect is powerful on Venus, and so Venus must have a very dense atmosphere of greenhouse gases (CO_2 in this case).

d It means that Mercury must have a very tenuous atmosphere. An atmosphere works against large day and night temperature variations by not letting so much heat escape at night.

14 a Imagine a sphere centred at the source of radius d. The power P radiated by the source is distributed over the surface area A of this imaginary sphere. The power per unit area, i.e. the intensity, is thus $I = \frac{P}{A} = \frac{P}{4\pi d^2}$.

b We have assumed that the radiation is uniform in all directions.

15 a Albedo is the ratio of the reflected intensity to the incident intensity on a surface.

b The albedo of a planet depends on factors such as cloud cover in the atmosphere, amount of ice on the surface, amount of water on the surface and color and nature of the soil and vegetation.

c The earth receives radiant energy from the sun and in turn radiates energy itself. The radiated energy is in the infrared region of the electromagnetic spectrum. Gases in the atmosphere absorb part of this radiated energy and re-radiate it in all directions. Some of this radiation returns to the earth's surface, warming it further.

d The main greenhouse gases are water vapour, carbon dioxide and methane.
See textbook for sources.

16 Radiation is the main mechanism by which the surface transfers energy to the atmosphere. In addition we have evaporation of water from the surface. To turn liquid water into vapour we must supply the latent heat of evaporation so this is energy leaving the surface. We also have convection currents that carry warm air higher into the atmosphere, thus removing energy from the surface.

17 a Dry subtropical land has a high albedo, around 0.4, whereas a warm ocean has an albedo of less than 0.2.

b Radiation and convection currents are the main mechanisms.

c Replacing dry land by water reduces the albedo of the region. Reducing the albedo means that less radiation is reflected and more is absorbed, and so an increase in temperature might be expected. The increase in temperature might involve additional evaporation and so more rain.

18 The rate of evaporation from water depends on the temperature of the water and the temperature of the surrounding air. These are both higher in the case of the tropical ocean water, and evaporation will be more significant in that case.

19 We have that the original average albedo of the area was $0.6 \times 0.10 + 0.4 \times 0.3 = 0.18$ and the new one is $0.7 \times 0.10 + 0.3 \times 0.3 = 0.16$ for a reduction in albedo of 0.02. Hence the expected temperature change is estimated to be 2 °C.

Chapter 9

Test your understanding

1 A mole of Ferraris is 6.02×10^{23} cars so $4 \times 6.02 \times 10^{23} = 2.4 \times 10^{24}$ tyres (not counting the spare).

2 28 g corresponds to 14 moles and so $14 \times 6.02 \times 10^{23} = 8.4 \times 10^{24}$ molecules.

3 $\frac{12}{28} = 0.43$ mol

4 $\frac{6.0}{4.0} = 1.5$ mol

5 $\frac{2.0 \times 10^{24}}{6.02 \times 10^{23}} = 3.3$ mol

6 $3 \times 32 = 96$ g

7 $\frac{3.0 \times 10^{24}}{6.02 \times 10^{23}} = 4.98$ mol so the mass is $4.98 \times 12 = 60$ g.

8 21 g of krypton is $\frac{21}{84} = 0.25$ mol. 0.25 mol of carbon is 3.0 g.

9 One mol of helium has a mass of 4.0 g and contains 6.02×10^{23} atoms. So one atom has a mass of $\frac{4.0 \times 10^{-3}}{6.02 \times 10^{23}} = 6.6 \times 10^{-27}$ kg.

10 From $\frac{P_1 V_1}{n_1 T_1} = \frac{P_2 V_2}{n_2 T_2}$ we deduce that $\frac{P_1}{T_1} = \frac{P_2}{T_2}$, i.e. that $\frac{12.0 \times 10^5}{295} = \frac{P_2}{393}$, hence $P_2 = 16.0 \times 10^5$ Pa. (Notice the change to kelvin.)

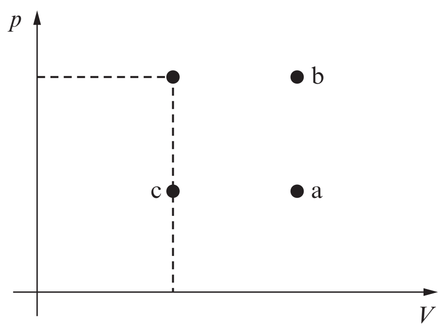
11 From $\frac{P_1 V_1}{n_1 T_1} = \frac{P_2 V_2}{n_2 T_2}$ we deduce that $P_1 V_1 = P_2 V_2$, i.e. that $8.2 \times 10^6 \times 2.3 \times 10^{-3} = 4.5 \times 10^6 \times V_2$ hence $V_2 = 4.2 \times 10^{-3}$ m^3.

12 A quantity of 12.0 kg of helium corresponds to $\frac{12 \times 10^3}{4} = 3.0 \times 10^3$ mol. Then from the gas law, $PV = nRT$, we get $P = \frac{nRT}{V} = \frac{3.00 \times 10^3 \times 8.31 \times 293}{5.00 \times 10^{-3}} = 1.46 \times 10^9$ Pa.

13 From the gas law, $PV = nRT$, we get $n = \frac{PV}{RT} = \frac{4.00 \times 1.013 \times 10^5 \times 12.0 \times 10^{-3}}{8.31 \times 293} = 1.998$ mol. Since the mass of one mole of carbon dioxide (CO_2) is 44 g, we need $44 \times 1.9998 = 87.9$ g.

14 We use $\frac{P_1 V_1}{n_1 T_1} = \frac{P_2 V_2}{n_2 T_2}$ to get $\frac{P}{n_1} = \frac{\frac{P}{2}}{n_2}$ and hence $n_2 = \frac{n_1}{2}$. In other words to reduce the pressure to half its original value, half the molecules must leave the container. The original number of molecules can be found using $pV = nRT$ to get $n = \frac{PV}{RT} = \frac{5.00 \times 10^5 \times 300 \times 10^{-6}}{8.31 \times 300} = 0.0602$ and hence $N = 0.0602 \times 6.02 \times 10^{23} = 3.62 \times 10^{22}$. So we will have to lose $\frac{N}{2} = 1.81 \times 10^{22}$ molecules. This will take $\frac{1.81 \times 10^{22}}{3.00 \times 10^{19}} = 603$ s ≈ 10 min.

15



16 Let there be n_1 moles of the gas in the left container and n_2 in the right. Then it must be true $\left(\text{using } n = \frac{PV}{RT}\right)$ that $n_1 = \frac{12 \times 10^5 \times 6.0 \times 10^{-3}}{RT}$ and $n_2 = \frac{6.0 \times 10^5 \times 3.0 \times 10^{-3}}{RT}$. When the gases mix we will have $n_1 + n_2$ moles in a volume of 9.0 dm^3 and so $n_1 + n_2 = \frac{P \times 9.0 \times 10^{-3}}{RT}$. Hence $\frac{12 \times 10^5 \times 6.0 \times 10^{-3}}{RT} + \frac{6.0 \times 10^5 \times 3.0 \times 10^{-3}}{RT} = \frac{P \times 9.0 \times 10^{-3}}{RT}$. This means that $P = \frac{12 \times 10^5 \times 6.0 + 6.0 \times 10^5 \times 3.0}{9.0} = 10 \times 10^5$ Pa $= 10$ atm.

17 a The cross-sectional area of the piston is $A = \frac{V}{h} = \frac{0.050}{0.500} = 0.10$ m^2. The pressure in the gas is constant and equal to $P = \frac{F}{A} = \frac{10.0 \times 9.81}{0.10} = 981 \approx 1.0 \times 10^3$ Pa.

b From the gas law, $n = \frac{PV}{RT} = \frac{1.0 \times 10^3 \times 0.050}{8.31 \times 292} = 0.020$. The number of molecules is then $N = 0.020 \times 6.02 \times 10^{23} = 1.2 \times 10^{22}$.

c From $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ we get $\frac{0.050}{292} = \frac{V_2}{425}$ hence $V_2 = 7.3 \times 10^{-2}$ m^3.

18 The mass is just $28 \times 2 = 56$ g. The volume is found from $V = \frac{nRT}{P} = \frac{2.0 \times 8.31 \times 273}{1.0 \times 10^5} = 0.045$ m^3.

19 The molar mass of helium is 4.00 g per mole. A mass of 70.0 kg of helium corresponds to $\frac{70.0 \times 10^3}{4.00} = 1.75 \times 10^4$ mol. Thus $P = \frac{nRT}{V} = \frac{1.75 \times 10^4 \times 8.31 \times 290}{404} = 1.04 \times 10^5$ Pa.

20 a $n = \frac{PV}{RT} = \frac{150 \times 10^3 \times 5.0 \times 10^{-4}}{8.31 \times 300} = 3.01 \times 10^{-2}$ mol.

b $N = nN_A = 3.01 \times 10^{-2} \times 6.02 \times 10^{23} = 1.8 \times 10^{22}$.

c $M = n\mu = 3.01 \times 10^{-2} \times 29 = 0.87$ g.

21 a $V = \frac{nRT}{P} = \frac{1.0 \times 8.31 \times 273}{1.0 \times 10^5} = 2.27 \times 10^{-2}$ m^3.

b We have 1 mol and so 4.00 g of helium. The density is thus $\rho = \frac{M}{V} = \frac{4.00 \times 10^{-3}}{2.27 \times 10^{-2}} = 0.176$ kg m^{-3}.

22 a The molecules are moving faster on average and hit the walls more frequently. Pressure has to do with the momentum change at each collision (which increases) and the frequency of the collisions (which also increases).

b The temperature increases so the molecules are moving faster on average. For pressure to stay the same, the collisions must be less frequent and so the volume must increase so that molecules have longer to travel before colliding with walls.

23 a $\frac{\bar{E}_{\text{He}}}{\bar{E}_{\text{Ne}}} = \frac{\frac{3}{2}kT}{\frac{3}{2}kT} = 1$

b $\frac{1}{2}m_{\text{He}}c_{\text{He}}^2 = \frac{1}{2}m_{\text{Ne}}c_{\text{Ne}}^2$ so $\frac{c_{\text{He}}}{c_{\text{Ne}}} = \sqrt{\frac{m_{\text{Ne}}}{m_{\text{He}}}} = \sqrt{\frac{\frac{20}{N_A}}{\frac{4}{N_A}}} = \sqrt{5}$.

24 Under the given changes, the volume will stay the same and so the density will be unchanged.

25 From $P = \frac{1}{3}\rho c^2$ we get $c = \sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3 \times 2.8 \times 10^5}{1.2}} = 836.7 \approx 840$ m s^{-1}.

26 a From $PV = nRT$, the temperature is quadrupled. From $\frac{1}{2}mc^2 = \frac{3}{2}k_BT$, c^2 is quadrupled so c is doubled.

b From $U = \frac{3}{2}RnT$, the internal energy is quadrupled.

27 a From $PV = nRT$, the temperature is halved. From $\frac{1}{2}mc^2 = \frac{3}{2}k_BT$, c^2 is halved so c is reduced to $\frac{c}{\sqrt{2}}$.

b From $U = \frac{3}{2}RnT$, the internal energy is halved.

28 a From $PV = nRT$, the pressure doubled. The density is also doubled. From $P = \frac{1}{3}\rho c^2$, c stays the same.

b From $\overline{E}_K = \frac{3}{2}k_BT$, we see that c stays the same.

29 The mass of an atom of krypton is $\frac{84 \times 10^{-3}}{6.02 \times 10^{23}} = 1.395 \times 10^{-25}$ kg. From $\frac{1}{2}mc^2 = \frac{3}{2}kT$ we get $c = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 350}{1.395 \times 10^{-25}}} = 322 \approx 320\ \text{m s}^{-1}$.

30 From $\frac{1}{2}mc^2 = \frac{3}{2}kT$ we get $c = \sqrt{\frac{3kT}{m}}$. The mass of a molecule (in kg) is $\frac{M}{N_A}$. Hence $c = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3kN_AT}{M}} = \sqrt{\frac{3RT}{M}}$ since $k = \frac{R}{N_A}$.

31 $c = \sqrt{\frac{3kT}{m}}$ so we need $\frac{300}{\frac{84}{N_A}} = \frac{T}{\frac{132}{N_A}}$, i.e. $T = \frac{132}{84} \times 300 = 471 \approx 470$ K.

32 $U = \frac{3}{2}RnT = \frac{3}{2}PV = \frac{3}{2} \times 1.3 \times 10^5 \times 0.25 = 4.9 \times 10^4$ J.

Chapter 10

Test your understanding

1 An isothermal process has to be slow to allow for heat to be exchanged in order to keep the temperature constant.

An adiabatic process has to be fast so that there is no time for heat to enter or leave the system.

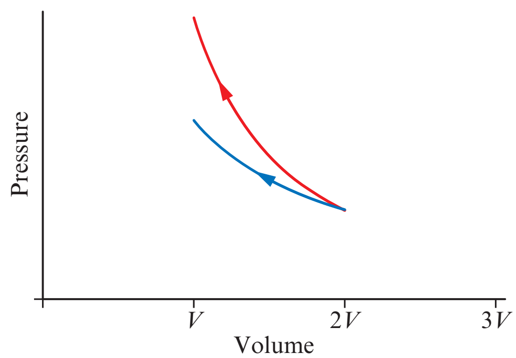
2 In an adiabatic expansion the piston moves out fast so that molecules bounce of it with a reduced speed. The average kinetic energy of the molecules decreases and so does temperature since $\overline{E}_K \propto T$.

3 Use $PV^{\frac{5}{3}} = c$ to get $8.1 \times 10^5 \times (2.5 \times 10^{-3})^{\frac{5}{3}} = P \times (4.6 \times 10^{-3})^{\frac{5}{3}}$, i.e. $P = 8.1 \times 10^5 \times \left(\frac{2.5 \times 10^{-3}}{4.6 \times 10^{-3}}\right)^{\frac{5}{3}} =$ 2.9×10^5 Pa.

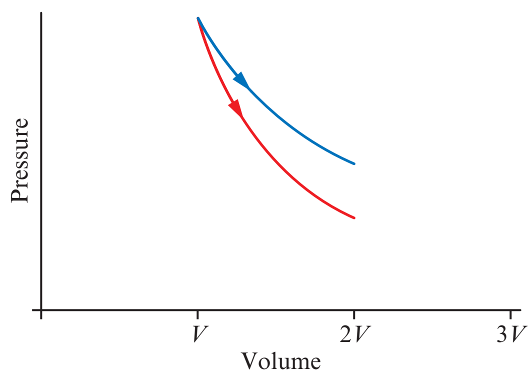
4 We know that $PV^{\frac{5}{3}} = c$ and also $PV = nRT$. From the ideal gas law we find $P = \frac{nRT}{V}$, and substituting in the first equation we find $\frac{nRT}{V}V^{\frac{5}{3}} = c$ or $TV^{\frac{2}{3}} = c'$ (another constant). Hence, $560 \times (2.8 \times 10^{-3})^{\frac{2}{3}} =$ $T \times (4.8 \times 10^{-3})^{\frac{2}{3}}$. Hence $T = 560 \times \left(\frac{2.8 \times 10^{-3}}{4.8 \times 10^{-3}}\right)^{\frac{2}{3}} = 391 \approx 390$ K

5 $W = P\Delta V = 5.4 \times 10^5 \times (4.3 - 3.6) \times 10^{-3} = 378 \approx 380$ J.

6 The blue curve is isothermal and the red is adiabatic. The area under the adiabatic curve is larger and so the work done is larger.



7 Isothermal is the blue curve, and the red curve is adiabatic. The area under the isothermal curve is larger and so the work done is larger.



8 $Q = \Delta U + W$ and $W = 0$. The temperature halved and so $\Delta U < 0$, hence $Q < 0$ and energy is removed.

9

	W	ΔU	ΔT
X	positive	0	0
Y	0	positive	positive
Z	positive	positive	positive

10 a $Q = \Delta U + W$ and $Q = 0$, so $\Delta U = -W$. The work is negative since the gas is compressed. Hence $\Delta U > 0$, i.e. temperature increases.

b Molecules bounce off the fast inward moving piston with increased speeds. Therefore the average kinetic energy of the molecules increases and so does temperature since $\bar{E}_K \propto T$.

11 a Use $W = P\Delta V = 6.00 \times 10^6 \times (0.600 - 0.200) = 2.4$ MJ.

b From the gas law, $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ and so $\frac{0.200}{300} = \frac{0.600}{T_2}$ giving $T_2 = 900$ K.

c The change in internal energy is $\Delta U = \frac{3}{2}Rn\Delta T = \frac{3}{2}\frac{PV}{T}\Delta T = \frac{3}{2} \times \frac{6.00 \times 10^6 \times 0.200}{300} = 3.6$ MJ. Therefore $Q = \Delta U + W = 3.6 + 2.4 = 6.0$ MJ.

12 No, it can all go to doing work: $Q = \Delta U + W$, if $Q = W$ the internal energy of the system stays the same.

13 In an isothermal expansion $\Delta U = 0$, $Q = W = 3.0$ kJ.

14 From $Q = \Delta U + W$, $500 = \Delta U + 500 \Rightarrow \Delta U = 0$.

15 From $Q = \Delta U + W$, $0 = \Delta U - 500 \Rightarrow \Delta U = +500$ J.

16 a From the graph we see that as T decreases P increases. From $PV = nRT$ this implies that V is decreasing as T decreases. Hence work is done on the gas.

b $Q = \Delta U + W$ with $W < 0$ and $\Delta U < 0$, so $Q < 0$.

17 For X (constant volume) we have $Q = \Delta U_X + W = \Delta U_X + 0$.

For Y (constant pressure) we have that $Q = \Delta U_Y + W$. Since the heats are equal

$\Delta U_X = \Delta U_Y + W$

which shows that $\Delta U_X > \Delta U_Y$ since $W > 0$. The internal energy for an ideal gas is a function only of temperature, and so the temperature increase is greater for the constant volume case.

18 Working as in the previous problem, $Q_X = \Delta U$ and $Q_Y = \Delta U + W$. The two changes in internal energy are the same because the temperature differences are the same: $Q_Y = Q_X + W$. Since $W > 0$, $Q_Y > Q_X$.

19 $Q_I = \Delta U + W_I$ and $Q_{II} = \Delta U + W_{II}$. ΔU is the same for both. Since $W_I > W_{II}$ we have that $Q_I > Q_{II}$.

20 $Q_I = \Delta U + W_I$ and $Q_{II} = \Delta U + W_{II}$. ΔU is the same for both. Since $W_I > W_{II}$ we have that $Q_I > Q_{II}$.

21 $Q_I = \Delta U + 0$ and $Q_{II} = \Delta U + W_{II}$. ΔU is the same for both. Since $W_{II} < 0$ (more work is done compressing than was done expanding) we have that $Q_I > Q_{II}$.

22 Let a quantity of heat Q_1 be supplied to an ideal gas at constant volume: then $Q_1 = nc_V\Delta T$. Similarly, supply a quantity of heat Q_2 to another ideal gas of the same mass at constant pressure so that the change in temperature will be the same: $Q_2 = nc_P\Delta T$. Q_1 and Q_2 will be different because in the second case some of the heat will go into doing work as the gas expands at constant pressure. From the first law of thermodynamics we have that $Q_1 = nc_V\Delta T = \Delta U + 0$ and $Q_2 = nc_P\Delta T = \Delta U + p\Delta V$. Subtracting these two equations gives $nc_P\Delta T - nc_V\Delta T = p\Delta V$. From the ideal gas law we find $p\Delta V = nR\Delta T$ and so $nc_P\Delta T - nc_V\Delta T = nR\Delta T$ OR $c_P - c_V = R$ as required.

23 **a** The work done is $W = P\Delta V = 3.0 \times 10^5 \times (1.8 - 1.4) \times 10^{-3} = 120\,\text{J}$. The change in internal energy is $\Delta U = \frac{3}{2}P\Delta V = \frac{3}{2} \times 120 = 180\,\text{J}$. Hence $Q = \Delta U + W = 120 + 180 = 300\,\text{J}$.

b $\Delta U = Q - W = (-490) - (-100) = -390\,\text{J}$.

c The area of the triangle is net work done, i.e. $W_{\text{net}} = 120 - 100 = 20\,\text{J}$.

d Area: $\frac{1}{2} \times (3.0 \times 10^5 - P_{\text{C}}) \times (1.8 - 1.4) \times 10^{-3} = 20 \Rightarrow P_{\text{C}} = 2.0 \times 10^5\,\text{Pa}$.

e The net work is equal to $W_{\text{net}} = Q_{\text{in}} - Q_{\text{out}}$ and so $20 = 300 - 490 + Q_{\text{AC}} \Rightarrow Q_{\text{AC}} = 210\,\text{J}$.

24 **a** $\frac{V_{\text{A}}}{T_{\text{A}}} = \frac{V_{\text{B}}}{T_{\text{B}}} \Rightarrow T_{\text{B}} = T_{\text{A}}\frac{V_{\text{B}}}{V_{\text{A}}} = 400 \times \frac{0.4}{0.1} = 1600\,\text{K}$

$\frac{P_{\text{A}}}{T_{\text{A}}} = \frac{P_{\text{D}}}{T_{\text{D}}} \Rightarrow T_{\text{D}} = T_{\text{A}}\frac{P_{\text{C}}}{P_{\text{A}}} = 400 \times \frac{2}{4} = 200\,\text{K}$

$\frac{V_{\text{C}}}{T_{\text{C}}} = \frac{V_{\text{B}}}{T_{\text{B}}} \Rightarrow T_{\text{C}} = T_{\text{B}}\frac{V_{\text{C}}}{V_{\text{B}}} = 200 \times \frac{0.4}{0.1} = 800\,\text{K}$.

b $\Delta U_{\text{AB}} = \frac{3}{2}P\Delta V = \frac{3}{2} \times 4.0 \times 10^5 \times 0.30 \times 10^{-3} = 180\,\text{J}$

$\Delta U_{\text{BC}} = \frac{3}{2}V\Delta P = -\frac{3}{2} \times 0.40 \times 10^{-3} \times 2.0 \times 10^5 = -120\,\text{J}$

$\Delta U_{\text{CD}} = \frac{3}{2}P\Delta V = -\frac{3}{2} \times 2.0 \times 10^5 \times 0.30 \times 10^{-3} = -90\,\text{J}$

$\Delta U_{\text{DA}} = \frac{3}{2}V\Delta P = \frac{3}{2} \times 0.10 \times 10^{-3} \times 2.0 \times 10^5 = 30\,\text{J}$.

(We can check that $\sum \Delta U = 0$.)

c $Q_{\text{AB}} = 180 + 4.0 \times 10^5 \times 0.30 \times 10^{-3} = 300\,\text{J}$

$Q_{\text{BC}} = -120\,\text{J}$

$Q_{\text{CD}} = -9.0 - 2.0 \times 10^5 \times 0.30 \times 10^{-3} = -150\,\text{J}$

$Q_{\text{DA}} = 30\,\text{J}$.

d The net work is $W_{\text{net}} = 2.0 \times 10^5 \times 0.30 \times 10^{-3} = 60\,\text{J}$. The heat into the system is 330 J so ratio is $\frac{60}{330} = 0.18$.

25 Heat has been taken out of the gas and so the entropy of the gas decreased. The heat extracted from the gas goes into the surroundings, increasing the entropy by the same amount as the decrease in entropy of the gas. This is an ideal reversible process and so the total entropy of the universe stayed the same, consistently with the second law.

26 No, because the conversion of thermal energy to mechanical work with 100% efficiency is forbidden for cyclic processes, and we don't have a cyclic process here.

27 The volume increases and so the number of microstates available also increases.

28 The temperature is reduced.

29 No, since all the heat extracted from within the refrigerator is put back into the room.

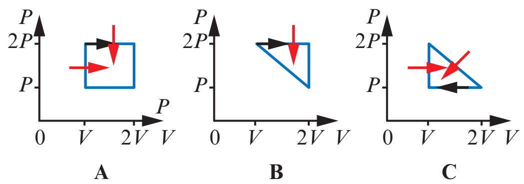
30 $\Delta S = \frac{Q}{T} = \frac{0.025 \times 334 \times 10^3}{273} = 31\,\text{J K}^{-1}$.

31 $\Delta S = \frac{50}{273} - \frac{50}{373} = 4.9 \times 10^{-2}\,\text{J K}^{-1}$.

32 a $\Delta S = \frac{Q}{T} = \frac{0.015 \times 334 \times 10^3}{273} = 18.4\,\text{J K}^{-1}$.

b The temperature of the water will not change much since there is lots of water. The heat the water gave to melt the ice and warm it to 20 °C is $0.015 \times 334 \times 10^3 + 0.015 \times 4200 \times 20 = 6.27 \times 10^3\,\text{J}$. So the entropy change of the water is $-\frac{6.27 \times 10^3}{293} = -21.4\,\text{J K}^{-1}$. The overall change is then $18.4 + 4.5 - 21.4 = +1.5\,\text{J K}^{-1}$.

33 The net work is the area of the loop so it is PV for A and $0.5PV$ for B and C. We need to find the heat into each engine.



A: $Q = 2PV + \frac{3}{2}2PV = 5PV$ and $Q = \frac{3}{2}PV$ for a total $Q_{in} = \frac{13}{2}PV$. Hence $e_A = \frac{PV}{\frac{13}{2}PV} = \frac{2}{13} = 0.15$.

B: $Q_{in} = 2PV + \frac{3}{2}2PV = 5PV$. Hence $e_B = \frac{0.5PV}{5PV} = \frac{1}{10} = 0.10$.

C: $Q = \frac{2P + P}{2}V = \frac{3}{2}PV$ and $Q = \frac{3}{2}PV$ for a total $Q_{in} = 3PV$. Hence $e_C = \frac{0.5PV}{3PV} = \frac{1}{6} = 0.17$.

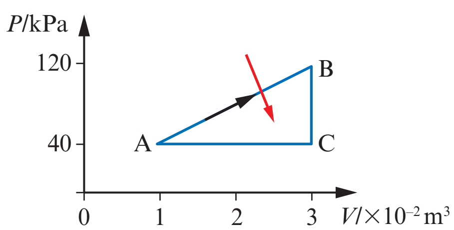
34 The Carnot efficiency for these temperatures is $e_C = 1 - \frac{300}{500} = 0.40$, so no.

35 A is impossible since it violates energy conservation.

B has an efficiency $\frac{60}{100} = 0.60$. A Carnot engine at these temperatures has an efficiency $1 - \frac{300}{600} = 0.50$ so B is impossible.

C is possible.

36 a A to B.



b The net work is the area of the cycle $W_{net} = \frac{1}{2} \times 2.0 \times 10^{-2} \times 80 \times 10^3 = 800\,\text{J}$. To find Q we need the work done and change of internal energy from A to B: $W_{AB} = \frac{40 + 120}{2} \times 10^3 \times 2.0 \times 10^{-2} = 1600\,\text{J}$. If the temperature is T at A, it is $9T$ at B. The change in internal energy is then $\Delta U_{AB} = \frac{3}{2}Rn\Delta T = \frac{3}{2}Rn(9T - T) = 12RnT = 12P_AV_A = 12 \times 40 \times 10^3 \times 1.0 \times 10^{-2} = 4800\,\text{J}$. So $Q_{AB} = 1600 + 4800 = 6400$ J. Finally, $e = \frac{800}{1600} = \frac{1}{8}$

37 a The entropy of the universe always increases.

b The Carnot efficiency always has efficiency less than 1, and no real engine can have a larger efficiency at the same operating temperatures. Hence any real engine has efficiency less than 1 and so must reject heat.

38 a $Q = W + \Delta U = 900 + 0 = 900\,\text{J}$. Hence $\Delta S = \frac{Q}{T} = \frac{900}{300} = 3.0\,\text{J K}^{-1}$.

b For the entire cycle $\Delta S = 0$ and $\Delta S = 0$ for the adiabatic compression since $Q = 0$. Hence $\Delta S = -3.0\,\text{J K}^{-1}$ for the isovolumetric change.

39 a An adiabatic curve represents a change in a gas in which no heat enters or leaves the gas.

b BC is isobaric and DA is isovolumetric.

c BC

d i $Q = \Delta U + W = \Delta U + 0$; $\Delta U = -\frac{3}{2} V\Delta P = -\frac{3}{2} \times 8.6 \times 10^{-3} \times 2.6 \times 10^{6} = -3.35 \times 10^{4}\,\text{J}$.

ii $e = \frac{W}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$. So $0.36 = 1 - \frac{3.35 \times 10^{4}}{Q_{in}} \Rightarrow Q_{in} = 5.23 \times 10^{4}\,\text{J}$.

iii This the net work done, i.e. $eQ_{in} = 0.36 \times 5.23 \times 10^{4} = 1.88 \times 10^{4}\,\text{J}$.

40 a A steeper curve.

b In the adiabatic case since there is more area under the adiabatic curve.

c $Q = W + \Delta U = 25 + 0 = -25\,\text{kJ}$. Hence $\Delta S = \frac{Q}{T} = -\frac{25 \times 10^{3}}{300} = -83\,\text{J K}^{-1}$.

41 a $e = \frac{W}{Q_{in}}$ so $0.25 = \frac{300}{Q_{in}} \Rightarrow Q_{in} = 1200\,\text{J}$.

b 1200 – 300 = 900 J.

42 a $e = \frac{W}{Q_{in}}$ so $0.30 = \frac{W}{900} \Rightarrow W = 270\,\text{J}$ so $Q_{out} = 900 - 270 = 630$ J.

b $e = 1 - \frac{T_C}{T_H}$ so $0.30 = 1 - \frac{T_C}{600} \Rightarrow T_C = 420\,\text{K}$.

43 a Zero.

b The entropy of the surroundings increases by the same amount because heat is removed from the gas to the surroundings.

c So that there is a temperature difference for the heat to move from the gas to the surroundings.

d Since the temperature difference is infinitesimally small, it will take a very long time for the heat to be removed and still maintain the infinitesimally small temperature difference.

Chapter 11

Test your understanding

1 $W = q\Delta V \Rightarrow \Delta V = \frac{W}{q} = \frac{240}{5.0} = 48$ V.

2 $\frac{E_\alpha}{E_p} = \frac{q_\alpha \Delta V}{q_p \Delta V} = 2$

$E_K = \frac{1}{2}mv^2$ so $\frac{E_\alpha}{E_p} = 2 = \frac{\frac{1}{2}m_\alpha v_\alpha^2}{\frac{1}{2}m_p v_p^2} \Rightarrow 2 = \frac{m_\alpha v_\alpha^2}{m_p v_p^2} \Rightarrow \frac{v_\alpha^2}{v_p^2} = 2 \times \frac{m_p}{m_\alpha}$. Hence $\frac{v_\alpha}{v_p} = \sqrt{2 \times \frac{1}{4}} = \frac{1}{\sqrt{2}}$.

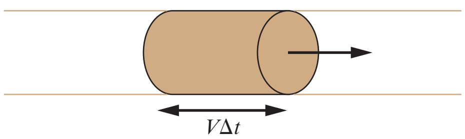
3 The charge that goes through is $Q = It = 10$ C. The number of electrons is then $\frac{10}{1.6 \times 10^{-19}} = 6.25 \times 10^{19} \approx 6 \times 10^{19}$.

4 The charge in 1 s is I coulomb. Hence the number of electrons is $\frac{I}{e}$.

5 The resistance is constant and so $\frac{6.0}{1.5} = \frac{V}{3.5} \Rightarrow V = 14$ V.

6 The resistance is constant and so $\frac{240}{0.80} = \frac{160}{I} \Rightarrow I = 0.53$ A.

7 **a** The electrons in the conductor which will be able to go through the cross-sectional area of the conductor within a time Δt are only those within the shaded cylinder of height $v\,\Delta t$.



The number of these electrons is then $nAv\Delta t$ and so the total charge going through the cross-sectional area in time Δt is $enAv\Delta t$. The current is then $I = \frac{enAv\Delta t}{\Delta t} = nAve$.

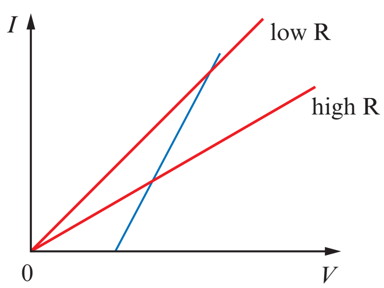
b $I = 10^{28} \times \pi \times (0.002)^2 \times v \times 1.6 \times 10^{-19} \Rightarrow v \approx 5 \times 10^{-5}\ \text{m s}^{-1} \approx 0.05\ \text{mm s}^{-1}$.

c The current is the same at A and B and from $I = nAve$, since the area is greater at B, the speed at B will be less.

8 **a** Electrons collide with the atoms of the metal and transfer kinetic energy to them. The atoms vibrate about their equilibrium positions with increased average kinetic energy. Since the average kinetic energy is proportional to the kelvin temperature, the temperature of the metal increases.

b The increased vibration of the atoms impedes the motion of electrons by decreasing their drift speed. From $I = nAve$, a decreased speed means a decreased current and so from $R = \frac{V}{I}$ an increased resistance.

9 As the voltage increases the resistance decreases so it is not ohmic. We can see why resistance decreases with the help of the graphs for two ohmic resistors (red lines). The steeper line has lower resistance.



10 The resistance decreases, and since the current stays the same the voltmeter reading will decrease.

11 When it is turned on, the filament is cold and so its resistance is low. Hence it takes a lot of current initially.

12 **a** It does, because we have straight line graphs through the origin implying a constant resistance at a fixed temperature.

b Wire B has higher resistance so it corresponds to a higher temperature.

13 $R = \rho\frac{L}{A}$. The new wire will have the same volume so if the length is doubled the cross-sectional area is halved. Thus, $R' = \rho\frac{2L}{\frac{A}{2}} = 4\rho\frac{L}{A} = 4R$.

14 We will have the smallest current for the largest resistance.

Across A and the opposite face: $R = \rho\frac{0.10}{0.40 \times 0.20} = 1.25\rho$.

Across B and the opposite face: $R = \rho\frac{0.40}{0.10 \times 0.20} = 20\rho$.

Across C and the opposite face: $R = \rho\frac{0.20}{0.40 \times 0.10} = 5\rho$.

So, across B and the opposite face.

15 $R = \rho\frac{L}{A} = \frac{2.5 \times 10^{-8} \times 2.2}{\pi \times (0.25 \times 10^{-3})^2} = 0.28\ \Omega$

16 $R = \rho\frac{L}{A} = \rho\frac{L}{\pi r^2}$ so $R' = \rho\frac{3L}{\pi(3r)^2} = \frac{1}{3}\rho\frac{L}{\pi r^2} = \frac{1}{3}R$.

17 $R = \rho\frac{L}{A} \Rightarrow L = \frac{RA}{\rho} = \frac{R\pi r^2}{\rho} = \frac{5.0 \times \pi \times (0.30 \times 10^{-3})^2}{1.7 \times 10^{-8}} = 83$ m.

18 $R = \rho\frac{L}{A} \Rightarrow A = \frac{\rho L}{R} = \pi r^2 \Rightarrow r = \sqrt{\frac{\rho L}{\pi R}} = \sqrt{\frac{5.6 \times 10^{-8} \times 3.5 \times 10^3}{\pi \times 4.0}} \approx 3.9$ mm.

19 $R = \rho\frac{L}{A} = \rho\frac{L}{\pi r^2}$. So $\rho\frac{L_X}{\pi r_X^2} = \rho\frac{L_Y}{\pi r_Y^2}$ and thus $\frac{r_X}{r_Y} = \sqrt{\frac{L_X}{L_Y}} = \sqrt{2}$.

20 **a** $R = \rho\frac{L}{A} \Rightarrow \frac{R}{L} = \frac{\rho}{A} = \frac{\rho}{\pi r^2} = \frac{1.7 \times 10^{-8}}{\pi \times (4.0 \times 10^{-3})^2} = 3.38 \times 10^{-4}\ \Omega\ \text{m}^{-1}$.

b $P = RI^2 = 150 \times 10^3 \times 3.38 \times 10^{-4} \times 7.5^2 = 2.85 \times 10^3$ W.

c $V = RI = 150 \times 10^3 \times 3.38 \times 10^{-4} \times 7.5 = 3.8 \times 10^2$ V.

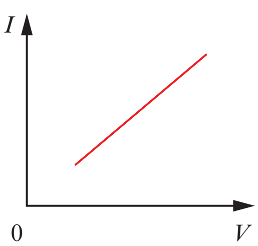
21 $P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = \frac{230^2}{1.1 \times 10^3} \approx 48\ \Omega$.

22 **a** $P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P} = \frac{120^2}{60} = 240\ \Omega$.

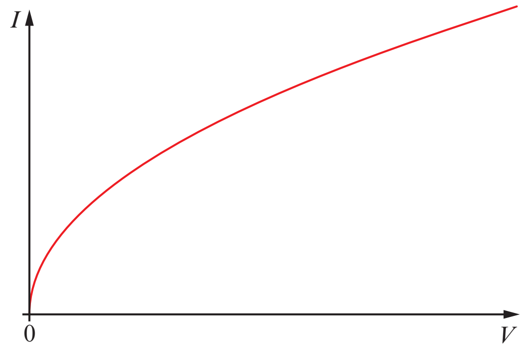
b $R = \rho\frac{L}{A} \Rightarrow L = \frac{RA}{\rho} = \frac{R\pi r^2}{\rho} = \frac{240 \times \pi \times (0.042 \times 10^{-3})^2}{3.6 \times 10^{-7}} = 3.7$ m.

c It is coiled.

23



24



25 **a** 8.0 V and 12 V.

b 0 and 20 V.

26 $P = VI \Rightarrow I = \frac{P}{V} = \frac{60}{120} = 0.5$ A.

27 $P = \frac{V^2}{R} \Rightarrow \frac{P}{12} = \frac{\frac{9.0^2}{R}}{\frac{6.0^2}{R}} = \frac{9.0^2}{6.0^2} \Rightarrow P = 12 \times \frac{9.0^2}{6.0^2} = 27$ W.

28 $P = \frac{V^2}{R} \Rightarrow P' = \frac{V^2}{\frac{R}{2}} = \frac{2V^2}{R} = 2P = 2 \times 4.0 = 8.0$ W.

29 $P = \frac{V^2}{R} \Rightarrow P' = \frac{(2V)^2}{2R} = \frac{4V^2}{2R} = 2P = 2 \times 32 = 64$ W.

30 $P = \varepsilon I = 6.0 \times 2.5 = 15$ W.

31 $\varepsilon = \frac{W}{Q} = \frac{48}{12} = 4.0$ V.

32 Each piece will have resistance $\frac{R}{2}$. In parallel: $\frac{1}{\frac{R}{2}} + \frac{1}{\frac{R}{2}} = \frac{2}{R} + \frac{2}{R} = \frac{4}{R} = \frac{1}{R_T} \Rightarrow R_T = \frac{R}{4}$.

33 The pieces will have resistance $\frac{R}{3}$ and $\frac{2R}{3}$. In parallel: $\frac{1}{\frac{R}{3}} + \frac{1}{\frac{2R}{3}} = \frac{3}{R} + \frac{3}{2R} = \frac{9}{2R} = \frac{1}{R_T} \Rightarrow R_T = \frac{2R}{9}$.

34 **a** The top line gives 8.0 Ω and the lower line 4.0 Ω for a total of 2.7 Ω.

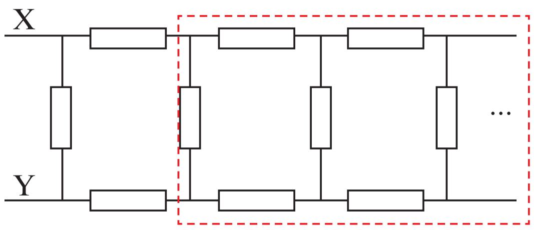
b The parallel connection gives 2.4 Ω and so the total is 12.4 Ω.

c 1.0 Ω

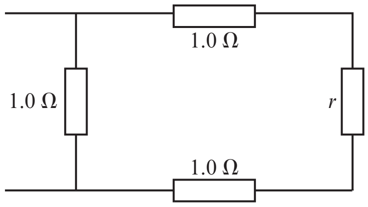
35 **a** The middle block has resistance $\frac{R}{2}$ and the third resistance $\frac{R}{3}$. Hence the total is $R + \frac{R}{2} + \frac{R}{3} = \frac{6R}{6} + \frac{3R}{6} + \frac{2R}{6} = \frac{11R}{6}$. This is 11 Ω and so $R = 6.0$ Ω.

b The total resistance would be $R\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots\right)$, and this series does not have a finite sum.

36 Let us call the total resistance r. But because the system extends forever the resistors in the dotted box also have a total resistance r.



So we have the connection:



Hence

$\frac{1}{r} = \frac{1}{1} + \frac{1}{2+r}$, from which we get $r^2 + 2r - 2 = 0$ and so $r = -1 + \sqrt{3}\ \Omega$.

37 $\frac{1}{R_T} = \frac{1}{3.0 + 2.0 + 1.0} + \frac{1}{3.0} = \frac{1}{6.0} + \frac{1}{3.0} = \frac{1}{2.0}$ so $R_T = 2.0\ \Omega$.

38 The current is $I = \frac{\varepsilon}{R_1 + R_2}$. Then, $V_1 = R_1 I = \frac{\varepsilon R_1}{R_1 + R_2}$ and similarly, $V_2 = R_2 I = \frac{\varepsilon R_2}{R_1 + R_2}$.

39 The total resistance of the parallel combination is 3.0 kΩ and so the total for the circuit is 9.0 kΩ. The current leaving the cell is then 1.0 mA. Each of the parallel resistors gets 0.50 mA, and the other resistor gets 1.0 mA.

40 Reduce R_1 and increase R_2.

41 The total resistance will increase and so the current leaving the cell will decrease. Thus the voltage across L decreases.

42 The total resistance of the circuit will decrease and so the current leaving the cell will increase. Hence Z will be brighter.

43 **a** With the switch open the total resistance is $2R$. The total power is then $P = \frac{\varepsilon^2}{2R}$. With the switch closed the total resistance is $\frac{3R}{2}$ and so the total power is $P' = \frac{2\varepsilon^2}{3R} = \frac{4}{3} \times \frac{\varepsilon^2}{2R} = \frac{4}{3}P$.

b The voltage across X and Y was $\frac{\varepsilon}{2}$. But with the switch closed, the voltage across X is $\frac{2\varepsilon}{3}$ and across Y it is $\frac{\varepsilon}{3}$. So the brightness of X will increase and that of Y will decrease.

44 The voltage across the 8.0 Ω and 2.0 Ω resistors is the same. The same is true for the 4.0 Ω resistor and X. No current goes through the voltmeter and so the 8.0 Ω and 4.0 Ω resistors take the same current x and the 2.0 Ω resistor and X take the same current y. Then $8.0x = 2.0y \Rightarrow y = 4x$. And $4.0x = Ry$. Hence $4.0x = R \times 4x \Rightarrow R = 1.0\ \Omega$.

45 In a the power is $P = \frac{\varepsilon^2}{R}$. In b it is $P_B = \frac{\varepsilon^2}{2R} = \frac{P}{2}$. In c it is $P_C = \frac{\varepsilon^2}{\frac{R}{2}} = \frac{2\varepsilon^2}{R} = 2P$.

46 **a** The voltage across each resistor is 1.5 V, and from the graph we have a current 1.55 A in Y and 2.65 A in X for a total current of about 4.2 A.

b X and Y take the same current and their voltages must add up to 1.5 V. A current of 1.1 A means a voltage of 0.5 V for X and 1.0 V for Y, which works.

47 **a** The voltage across X is 1.2 V so the voltage across Y is 3.0 – 1.2 = 1.8 V. From the graph, the current in Y is about 0.117 A.

b The current in Y is about 0.117 A so the current in R is 0.033 A. Since the voltage is 1.8 V the resistance is $R = \frac{1.8}{0.033} \approx 55\ \Omega$.

c The total resistance will be at least 55 Ω, greater than before and so the current leaving the cell will decrease.

d The resistance of X decreases as the current and voltage are reduced. Since the current in X will now be smaller, the resistance of X will decrease.

48 a $I = \frac{\varepsilon}{R + r}$ i.e. $1.0 = \frac{6.0}{5.0 + r} \Rightarrow r = 1.0\ \Omega$.

b $V = \varepsilon - Ir = 6.0 - 1.0 \times 1.0 = 5.0$ V.

49 a $V = \varepsilon - Ir$ so

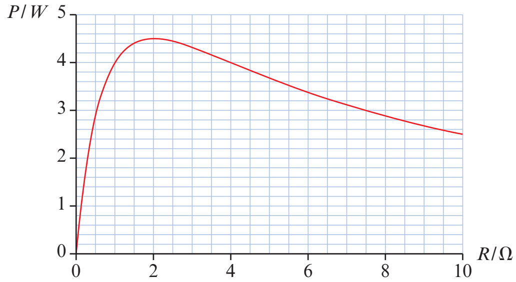
$5.8 = \varepsilon - 0.50r$

$5.7 = \varepsilon - 0.60r$

Subtracting, $0.10 = 0.10r$ so $r = 1.0\ \Omega$.

b Then, $\varepsilon = 6.3$ V.

50 a The power dissipated in the external resistor is $P = RI^2$ and $I = \frac{\varepsilon}{R + r} = \frac{6.0}{R + 2.0}$ so that $P = R\left(\frac{6.0}{R + 2.0}\right)^2$. Graphing this gives:



This shows a maximum power at $R = 2.0\ \Omega$.

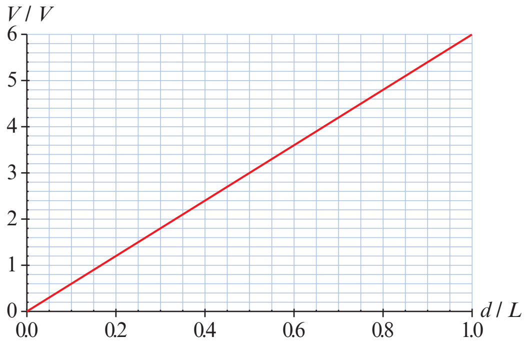
b The maximum power is read from the graph to be 4.5 W.

51 a The total resistance of the circuit would decrease, thus the reading of the ammeter would increase. The voltmeter reads $V = \varepsilon - Ir$ and since the current decreases, V will increase.

b The current would still decrease. The voltmeter would read the emf before and after the closing of the switch so there would be no change.

52 a **i** 0, **ii** 6.0 V.

b L is the length XY:



53 a When the variable resistor is set to 0 Ω, the voltmeter reads 0. When set at 6.0 Ω it is 6.0 V; this is because the voltage across the variable resistor would be double that across the 3.0 Ω resistor, and their sum has to be 9.0 V so 6.0 V and 3.0 V. The range is 0–6 V.

b When the variable resistor is set to 0 Ω, the voltmeter still reads 0. When set at 6.0 Ω the total resistance is 10 Ω and the current is $I = \frac{9.0}{10} = 0.90$ A. The voltmeter would then read $V = RI = 6.0 \times 0.9 = 5.4$ V, so a range of 0–5.4 V.

54 **a** The voltage drop across the connecting wires is $RI = 16 \times 25 = 400$ V. Hence the emf of the source must be $\varepsilon = 400 + 120 = 520$ V.

b The total power produced is $\varepsilon I = 520 \times 25 = 13$ kW. The power wasted in the wires is $RI^2 = 16 \times 25^2 = 10$ kW. The fraction of wasted power is $\frac{10}{13} = 0.77$.

55 **a** A: 6.0 V; B: 0 V.

b A: $I = \frac{6.0}{3.0 + 2.0} = 1.2$ A. $V = \varepsilon - Ir = 6.0 - 1.2 \times 2.0 = 3.6$ V. B shows the same: $V = IR = 1.2 \times 3.0 = 3.6$ V.

Chapter 12

Test your understanding

1 An oscillation is any motion in which the displacement of a particle from a fixed point keeps changing direction and there is a periodicity in the motion, i.e. the motion repeats in some way. In simple harmonic motion, the displacement from an equilibrium position and the acceleration are proportional and opposite to each other.

2 It is an oscillation since we may define the displacement of the particle from the middle point and in that case the displacement changes direction and the motion repeats. The motion is not simple harmonic, however, since there is no acceleration that is proportional (and opposite) to the displacement.

3 It is an oscillation since the motion repeats. The motion is not simple harmonic however since the acceleration is constant and is not proportional (and opposite) to the displacement.

4 a The acceleration is opposite to the displacement so every time the particle is displaced there is a force towards the equilibrium position.

b The acceleration is not proportional to the displacement; if it were the graph would be a straight line through the origin.

c For small amplitudes the graph is essentially a straight line through the origin with negative slope so acceleration is opposite and proportional to displacement so oscillations are approximately simple harmonic.

5 $T = 2\pi\sqrt{\frac{L}{g}}$ hence $L = \frac{gT^2}{4\pi^2} = \frac{9.81 \times 1}{4\pi^2} = 0.248$ m.

6 From $T = 2\pi\sqrt{\frac{L}{g}}$ we find $\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta L}{L}$ so period increases by 2.0%.

7 $T = 2\pi\sqrt{\frac{m}{k}}$ hence $m = \frac{kT^2}{4\pi^2} = \frac{240 \times 1.2^2}{4\pi^2} = 8.8$ kg.

8 By attaching herself to a spring, measuring the period of oscillation and using $T = 2\pi\sqrt{\frac{m}{k}}$.

9 a $\sqrt{L}$

b $\frac{2\pi}{\sqrt{g}}$

c L

d $\frac{4\pi^2}{g}$

10 a π

b The period is 2.0 s so $\omega = \frac{2\pi}{T} = \frac{2\pi}{2.0} = \pi \approx 3.1\ \text{rad s}^{-1}$.

c In SHM $a = -\omega^2 x$ so $a = -\pi^2 \times 0.040 \approx -0.39\ \text{m s}^{-2}$.

11 a i 0.5 s, 1.5 s, 2.5 s

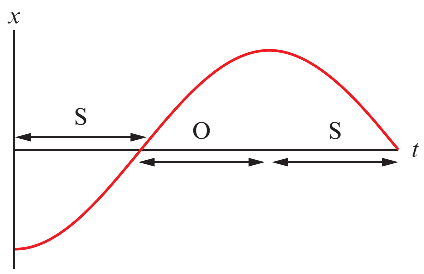
ii 0, 2 s

iii 0, 1 s, 2 s

iv 1.5 s

b The period is 2.0 s so $f = \frac{1}{T} = 0.50$ Hz and $\omega = \frac{2\pi}{T} = \frac{2\pi}{2.0} = \pi \approx 3.1\ \text{rads}^{-1}$.

12



13 a The amplitude is the area from $t = 0$ to $t = 0.3$ s. This is approximately (counting squares) 2 cm.

b M: 0.9 s

c E: 0, 0.6 s, 1.2 s

d Negative: displacement is the area under the curve from $t = 0$ to $t = 1.0$ s and is negative.

14 a The acceleration is proportional and opposite to the displacement because the graph is a straight line through the origin with negative gradient.

b V: at the origin.

c The gradient is $-\omega^2$. The gradient of the graph is $\frac{-0.10 - 0.10}{0.5 - (-0.5)} \frac{\text{m s}^{-2}}{\text{m}} = -0.20\ \text{s}^{-2}$. So $-\omega^2 = -0.20\ \text{s}^{-2}$ hence $\omega = 0.447\ \text{s}^{-1}$. The period is then $T = \frac{2\pi}{\omega} = 14$ s.

d The acceleration is $+\ 0.04\ \text{m s}^{-2}$ and so the restoring force is $0.15 \times 0.04 = 6.0$ mN.

e The only change is that the graph would extend from $x = -0.25$ m to $x = +0.25$ m.

15 a In all cases the acceleration is not proportional and opposite to the displacement.

b For small x (small amplitude) $F = -k^2 \sin x \approx -k^2 x$, $F = k^2 \frac{x}{x-1} \approx -k^2 x$ and $F = (1 - e^{k^2 x}) \approx -k^2 x$ so these are approximately SHM.

16 a When the block is displaced by a distance x to the right, the net force is $2kx$ to the left. This means $ma = -2kx$ OR $a = -\frac{2k}{m}x$. So we have SHM with $\omega^2 = \frac{2k}{m}$. Hence, $T = 2\pi\sqrt{\frac{m}{2k}} = 2\pi\sqrt{\frac{2.0}{240}} = 0.57$ s.

b When the block is displaced by a distance x to the right, the left spring exerts a force kx to the left and the right spring also exerts kx to the left, so nothing changes.

17 a $a_{\text{max}} = \omega^2 x_0$ and $v_{\text{max}} = \omega x_0$ and so $\frac{a_{\text{max}}}{v_{\text{max}}} = \omega$. Thus $\omega = 4.0\ \text{s}^{-1}$. Therefore $T = \frac{2\pi}{\omega} = \frac{2\pi}{4.0} = 1.6$ s.

b $x_0 = \frac{v_{\text{max}}}{\omega} = \frac{12}{4.0} = 3.0$ m.

18 a $\sin(\omega t + \phi) = \frac{x}{x_0}$ and $\cos(\omega t + \phi) = \frac{v}{\omega x_0}$ so that $\left(\frac{x}{x_0}\right)^2 + \left(\frac{v}{\omega x_0}\right)^2 = 1$. Simplifying:

$$\frac{x^2}{x_0^2} + \frac{v^2}{\omega^2 x_0^2} = 1$$

$$\frac{\omega^2 x^2}{\omega^2 x_0^2} + \frac{v^2}{\omega^2 x_0^2} = 1$$

$$\omega^2 x^2 + v^2 = \omega^2 x_0^2$$

$$v^2 = \omega^2 x_0^2 - \omega^2 x^2 = \omega^2 (x_0^2 - x^2)$$

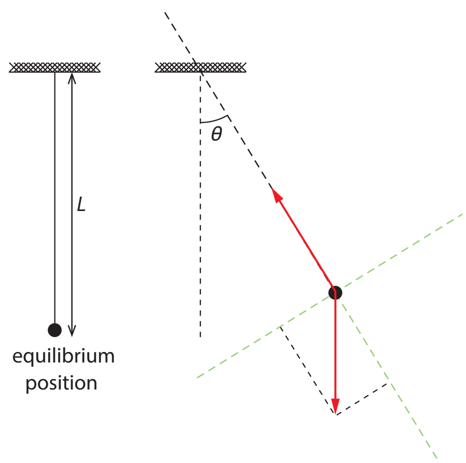
b The speed is $\omega\sqrt{\left(x_0^2 - \frac{x_0^2}{4}\right)} = \omega\sqrt{\frac{3x_0^2}{4}} = \underbrace{(\omega x_0)}_{v_{\text{max}}}\frac{\sqrt{3}}{2} = v\frac{\sqrt{3}}{2}$.

19 a When displaced downwards by a distance x the net force on the block is $B - Mg = \rho V_{imm} g - Mg = \rho A(h + x)g - Mg$.

At equilibrium $Mg = \rho Ahg$ and so the net force simplifies to ρAgx. The net force is directed opposite to the displacement and so $Ma = -\rho Agx$ OR $a = -\frac{\rho Ag}{M}x$, which is the defining relation for SHM with $\omega^2 = \frac{\rho Ag}{M}$.

b The period is then $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{M}{\rho g A}}$.

20 a



b The force bringing the bob back to equilibrium is $mg\sin\theta$ directed opposite to the displacement hence $a = -g\sin\theta$.

c $a = -g\sin\theta = -g\sin\left(\frac{x}{L}\right)$. For small angles, $\sin\theta \approx \theta$ so $a = -g\sin\left(\frac{x}{L}\right) \approx -\frac{g}{L}x$, and we have the defining relation for SHM with $\omega^2 = \frac{g}{L}$.

d Hence $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$.

21 a $v_{max} = \omega x_0 = \frac{2\pi}{0.25} \times 0.050 = 1.3\ \text{m s}^{-1}$

b $a_{max} = \omega^2 x_0 = \left(\frac{2\pi}{0.25}\right)^2 \times 0.050 = 32\ \text{m s}^{-2}$

22 $\omega = \frac{2}{3}\ \text{s}^{-1}$, $T = \frac{2\pi}{\omega} = 9.4\ \text{s}$

23 a 0

b 1.0 s

c 4.76 mm

d On your graphics calculator plot $x = 5\sin(2\pi t)$ and $x = -2$ and see that they intersect at $t = 0.57$ s.

24 a We see that $\omega = \frac{\pi}{2}$ and then use $v = \pm\omega\sqrt{x_0^2 - x^2} = \pm\frac{\pi}{2} \times \sqrt{(0.12^2 - 0.06^2)} = \pm 0.16\ \text{m s}^{-1}$.

b The acceleration is $a = -\omega^2 x = -\left(\frac{\pi}{2}\right)^2 \times 0.06 = -0.15\ \text{m s}^{-2}$.

25 a $x = 8.0\cos(2\pi \times 14t) = 8\cos(28\pi t)$ OR $x = 8\sin\left(28\pi t + \frac{\pi}{2}\right)$.

b At $t = 0.025$ s, $x = 8.0\cos(28\pi \times 0.025) = -4.702 \approx -4.7$ cm; the velocity is given by $v = 28\pi \times 8\cos\left(28\pi t + \frac{\pi}{2}\right) = 28\pi \times 8\cos\left(28\pi \times 0.025 + \frac{\pi}{2}\right) = -5.7\ \text{m s}^{-1}$.

$a = -\omega^2 x = -(28\pi)^2 \times (-4.7) = 364\ \text{m s}^{-2}$

26 $\omega = 2\pi f = 2\pi \times 460 = 2890\ \text{s}^{-1}$. So $v_{max} = \omega x_0 = 2890 \times 5.0 \times 10^{-3} = 14.5\ \text{m s}^{-1}$

$a_{max} = \omega^2 x_0 = (2890)^2 \times 5.0 \times 10^{-3} = 4.2 \times 10^4\ \text{m s}^{-2}$

27 **a** The displacement can be written as $y = (6 \sin(\pi x)) \sin\left(1040\pi t + \frac{\pi}{2}\right)$ which is the standard equation of SHM with $\omega = 1040\pi\ \text{s}^{-1}$, phase angle $\frac{\pi}{2}$ and an amplitude that varies with position $6 \sin(\pi x)$. The frequency is 520 Hz.

b Answered in **a**.

c The maximum value of $6 \sin(\pi x)$ is 6.0 mm.

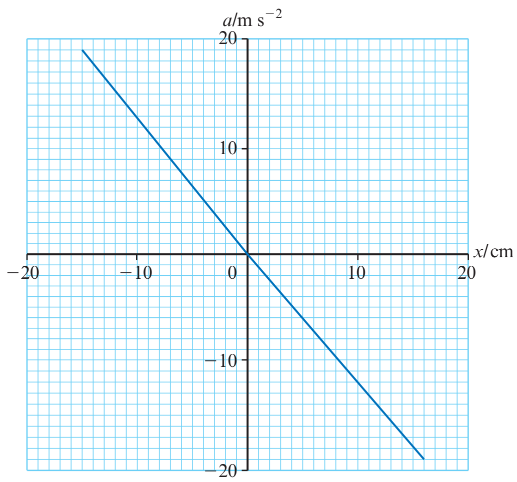
d At $x = L$, $y = 0$. So $\sin(\pi L) = 0$ hence $\pi L = \pi$, so $L = 1$ m.

e When $x = 0.25$ m the amplitude is $6 \sin(\pi \times 0.25) = 4.2$ mm.

28 **a** The area is approximately 0.50 cm (the exact value is 0.51 cm).

b This is the displacement from when the velocity is zero to when it is zero again, i.e. from one extreme position until the other, i.e. twice the amplitude.

29 We need to graph the equation $a = -\omega^2 x$ where $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.5} = 12.57\ \text{s}^{-1}$.



30 **a** The block is likely to lose contact at the highest point in the oscillation. At that point the acceleration is directed downwards and the net force is $mg - N = ma$. Since we have simple harmonic motion $a = \omega^2 x = 4\pi^2 f^2 x$ in magnitude, and the largest acceleration is obtained when $x = x_0$, the amplitude of the oscillation. The frequency is 5.0 Hz. The critical point is when $N = 0$, i.e. $g = 4\pi^2 f^2 x_0$ and so $x_0 = \frac{g}{4\pi^2 f^2} = \frac{9.8}{4\pi^2 \times 25} = 0.0099$ m. The amplitude must not exceed this value.

b At the lowest point:

$N - mg = ma = m4\pi^2 f^2 A \Rightarrow N = mg + m4\pi^2 f^2 A$

$N = 0.120 \times 9.8 + 0.120 \times 4 \times \pi^2 \times 25 \times 0.0099$

$N = 2.35$ N

31 $x = x_0 \sin(\omega t + \phi)$. At $t = 0$, $0.5 = 1.0 \sin\phi$ so $\phi = \frac{\pi}{6}$ OR $\phi = \frac{5\pi}{6}$. To see which is which notice that at approximately $t = 0.4$ s the top graph has x about zero. Indeed $x = x_0 \sin\left(2\pi \times 0.4 + \frac{\pi}{6}\right) \approx 0$ and $x = x_0 \sin\left(2\pi \times 0.4 + \frac{5\pi}{6}\right) \approx -0.9$ so $\phi = \frac{\pi}{6}$ goes with the top graph and $\phi = \frac{5\pi}{6}$ with the bottom. But see the next problem.

32 $x = x_0 \sin(\omega t + \phi)$ and $v = \omega x_0 \cos(\omega t + \phi)$. Then $x(0) = x_0 \sin\phi$ and $v(0) = \omega x_0 \cos\phi$. Dividing side by side we get

$$\frac{x_0 \sin\phi}{\omega x_0 \cos\phi} = \frac{x(0)}{v(0)}$$

$$\tan\phi = \frac{\omega x(0)}{v(0)}$$

This helps resolve the ambiguity of the previous problem. Notice the top graph of the previous problem has positive initial velocity and so $\tan\phi > 0$ implying the angle is in the first quadrant, i.e. $\phi = \frac{\pi}{6}$.

33 After time t the angular position of the particle will increase by ωt. Then $x = R\cos(\omega t + \phi)$ and $y = R\sin(\omega t + \phi)$. These are the equations of SHM.

We may rewrite these as

$$x = R\cos(\omega t + \phi) = R\sin\left(\omega t + \phi + \frac{\pi}{2}\right)$$

$$y = R\sin(\omega t + \phi)$$

which tells us that the phase angle for y is ϕ and that for x is $\phi + \frac{\pi}{2}$. The *phase difference* between the two motions is then $\phi + \frac{\pi}{2} - \phi = \frac{\pi}{2}$.

34 **a** The acceleration is proportional and opposite to the displacement because the graph is a straight line through the origin with negative gradient.

b The gradient is $-\omega^2$. The gradient of the graph is $\frac{-1.5 - 1.5}{0.10 - (-0.10)} = -15\ \text{s}^{-2}$. So $-\omega^2 = -15\ \text{s}^{-2}$ hence $\omega = 3.87\ \text{s}^{-1}$. The period is then $T = \frac{2\pi}{\omega} = 1.6$ s.

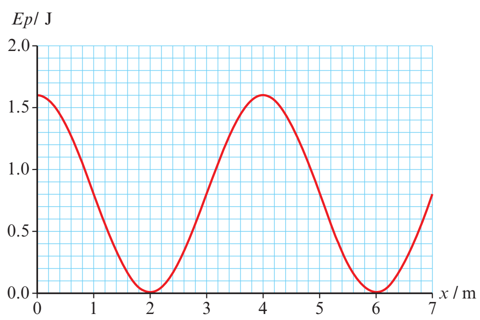
c $v_{max} = \omega x_0 = 3.87 \times 0.10 = 0.39\ \text{m s}^{-1}$

d $F_{max} = ma_{max} = m\omega^2 x_0 = 0.150 \times 15 \times 0.10 = 0.22$ N

e $E_T = \frac{1}{2}m\omega^2 x_0^2 = \frac{1}{2} \times 0.150 \times 15 \times 0.10^2 = 0.011$ J

35 **a** 8.0 s

b



c **i** 0.8 J **ii** 0

36 a $E_P = \frac{1}{2}m\omega^2x^2 = \frac{1}{2} \times 0.12 \times \left(\frac{2\pi}{2.0}\right)^2 \times 0.09^2 = 4.8\text{ mJ}$

b $E_K = \frac{1}{2}m\omega^2(x_0^2 - x^2) = \frac{1}{2} \times 0.12 \times \left(\frac{2\pi}{2.0}\right)^2 \times (0.15^2 - 0.06^2) = 11\text{ mJ}$

37 $E_P = \frac{1}{2}m\omega^2x^2 = \left(\frac{1}{2}m\omega^2x_0^2\right)\frac{x^2}{x_0^2} = 18 \times \left(\frac{8.0}{12}\right)^2 = 8.0\text{ J}$

38 $E_K = \frac{1}{2}m\omega^2(x_0^2 - x^2) = \left(\frac{1}{2}m\omega^2x_0^2\right)\left(1 - \frac{x^2}{x_0^2}\right) = 12 \times \left(1 - \left(\frac{4.0}{8.0}\right)^2\right) = 9.0\text{ J}$

39 $E_K = \frac{1}{2}m\omega^2(x_0^2 - x^2) = \left(\frac{1}{2}m\omega^2x_0^2\right)\left(1 - \frac{x^2}{x_0^2}\right)$

$6.0 = 8.0 \times \left(1 - \frac{x^2}{x_0^2}\right) \Rightarrow 1 - \frac{x^2}{x_0^2} = \frac{3}{4} \Rightarrow \frac{x^2}{x_0^2} = \frac{1}{4} \Rightarrow \frac{x}{x_0} = \frac{1}{2} \Rightarrow x = 2.5\text{ cm}$

40 $v = \omega\sqrt{x_0^2 - x^2} \Rightarrow 0.18 = \frac{2\pi}{1.5}\sqrt{x_0^2 - 0.04^2}$ Solving, $x_0 = 5.9$ cm.

41 $\frac{1}{2}m\omega^2(x_0^2 - x^2) = \frac{1}{2}m\omega^2x^2$

$x_0^2 - x^2 = x^2$

$2x^2 = x_0^2 \Rightarrow x = \frac{x_0}{\sqrt{2}} = \frac{4.2}{\sqrt{2}} = 3.0\text{ cm.}$

Chapter 13

Test your understanding

1 The delay time between you seeing the person next to you stand up and you standing up and the number density of the people, i.e. how many people per unit meter. For a fixed delay time, the closer the people are the faster the wave.

2 There is a disturbance that travels through the line of dominoes just as a disturbance travels through a medium when a wave is present. You can increase the speed by placing them closer together. An experiment to investigate this might be to place a number of dominoes on a line of fixed length such that the dominoes are a fixed distance d apart. We must give the same initial push to the first domino (for example using a pendulum that is released from a fixed height and strikes the domino at the same place). We then measure time from when the first domino is hit until the last one is hit. Dividing the fixed distance by the time taken gives the speed of the pulse. We can then repeat with a different domino separation and see how the speed depends on the separation d.

3 See textbook.

4 $c = f\lambda \Rightarrow f = \frac{c}{\lambda} = \frac{3 \times 10^8}{656 \times 10^{-9}} = 4.57 \times 10^{14} \approx 4.6 \times 10^{14}$ Hz

5 **a** $\lambda = \frac{v}{f} = \frac{330}{256} = 1.29$ m.

b $\lambda = \frac{v}{f} = \frac{330}{25 \times 10^3} = 1.32 \times 10^{-2}$ m.

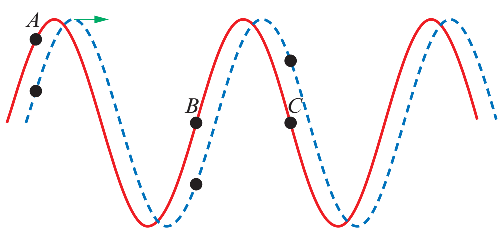
6 Distance travelled is 500 m in 1.5 s so speed is $\frac{500}{1.5} = 333 \approx 330\ \text{m s}^{-1}$.

7 $c = f\lambda \Rightarrow f = \frac{c}{\lambda}$

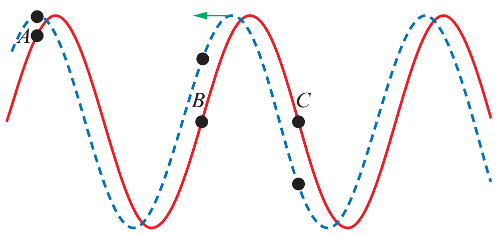
$\frac{f_{\text{light}}}{f_{\text{sound}}} = \frac{3 \times 10^8}{330} = 9.1 \times 10^5$

8 **a** The displacement is at right angles to the direction of energy transfer.

b Draw a copy of the wave displaced slightly to the right (dotted line). The wave is transverse so the points will move up or down. Find the point on the displaced wave that is on the same vertical line as the original points to find the new positions: down, down and up.

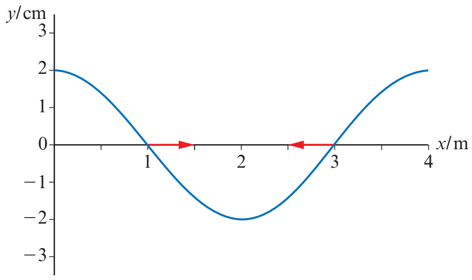


c Now draw a wave slightly displaced to the left and repeat the procedure in **a** to get up, up and down.



9 a The displacement is parallel to the direction of energy transfer.

b



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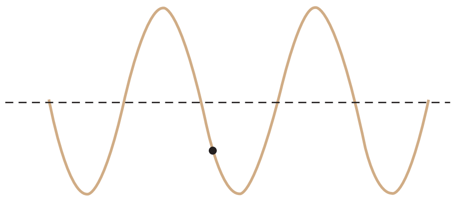


11 a The crest moved forward a distance 0.20 m in 1.25 ms so the speed is $\frac{0.20}{1.25 \times 10^{-3}} = 160\ \text{m s}^{-1}$.

b The wavelength is 1.2 m so the frequency is $\frac{160}{1.2} = 133 \approx 130$ Hz.

12 $d = 8.0t = 3.4(t + 4.2)$; thus $t = 3.104$ s. Hence $d = 8.0 \times 3.104 \approx 25$ km.

13 a The particle is performing simple harmonic oscillations. In SHM the speed is maximum at the equilibrium position.

b

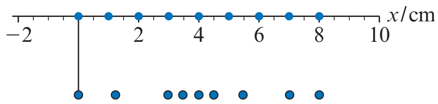
c It is zero at the equilibrium position.

d At $x = 1.0$ m.

e $\Delta\phi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{2.0} \times 1.0 = \pi$.

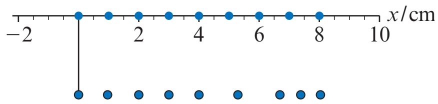
14 a The displacement is parallel to the direction of energy transfer.

b i



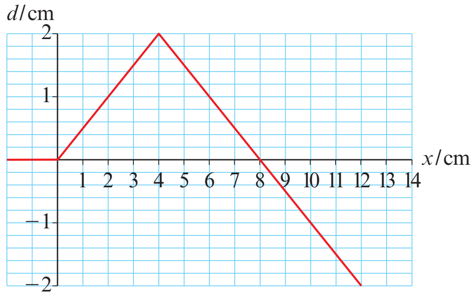
ii At $x = 4$ m.

c i

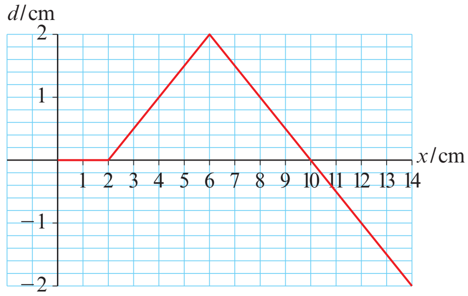


ii It has moved forward by 4 cm.

15 a



b



16 a The wavelength is 0.40 m so the frequency is $\frac{340}{0.40} = 850$ Hz.

b **i** compression at $x = 0.3$ m;

ii rarefaction at $x = 0.1$ m.

17 The sun warms the earth, implying that the electromagnetic waves from the sun carry energy through the vacuum of space.

18 The thing that oscillates in an EM wave is a pair of fields, an electric field and a magnetic field at right angles to each other and both at right angles to the direction of energy transfer. This makes EM waves transverse. (Transverse waves show a phenomenon called polarisation, and light can be polarised.)

19 With $\lambda_0 T = 2.9 \times 10^{-3}$ we find $\lambda_0 = \frac{2.9 \times 10^{-3}}{2.7} = 1.1$ mm, which is a microwave wavelength.

20 From $t = \frac{d}{c}$ we find $t = \frac{1.5 \times 10^{11}}{3.0 \times 10^{8}} = 500$ s ≈ 8.3 min.

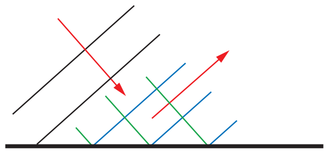
Chapter 14

Test your understanding

1 **a** A wavefront is a surface through crests at right angles to the direction of energy transfer.

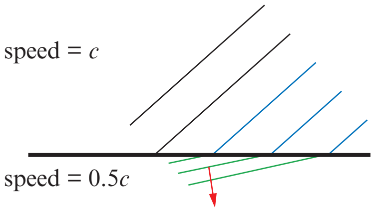
b A ray is the direction of energy transfer in a wave (and so is normal to wavefronts).

2



Reflected wavefronts in green.

3



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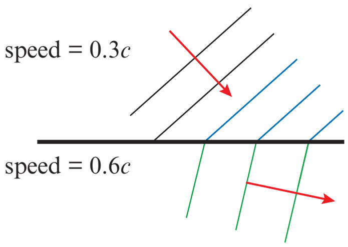

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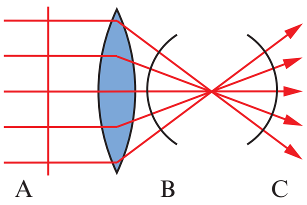

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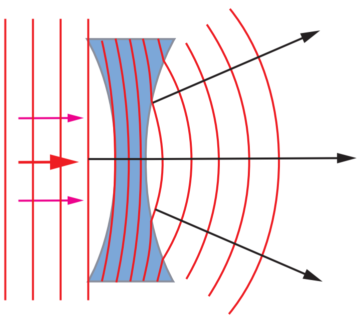
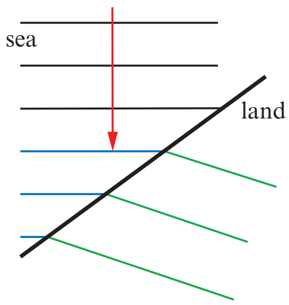


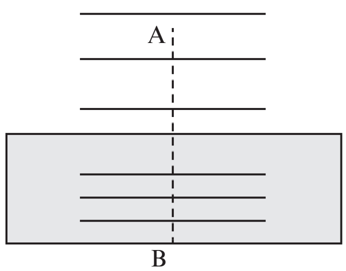
7 Sound travels faster in warmer air. Hence the diagram below.



8 **a** Time in vacuum: $\frac{1.5}{3.0 \times 10^8} = 5.0 \times 10^{-9}$ s; time in glass: $\frac{1.5}{2.0 \times 10^8} = 7.5 \times 10^{-9}$ s. Total time 12.5 ns.

b The wavelength in vacuum is $\frac{3.0 \times 10^8}{6.0 \times 10^{14}} = 5.0 \times 10^{-7}$ m. Hence $\frac{1.5}{5.0 \times 10^{-7}} = 3.0 \times 10^6$ full waves fit. The wavelength in glass is $\frac{2.0 \times 10^8}{6.0 \times 10^{14}} = 3.3 \times 10^{-7}$ m since the frequency stays the same. Hence $\frac{1.5}{3.3 \times 10^{-7}} = 4.5 \times 10^6$ full waves fit. Doing the same for the glass, the total is then 7.5×10^6.

c



9 The wavelength changes from 1 unit to 0.67 units so the speed of light must be 2.0×10^8 m s^{-1}.

10 When a wave travels from a medium where the speed is low into a medium where the speed is higher and the angle of incidence is greater than a certain critical value, the wave is reflected at the boundary with no refraction. One application is the propagation of digital signals carrying coded information securely in optical fibres.

11 **a** $1.20 \times \sin 35° = 1.80 \times \sin\theta$; $\sin\theta = \frac{1.20 \times \sin 35°}{1.80} = 0.3824$ so $\theta = 22°$.

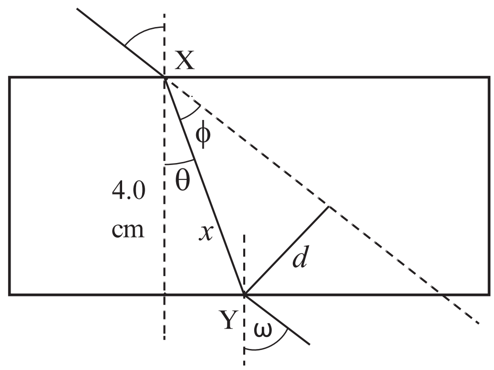
b The frequency is the same in both media so $f = \frac{c_X}{\lambda_X} = \frac{c_Y}{\lambda_Y}$. Thus, $\lambda_Y = \frac{c_Y}{c_X}\lambda_X = \frac{\frac{c}{n_Y}}{\frac{c}{n_X}}\lambda_X = \frac{n_X}{n_Y}\lambda_X = \frac{1.20}{1.80} \times 680 = 453$ nm.

c $1.80 \times \sin 32° = 1.20 \times \sin\theta$; $\sin\theta = \frac{1.80 \times \sin 32°}{1.20} = 0.79488$ so $\theta = 53°$.

d $1.80 \times \sin\theta_c = 1.20 \times \sin 90°$; $\sin\theta_c = \frac{1.20 \times \sin 90°}{1.80} = 0.66667$ so $\theta = 42°$.

12 a The angle of incidence in glass at Y (see the following figure) is the same as the angle of refraction at X. Hence the angle of refraction in air at Y (ω) will be the same as the angle of incidence at X (ω).

b First we find the angle of refraction (angle θ in the diagram).



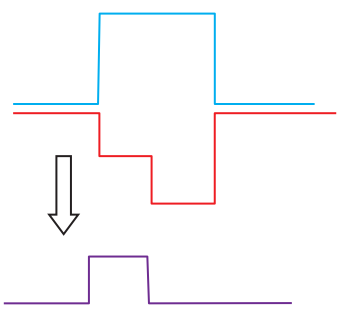
$1.00 \times \sin 40° = 1.450 \times \sin\theta$, hence $\theta = 26.3°$. This means that $x = \frac{4.0}{\cos 26.3°} = 4.46$ cm.

Now $\varphi = 40° - 26.3° = 13.7°$ and so $d = 4.46 \times \sin 13.7° = 1.06$ cm.

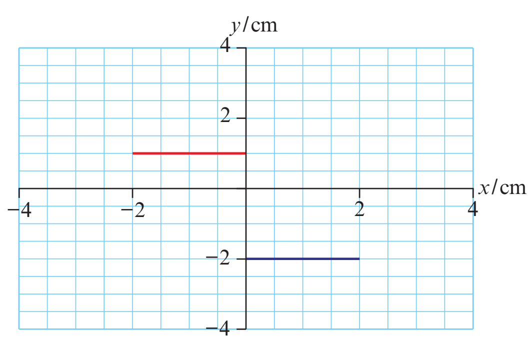
13 It is in the form of kinetic energy; the string is flat for an instant but it is moving.

14

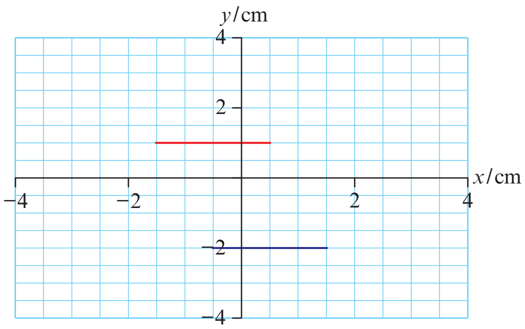
15

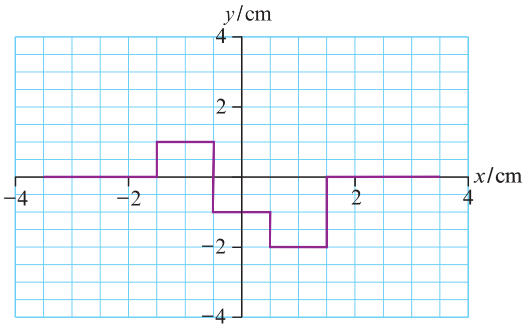


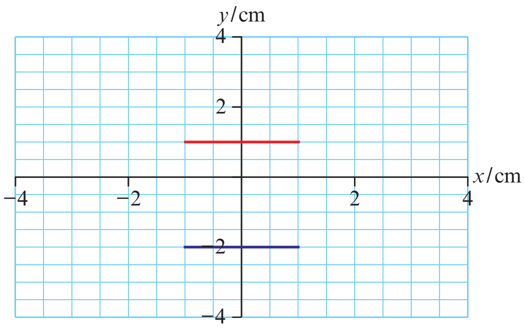
16 The pulses at time zero:

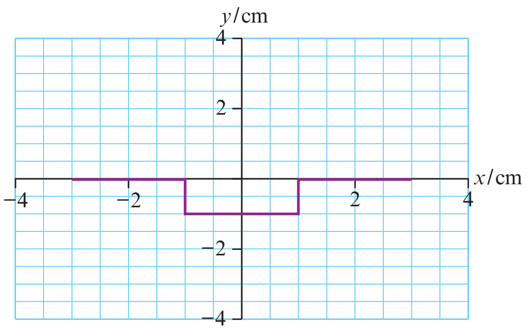


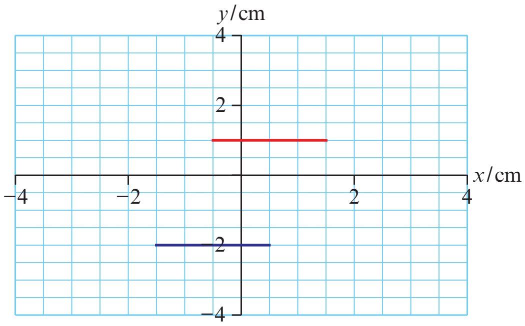
The pulses at t = 0.5 s, 1.0 s and 1.5 s:

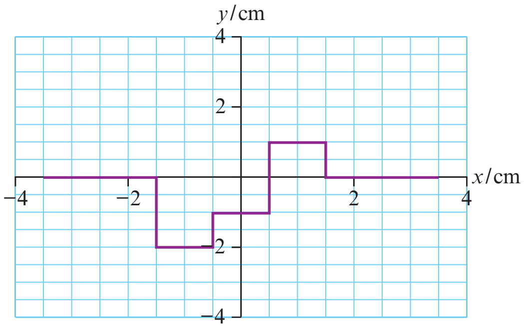









y / cm
4
2
x / cm
−4
−2
2
4
−2
−4


y / cm
4
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x / cm
−4
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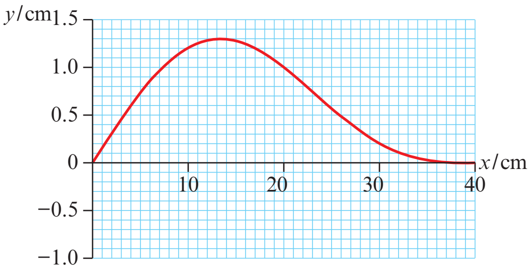
17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0.5
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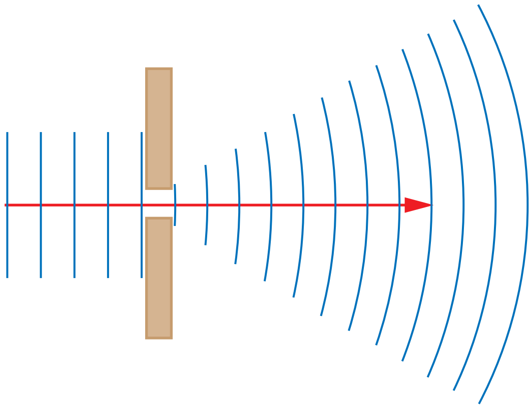
x / cm
10
20
30
40

18



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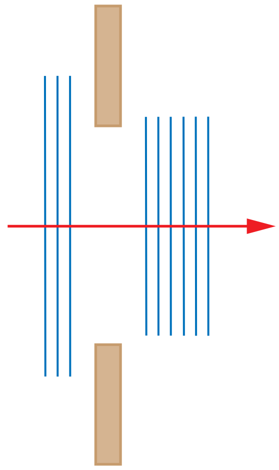


20 The path difference is $9 - 7 = 2$ m. This is 2.5 times the wavelength, i.e. a half integral multiple, so we observe destructive interference.

21 **a** The path difference is $\sqrt{12^2 + 16^2} - 16 = 4.0$ m, which is 2 times the wavelength, i.e. we have constructive interference.

b The path difference of 4.0 m is now 2.5 times the wavelength, i.e. we have destructive interference.

22 The path difference at B is $2x$. For destructive interference we have that $2x = \left(n + \frac{1}{2}\right)\lambda$. The smallest distance is then $x = \frac{\lambda}{4} = 6.0$ cm.

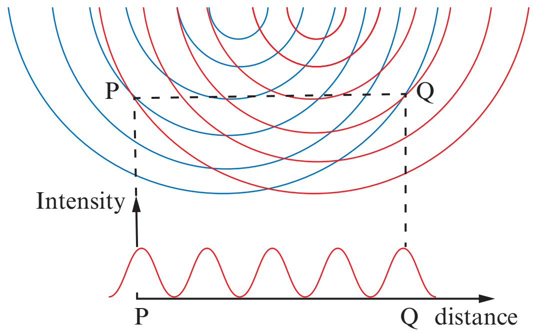
23 P: crests meet so constructive interference.

Q: crest meets trough so destructive interference.

R: crest meets trough so destructive interference.

S: trough meets trough so constructive interference.

24



25 The path difference is $2d$ where d is the distance between the house and the mountain. For destructive interference, $2d = n\lambda$ (there is a phase change) and so $d_{min} = \frac{\lambda}{2} = 800$ m.

26 Energy is redistributed to the maxima.

27 From $s = \frac{\lambda D}{d}$ we find $s = \frac{680 \times 10^{-9} \times 1.50}{0.12 \times 10^{-3}} = 8.5$ mm.

28 They are not coherent; the phase difference between them changes very fast (within nanoseconds) and so positions of maxima get interchanged with minima so fast that we can only observe an average effect, i.e. uniform illumination on the screen without interference fringes.

29 From $d\sin\theta = n\lambda$ we deduce that $n \times 680 = (n + 1) \times 510$ and so $n = \frac{510}{170} = 3$.

30 a From $d\sin\theta = n\lambda$ we deduce that $d\sin 20° = 1 \times \lambda$ and so $d = \frac{\lambda}{\sin 20°} = 2.9238\lambda \approx 2.9\lambda$.

b The maxima would be further apart. The first maximum, for example, would be observed at an angle given by $\frac{2.9238\lambda}{2}\sin\theta = 1 \times \lambda$, i.e. $\sin\theta = \frac{1}{1.4619} \Rightarrow \theta \approx 43°$.

31 a The distance between consecutive maxima is $s = \frac{3.1}{4} = 0.775$ mm. From $s = \frac{\lambda D}{d}$ we find $\lambda = \frac{sd}{D} = \frac{0.775 \times 10^{-3} \times 1.0 \times 10^{-3}}{1.2} = 646$ nm.

b The wavelength of light would be smaller, and so the distance of 3.1 mm would decrease.

32 a Maxima are given at angles $d\sin\theta = n\lambda$. The maximum of $\sin\theta$ is 1 and so $n = \frac{d}{\lambda} = \frac{3.0}{0.60} = 5$. The $n = \pm 5$ maxima would be observed at $\theta = \pm 90°$ which is impossible. Hence we would record 4 maxima on either side of the central maximum for a total of 9.

b In this case we have 9 maxima to the left and right of the sources and now the $n = \pm 5$ maxima are observable for a total of $9 + 9 + 2 = 20$.

33 a $d\sin\theta = \left(n + \frac{1}{2}\right)\lambda$. $d\sin 0.012 = \frac{1}{2} \times 656 \times 10^{-7}$ and so $d = 2.7 \times 10^{-5}$ m.

b The angular separation θ would increase since $\sin\theta \propto \frac{1}{d}$.

34 The separation of two consecutive minima is $\frac{12.6}{3} = 4.2$ mm and so $\lambda = \frac{sd}{D} = \frac{4.2 \times 10^{-3} \times 0.150 \times 10^{-3}}{1.2} =$ 525 nm.

35 The diffraction angle is $\theta \approx \frac{\lambda}{b} = 0.333$ rad and so the angular width is double this, i.e. 0.666 rad or 38.2°.

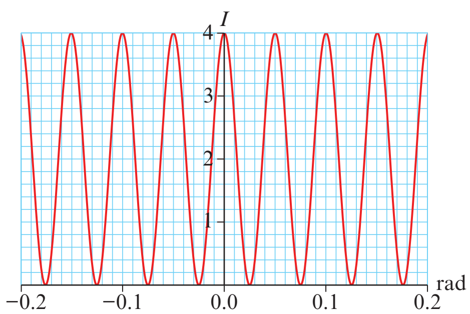
36 The diffraction angle is $\theta \approx \frac{\lambda}{b} = \frac{6.00 \times 10^{-7}}{0.12 \times 10^{-3}} = 0.0050$ rad and so the angular width is double this, i.e. 0.10 rad. The linear width is therefore $2d\theta = 2 \times 2.00 \times 0.0050 = 0.020$ m.

37 a The diffraction angle is about $\theta \approx \frac{\lambda}{b} = 0.041$ rad and so $b \approx \frac{\lambda}{0.04} \approx 25\lambda$.

b In **i** we have a smaller width and so a larger diffraction angle larger by a factor of 2. In **ii** there will be no change since both wavelength and width halve. Notice however that if we were to pay attention to the vertical axis scale, with a smaller slit width less light would go through so in both cases the intensity would be less (by a factor of 4).

38 a Light of a single wavelength in which all waves making up the light have the same phase on a cross section of the beam normal to the direction of energy transfer.

b Vertical axis units are arbitrary.



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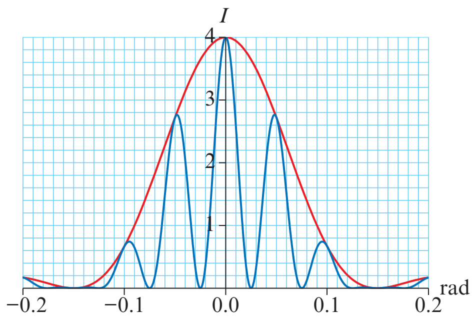
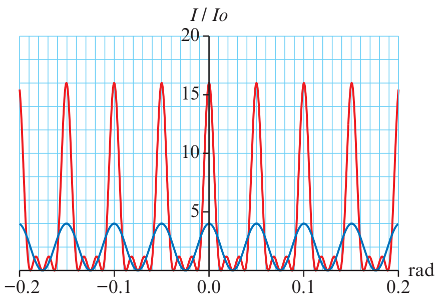
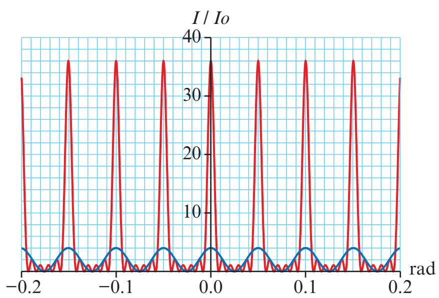


39 **a** A graph like that in **b** of the previous problem.

b **i** The primary maxima stay at the same angle but are thinner and more intense. There will be small intensity secondary maxima in between primary maxima. Shown is an example with two slits (blue) and four slits (red) and then two slits (blue) and six slits (red) showing these features.





ii The maxima are further apart.

40 **a** $d \sin\theta = n\lambda$ and so $\frac{1}{400} \times 10^{-3} \times \sin\theta = 1 \times 620 \times 10^{-9}$, giving $\theta = 14.4°$.

b Set $\frac{1}{400} \times 10^{-3} \times \sin 90° = n \times 620 \times 10^{-9}$ so $n = 4.03$. So $n = 4$ is the largest order (observed at $\theta = 83°$).

41 $\frac{1}{300} \times 10^{-3} \times \sin\theta = 2 \times 500 \times 10^{-9}$ so $\theta = 17.5°$.

$\frac{1}{300} \times 10^{-3} \times \sin\theta = 2 \times 600 \times 10^{-9}$ so $\theta = 21.1°$.

The difference is then $\Delta\theta = 3.6°$.

42 We must have $n \times 500 = (n + 1)400$ which gives $n = 4$. So the fourth order of 500 nm will coincide with the fifth order of 400 nm. The angle will be $\frac{1}{150} \times 10^{-3} \times \sin\theta = 4 \times 500 \times 10^{-9}$ and hence $\theta = 17.5°$.

43 $d \sin 17.46° = n \times 656 \times 10^{-9}$ and $d \sin 23.58° = (n + 1) \times 656 \times 10^{-9}$. This means $\frac{n \times 656 \times 10^{-9}}{\sin 17.46°} = \frac{(n+1) \times 656 \times 10^{-9}}{\sin 23.58°}$

i.e.

$3.3329n = 2.4998(n + 1)$ and solving, $n = 3$. Then, $d \sin 17.46° = 3 \times 656 \times 10^{-9}$ so that $d = 6.6 \times 10^{-6}$ m and finally about 152 lines per mm.

44 **a** $\frac{1}{400} \times 10^{-3} \times \sin 90° = n \times 700 \times 10^{-9}$ gives $n = 3.57$, i.e. $n = 3$. So we have three orders on either side of the central maximum, i.e. seven orders.

b $\frac{1}{400} \times 10^{-3} \times \sin 90° = 4\lambda$ giving $\lambda = 6.25 \times 10^{-7}$ m.

45 The $n = 4$ maximum must coincide with the first diffraction minimum of the one-slit diffraction pattern. So $\sin\theta = \frac{4\lambda}{d}$. This angle must be $\frac{\lambda}{b}$. Hence $d = 4b$.

46 We must have $d \sin\theta = n \times 630 = (n + 1) \times 420$ which gives $n = 2$. Then $\sin\theta = \frac{2 \times 630 \times 10^{-9}}{\frac{1}{160} \times 10^{-3}} = 0.2016$ and so $\theta = 11.6°$.

47 **a** $\frac{1}{300} \times 10^{-3} \times \sin 90° = n \times 700 \times 10^{-9}$ gives $n = 4.76$, i.e. $n = 4$.

b $\frac{1}{300} \times 10^{-3} \times \sin\theta_1 = 4 \times 400 \times 10^{-9}$ and so $\sin\theta_1 = 0.48$ i.e. $\theta_1 = 28.7°$.

$\frac{1}{300} \times 10^{-3} \times \sin\theta_2 = 4 \times 700 \times 10^{-9}$ and so $\sin\theta_2 = 0.84$ i.e. $\theta_2 = 57.1°$.

The angular width is then $\Delta\theta = 28.4°$.

Chapter 15

Test your understanding

1 A standing wave is a wave in which the particles of a medium oscillate as time goes on but the position of peaks and troughs does not move through space.

2 A standing wave is formed when two identical travelling waves moving in opposite directions meet and then superpose.

3 This wave, unlike a travelling wave, has nodes, i.e. points where the displacement is *always* zero. The antinodes, points where the displacement is the largest, do not appear to be moving. A standing wave differs from a travelling wave in that it does not transfer energy and the amplitude is variable. In a standing wave, points in between consecutive nodes have the same phase whereas in a travelling wave the phase changes from zero to 2π after a distance of one wavelength.

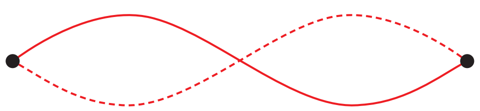
4 a A node is a point in the medium where the displacement is *always* zero.

b An antinode is a point in the medium where the displacement at some instant will assume its maximum value.

c Speed refers to the speed of the travelling waves whose superposition gives the standing wave.

5 a We must force the string to oscillate with a frequency that is equal to the frequency of the second harmonic by connecting one end of the string to an oscillator.

b



6 a $\mu = \frac{T}{v^2}$ so the units are: $[\mu] = \frac{\text{N}}{\text{m}^2\,\text{s}^{-2}} = \frac{\text{kg}\,\text{m}\,\text{s}^{-2}}{\text{m}^2\,\text{s}^{-2}} = \frac{\text{kg}}{\text{m}}$, i.e. mass per unit length.

b The speed will increase by $\sqrt{2}$ and since the wavelength will stay the same the frequency will become $250 \times \sqrt{2} = 354$ Hz.

7 $\lambda_X = 2L_x, f_X = \frac{c_X}{2L_x}$

$\lambda_Y = 2L_Y, f_Y = \frac{c_Y}{2L_Y}$

Hence

$$\frac{f_X}{f_Y} = \frac{\frac{c_X}{2L_x}}{\frac{c_Y}{2L_Y}} = \frac{c_X}{c_Y}\frac{L_Y}{L_x} = 2 \times 2 = 4.$$

8 a The wavelength is $2L = 1.00$ m. So the frequency is 225 Hz.

b The sound will have frequency 225 Hz so the wavelength is $\frac{340}{225} = 1.5$ m.

9 The wavelength is $\frac{122}{512} = 0.238$ m. Distance between nodes is half a wavelength i.e. 0.119 m.

10 (Many ways of doing this.) $\lambda_n = \frac{2L}{n} \Rightarrow f_n = \frac{cn}{2L} \cdot f_{n+1} - f_n = \frac{c(n+1)}{2L} - \frac{cn}{2L} = \frac{c}{2L} = f_1 = 425 - 340 = 85$ Hz.

OR

$\lambda_n = \frac{2L}{n} \Rightarrow f_n = \frac{cn}{2L}$ and $\frac{f_{n+1}}{f_n} = \frac{n+1}{n} = \frac{425}{340} \Rightarrow n = 4$. So $f_1 = \frac{340}{4} = 85$ Hz.

OR

Since frequencies are multiples of the first harmonic, find the **greatest** common factor of 340 and 425: {340, 170, 85, 42.5, …..} and {425, 215.5, 141, 7, 106,25, 85, 42.5, …..}. (Note: not taking the greatest common factor, e.g. taking 42.5 Hz would not make the harmonics of frequency 340 Hz and 425 Hz consecutive.)

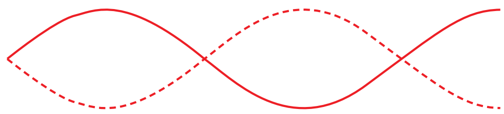
11 In one period the point moves a distance of 20 mm. The period is $\frac{1}{480} = 2.08 \times 10^{-3}$ s. The average speed is then $\frac{20 \times 10^{-3}}{2.08 \times 10^{-3}} = 9.6\ \text{m s}^{-1}$.

12 The wavelength is given by $\frac{4L}{n}$ where n is an odd integer. The frequency is then $\frac{c}{\frac{4L}{n}} = n\frac{c}{4L}$. The frequency of the first harmonic is $\frac{c}{4L}$.

One way: so two consecutive harmonics differ by $(n + 2)\frac{c}{4L} - n\frac{c}{4L} = \frac{c}{2L}$ which is double the frequency of the first harmonic $\left(\frac{c}{4L}\right)$. In our case the difference is 120 Hz. The first harmonic has frequency 60 Hz.

Another way: $300 = n\frac{c}{4L}$ and $420 = (n + 2)\frac{c}{4L}$. Thus $\frac{300}{n} = \frac{420}{n+2}$. Solving, $n = 5$. Then the first harmonic has frequency $\frac{300}{5} = 60$ Hz.

13



14 The wavelength is given by $\frac{4L}{n}$ where n is an odd integer. The wavelength here is $\frac{340}{425} = 0.80$ m. So $\frac{4L}{n} = 0.80$ and $L = 0.2n$. $n = 3$ and $n = 5$ give 0.60 m and 1.0 m.

15 **a** The wavelength is given by $\lambda = \frac{4L}{n} = \frac{0.800}{n}$ and also by $\lambda = \frac{c}{f} = \frac{c}{427}$. Hence $\frac{c}{427} = \frac{0.800}{n} \Rightarrow c = \frac{427 \times 0.800}{n} = \frac{342}{n}\ \text{m s}^{-1}$. The answer makes physical sense only if $n = 1$ (the first harmonic is established) in which case $c = 342\ \text{m s}^{-1}$.

b The next harmonic will have the same wavelength and so $\frac{4L'}{n} = 0.800 \Rightarrow L' = \frac{0.800n}{4} = 0.200n$. With $n = 3$ we get $L' = 0.600$ m.

OR

the extra length will be half a wavelength i.e. 0.400 m and so the new length is 0.600 m.

16 **a** The displacement of P is positive. The particle is returning to equilibrium position. So it is moving to the left.

b The displacement of Q is negative. The particle is returning to equilibrium position. So it is moving to the right.

17 **a** We must have the first harmonic of wavelength $2L$. The wavelength is $\frac{340}{586} = 0.58$ m and so $L = 0.29$ m.

b We will now have the second harmonic of wavelength L. Hence $L = 0.58$ m and the additional length we need is 0.29 m.

18 **a** The wavelengths in the open tube are given by $\lambda = \frac{2L}{n}$. The frequencies of two *consecutive* harmonics are then $\left(f = \frac{c}{\lambda} = \frac{cn}{2L}\right)$, $300 = \frac{cn}{2L}$ and $360 = \frac{c(n+1)}{2L}$. This means that $\frac{360}{300} = \frac{\frac{c(n+1)}{2L}}{\frac{cn}{2L}} \Rightarrow$ $\frac{n+1}{n} = 1.2 \Rightarrow n + 1 = 1.2n \Rightarrow 0.2n = 1 \Rightarrow n = 5$; we have the fifth and sixth harmonics.

(Notice that the difference in frequency of the consecutive harmonics is $\frac{c(n+1)}{2L} - \frac{cn}{2L} = \frac{c}{2L}$, which is the frequency of the first harmonic. So, the first harmonic frequency is 60 Hz. Since $300 = 5 \times 60$ and $360 = 6 \times 60$ we have the fifth and sixth harmonics in the pipe.)

b We get $300 = \frac{340 \times 5}{2 \times L} \Rightarrow L = 2.833 \approx 2.8$ m.

19 The wavelength of the second harmonic in X is $\lambda_X = L_X$. The wavelength of the first harmonic in Y is $\lambda_Y = 4L_Y$. The wavelengths must be the same so $L_X = 4L_Y \Rightarrow \frac{L_X}{L_Y} = 4$.

20 **a** A standing wave is made up of two travelling waves. By speed, we mean the speed of energy transfer of each of the two travelling waves.

b From $y = 5.0\cos(45\pi t)$ we deduce that the frequency of oscillation of point P and hence also of the wave is $\frac{45\pi}{2\pi} = 22.5$ Hz. The wavelength is then $\lambda = \frac{v}{f} = \frac{180}{22.5} = 8.0$ m. Since the diagram shows a second harmonic, this is also the length of the string.

c The phase difference is π and so $y = 5.0\cos(45\pi t + \pi) = -5.0\cos(45\pi t)$.

21 **a** The hit creates a longitudinal wave that travels down the length of the rod and reflects off the end. The reflected wave pushes the hammer back.

b $v = \frac{s}{t} = \frac{2.4}{0.37 \times 10^{-3}} = 6500\ \text{m s}^{-1}$

c We assume free–free end points and so the wavelength is given by 2.4 m. The frequency is then $f = \frac{c}{\lambda} = \frac{6500}{2.4} = 2.7$ kHz.

22 **a** The loss of energy in an oscillating system due to the presence of frictional, drag or resistance forces.

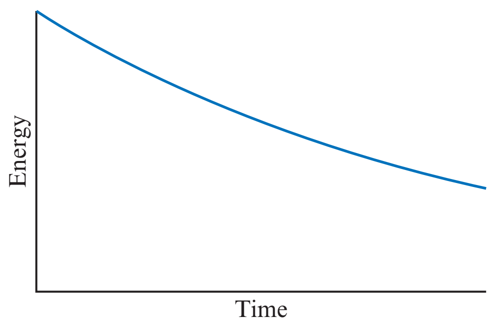
b Free oscillations are oscillations in which no external forces act on the system (the weight excepted). In driven oscillations an external periodic force acts on the system.

23 The condition in which the frequency of an external, periodic force acting on an oscillating system is about equal to the natural frequency of the system, resulting in large amplitude oscillations.

24 With one step per second you shake the cup with a frequency of about 1 Hz. In the first harmonic mode the wavelength would be about twice the diameter of the cup, i.e. 16 cm (we have antinodes at each end). This gives a speed of $v = 1 \times 16 = 16\ \text{cm s}^{-1}$.

25 **a** The period is 4.0 s so the frequency is 0.25 Hz.

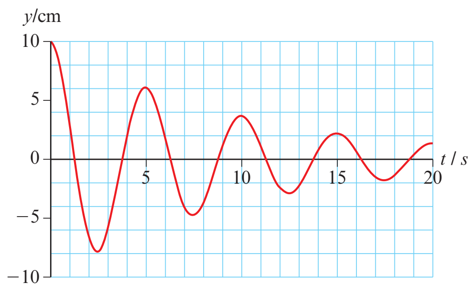
b An exponential decay curve:



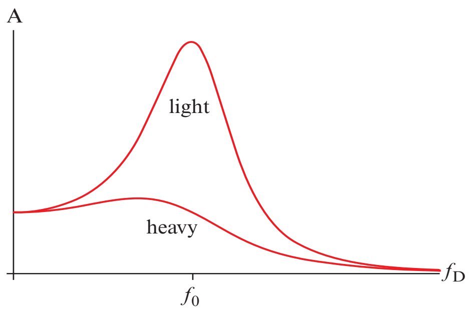
c The oscillations would die out faster.

26 a The acceleration must be opposite to and proportional to the displacement.

b



c **i** and **ii**



27 a The period is $T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{2.0}{9.81}} = 2.84$ s and so the frequency is 0.35 Hz. This is the resonant frequency.

b Integral multiples of 0.35 Hz will also do so.

28 a At the natural frequency, i.e. $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{220}{0.50}} = 3.3$ Hz.

b It will slightly decrease.

29 It is desirable in a radio that needs to tune to a particular station. It is undesirable for buildings, bridges and other structures.

30 In light damping the system is brought to rest after many oscillations. In critical damping the system is brought to rest as quickly as possible without oscillations. In heavy damping the system is brought to rest after a long time without oscillations.

Car shock absorbers should be critically damped so the car will return to its equilibrium state as fast as possible without uncomfortable oscillations.

Chapter 16

Test your understanding

1 See textbook.

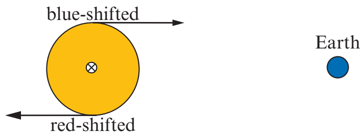
2 $\frac{\Delta\lambda}{\lambda} = \frac{v}{c} \Rightarrow \frac{5.65 \times 10^{-7} - 5.48 \times 10^{-7}}{5.48 \times 10^{-7}} = \frac{v}{c}$ and so $\frac{v}{c} = 0.031 \Rightarrow v = 9.3 \times 10^6\ \mathrm{m\,s^{-1}}$.

3 To make the red look green the speed must have been enormous. So he was fined for speeding.

4 a $\frac{\Delta\lambda}{\lambda} = \frac{v}{c} \Rightarrow \frac{6.57 \times 10^{-9} - 6.54 \times 10^{-9}}{6.57 \times 10^{-9}} = \frac{v}{c}$ and so $\frac{v}{c} = 0.00457 \Rightarrow v = 1.4 \times 10^6\ \mathrm{m\,s^{-1}}$.

b We are seeing a blue-shift so the galaxy is moving towards us. However, the speed calculated is only the component of velocity along the line of sight.

5 The linear speed of a point on the equator is $v = \omega R = \frac{2\pi}{24.5 \times 24 \times 60 \times 60} \times 7.0 \times 10^8 = 2.08 \times 10^3\ \mathrm{m\,s^{-1}}$.



Then there are blue-shifts and red-shifts given by $\Delta\lambda = \lambda\frac{v}{c} = 500 \times \frac{2.08 \times 10^3}{3.0 \times 10^8} \approx 3.5 \times 10^{-3}$ nm.

6 a The velocity is at right angles to the line of sight so there is no relative velocity towards or away from the observer and so no Doppler shift.

b $\frac{\Delta\lambda}{\lambda} = \frac{v}{c} \Rightarrow \frac{6.58 \times 10^{-7} - 6.50 \times 10^{-7}}{6.58 \times 10^{-7}} = \frac{v}{c}$ and so $\frac{v}{c} = 0.01216 \Rightarrow v = 3.6 \times 10^6\ \mathrm{m\,s^{-1}}$ and

$\frac{\Delta\lambda}{\lambda} = \frac{v}{c} \Rightarrow \frac{6.70 \times 10^{-7} - 6.58 \times 10^{-7}}{6.58 \times 10^{-7}} = \frac{v}{c}$ and so $\frac{v}{c} = 0.01824 \Rightarrow v = 5.5 \times 10^6\ \mathrm{m\,s^{-1}}$.

Star B is the faster star since it covers a longer distance than star A in the same time.

7 a There is double Doppler shift: the blood cells are moving away, say, so they receive a Doppler shifted wave and then send the wave back, acting now as a moving source.

b $\frac{\Delta f}{f} = \frac{2u}{v} \Rightarrow \frac{2.4 \times 10^3}{5.0 \times 10^6} = \frac{2u}{1500}$ and so $u = 0.36\ \mathrm{m\,s^{-1}}$.

c The blood cells do not all have the same velocity, and the ultrasound does not necessarily hit the blood cell along the direction of the velocity.

8 No, because there are cases when the wavelength remains unchanged even though the frequency changes.

9 See textbook.

10 a $f = 500 \times \frac{340}{340 - 40} = 566.7 \approx 570$ Hz

b i $\lambda_{\mathrm{source}} = \frac{340}{500} = 0.68$ m

ii $\lambda_{\mathrm{obs}} = \frac{340}{566.7} = 0.60$ m

11 a $f = 480 \times \frac{340}{340 + 32} = 438.709 \approx 440$ Hz

b i $\lambda_{\mathrm{source}} = \frac{340}{480} = 0.71$ m

ii $\lambda_{\mathrm{obs}} = \frac{340}{438.709} = 0.775 \approx 0.78$ m

12 **a** $f = 512 \times \frac{340 - 12}{340} = 493.9 \approx 490$ Hz

b **i** $\lambda_{source} = \frac{340}{512} = 0.66$ m

ii $\lambda_{obs} = \frac{328}{493.9} = 0.66$ m (no change).

13 **a** $f = 628 \times \frac{340 + 25}{340} = 674.2 \approx 670$ Hz

b **i** $\lambda_{source} = \frac{340}{628} = 0.54$ m

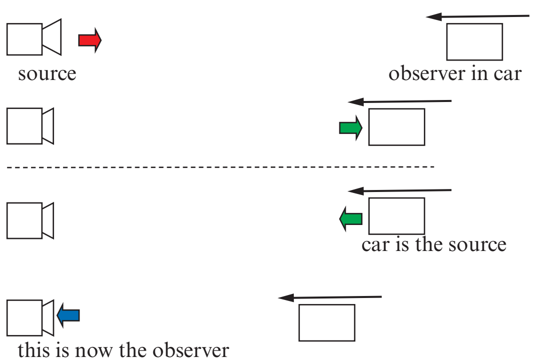
ii $\lambda_{obs} = \frac{365}{674.2} = 0.54$ m (no change).

14 In the first row of diagrams the source emits waves towards the approaching car.

In the second row the car receives the waves. The car is the observer, and it will measure a Doppler shift, measuring a higher frequency.

In the third row the roles are reversed. The car reflects the wave and acts as a source. The reflected wave has the same frequency as the incident wave.

In the fourth row, the original source acts as an observer and will measure a Doppler shift on top of the first Doppler shift, measuring an even higher frequency.

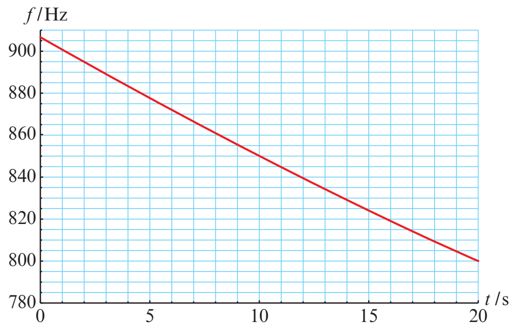


15 The observer will measure a frequency $f' = 500 \times \frac{340 - v}{340}$. The observer will reflect this frequency acting as a moving source. The reflected wave is received at a frequency $f'' = f' \times \frac{340}{340 + v} = 500 \times \frac{340 - v}{340} \times \frac{340}{340 + v} = 500 \times \frac{340 - v}{340 + v} = 480$ Hz. Hence, $\frac{340 - v}{340 + v} = \frac{480}{500}$. This gives $v = 6.9\ \mathrm{m\,s^{-1}}$.

16 The observer will measure a frequency $f' = 500 \times \frac{340}{340 - v}$. The observer will reflect this frequency acting as a stationary source. The reflected wave is received at a frequency $f'' = f' \times \frac{340 + v}{340} = 500 \times \frac{340}{340 - v} \times \frac{340 + v}{340} = 500 \times \frac{340 + v}{340 - v} = 512$ Hz. Hence, $\frac{340 + v}{340 - v} = \frac{512}{500}$. This gives $v = 4.0\ \mathrm{m\,s^{-1}}$.

17 The speed of the train is $v = 40 - 2t$ and so $f = 800 \times \frac{340}{340 - v} = 800 \times \frac{340}{340 - (40 - 2t)} = \frac{800 \times 340}{300 + 2t}$.

We get the following graph:



18 As far as the observer is concerned the velocity of the source is $v_S + v_0$ and the speed of the wave is $v_0 + c$.

So using the formula of the stationary observer and an approaching source we have $f_0 = \frac{f_S}{1 - \frac{v_S + v_0}{c + v_0}} = \frac{f_S}{\frac{c + v_0}{c + v_0} - \frac{v_S + v_0}{c + v_0}} = f_S \frac{c + v_0}{c - v_S}$.

Chapter 17

Test your understanding

1 **a** $F = G\frac{Mm}{R^2} = 6.67 \times 10^{-11} \times \frac{6.0 \times 10^{24} \times 7.3 \times 10^{22}}{(3.8 \times 10^8)^2} = 2.0 \times 10^{20}$ N.

b $F = G\frac{Mm}{R^2} = 6.67 \times 10^{-11} \times \frac{1.67 \times 10^{-27} \times 9.11 \times 10^{-31}}{(1.0 \times 10^{-10})^2} = 1.0 \times 10^{-47}$ N.

2 **a** Zero since it is being pulled equally from all directions.

b Zero, by Newton's third law.

c $F = G\frac{m^2}{4r^2}$

d $F = G\frac{m^2}{4r^2} + G\frac{Mm}{4r^2} = G\frac{m(m+M)}{4r^2}$.

3 $\frac{g_A}{g_B} = \frac{\frac{GM}{(9R)^2}}{\frac{GM}{R^2}} = \frac{1}{81}$

4 $\frac{g_A}{g_B} = \frac{\frac{G2M}{(2R)^2}}{\frac{GM}{R^2}} = \frac{1}{2}$

5 Since star A is 27 times as massive and the density is the same, the volume of A must be 27 times as large. Its radius must therefore be 3 times as large. Hence $\frac{g_A}{g_B} = \frac{\frac{G27M}{(3R)^2}}{\frac{GM}{R^2}} = 3$.

6 $\frac{g_{\text{new}}}{g_{\text{old}}} = \frac{\frac{GM/2}{(R/2)^2}}{\frac{GM}{R^2}} = 2$

7 Let this point be a distance x from the centre of the earth and let d be the centre-to-centre distance between the earth and the moon. Then $\frac{G81M}{x^2} = \frac{GM}{(d-x)^2}$

$81(d-x)^2 = x^2$

$9(d-x) = x$

$\frac{x}{d} = \frac{9}{10} = 0.9$

8 At point P the gravitational field strength is obviously zero.

The gravitational field strength at Q from each of the masses is $g = \frac{GM}{R^2} = 6.67 \times 10^{-11} \times \frac{3.0 \times 10^{22}}{(\sqrt{2} \times 10^9)^2} =$ $1.0 \times 10^{-6}\ \text{N kg}^{-1}$. The net field, taking components, is directed from Q to P and has magnitude $2g\cos 45° = 2 \times 1 \times 10^{-6} \cos 45° = 1.4 \times 10^{-6}\ \text{N kg}^{-1}$.

9 The force is larger for the mass $2m$.

The acceleration is the same: $F = \frac{GMm}{r^2} = ma \Rightarrow a = \frac{GM}{r^2}$.

10 $F = \frac{GMm}{r^2} = ma \Rightarrow a = \frac{GM}{r^2}$, hence $\frac{a_X}{a_Y} = \frac{\frac{GM}{r_X^2}}{\frac{GM}{r_Y^2}} = \frac{r_Y^2}{r_X^2} = \frac{1}{4}$.

11 We know that $\frac{GMm}{r^2} = m\frac{v^2}{r} \Rightarrow v^2 = \frac{GM}{r}$. But $v = \frac{2\pi r}{T}$ and so we deduce that $T^2 = \frac{4\pi^2 r^3}{GM}$. Therefore $r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = \sqrt[3]{\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times (24 \times 60 \times 60)^2}{4\pi^2}} = 4.2 \times 10^7$ m. So the height is 3.6×10^7 m.

12 **a** From $v^2 = \frac{GM}{r}$ we calculate $v = \sqrt{\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{(6.4 + 0.560) \times 10^6}} = 7.5828754 \times 10^3 \approx 7.6 \times 10^3\ \text{m s}^{-1}$.

b The shuttle speed is $v = \sqrt{\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{6.9595 \times 10^6}} = 7.5831478 \times 10^3\ \text{m s}^{-1}$. The relative speed of the shuttle and Hubble is $0.2724\ \text{m s}^{-1}$ and so the distance of 10 km will be covered in $\frac{10^4}{0.2724} = 36\,711$ s ≈ 10 hrs.

13 **a** 1 ly $= 3.0 \times 10^8 \times 365 \times 24 \times 60 \times 60 = 9.46 \times 10^{15}$ m.

b $v = \frac{2\pi r}{T} \Rightarrow T = \frac{2\pi r}{v} = \frac{2\pi \times 27\,000 \times 9.46 \times 10^{15}}{200 \times 10^3} = 8.02 \times 10^{15}\ \text{s} = 2.5 \times 10^8$ years.

c $v^2 = \frac{GM}{r} \Rightarrow M = \frac{v^2 r}{G} = \frac{(200 \times 10^3)^2 \times 27\,000 \times 9.46 \times 10^{15}}{6.67 \times 10^{-11}} = 1.5 \times 10^{41}$ kg.

14 From $v^2 = \frac{GM}{r}$ it has to be the planet closest to the sun i.e. Mercury.

15 **a** $\frac{Gm_1 m_2}{r^n} = m_2\frac{v^2}{r} \Rightarrow v^2 = \frac{Gm_1}{r^{n-1}}$. But $v = \frac{2\pi r}{T}$ and so $\left(\frac{2\pi r}{T}\right)^2 = \frac{Gm_1}{r^{n-1}}$ giving $\frac{4\pi^2 r^2}{T^2} = \frac{Gm_1}{r^{n-1}} \Rightarrow T^2 = \frac{4\pi^2 r^{n+1}}{Gm_1}$

b Kepler's law states that $T^2 \propto r^3$ and so $n + 1 = 3$, so $n = 2$.

16 **a** $F = \frac{GMm}{r^2} = m\frac{v^2}{r} \Rightarrow v^2 = \frac{GM}{r}$. But also $v = \frac{2\pi r}{T}$ and so $\left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r} \Rightarrow T^2 = \frac{4\pi^2}{GM}r^3$.

b From $T^2 = \frac{4\pi^2}{GM}r^3$ we get $M = \frac{4\pi^2}{GT^2}r^3$ and so $\frac{M_J}{M_N} = \frac{\frac{4\pi^2}{GT_E^2}r_E^3}{\frac{4\pi^2}{GT_T^2}r_T^3} = \frac{r_E^3\, T_T^2}{r_T^3\, T_E^2} = \left(\frac{6.7}{3.5}\right)^3 \times \left(\frac{5.9}{3.6}\right)^2 \approx 19$.

17 Working as in the previous problem we get: $M_E = \frac{4\pi^2}{GT^2}r^3 = \frac{4\pi^2}{6.67 \times 10^{-11} \times (27 \times 24 \times 3600)^2} \times (3.8 \times 10^8)^3 = 5.968 \times 10^{24}$ kg. The density is then: $\rho = \frac{5.968 \times 10^{24}}{\frac{4\pi}{3} \times (6.4 \times 10^6)^3} = 5.4 \times 10^3\ \text{kg m}^{-3}$.

18 From $T^2 = \frac{4\pi^2}{GM}r^3$, $\frac{T^2_{\text{Tethys}}}{T^2_{\text{Titan}}} = \frac{\frac{4\pi^2}{GM}r^3_{\text{Tethys}}}{\frac{4\pi^2}{GM}r^3_{\text{Titan}}} = \frac{r^3_{\text{Tethys}}}{r^3_{\text{Titan}}} \Rightarrow r^3_{\text{Titan}} = r^3_{\text{Tethys}}\frac{T^2_{\text{Titan}}}{T^2_{\text{Tethys}}}$, i.e. $r_{\text{Titan}} = r_{\text{Tethys}}\sqrt[3]{\frac{T^2_{\text{Titan}}}{T^2_{\text{Tethys}}}} = 2.9 \times 10^5 \times \sqrt[3]{\frac{16^2}{1.9^2}} = 1.2 \times 10^6$ km.

19 **a** The net field at P is $g = \frac{Gm}{(d/5)^2} - \frac{16Gm}{(4d/5)^2} = \frac{25Gm}{d^2} - \frac{16 \times 25Gm}{16d^2} = 0$.

b The net potential at P is $V = -\frac{Gm}{d/5} - \frac{16Gm}{4d/5} = -\frac{5Gm}{d} - \frac{16 \times 5Gm}{4d} = \frac{5Gm}{d} - \frac{20Gm}{d} = -\frac{25Gm}{d}$.

20 **a** $V = -\frac{GM}{5R_e} = -\frac{6.67 \times 10^{-11} \times 6.0 \times 10^{24}}{5 \times 6.4 \times 10^6} = -1.251 \times 10^7 \approx -1.25 \times 10^7\ \text{J kg}^{-1}$

b $E_p = mV = 500 \times -1.251 \times 10^7 \approx -6.3 \times 10^9$ J.

21 **a** $E_P = -\dfrac{GM_{\text{earth}}M_{\text{moon}}}{r} = -\dfrac{6.67\times10^{-11}\times6.0\times10^{24}\times7.3\times10^{22}}{3.8\times10^{8}} = -7.7\times10^{28}$ J.

b $E_P = M_{\text{moon}}V \Rightarrow V = \dfrac{E_P}{M_{\text{moon}}} = -\dfrac{7.7\times10^{28}}{7.3\times10^{22}} = -1.1\times10^{6}\ \text{J kg}^{-1}$.

c $v = \sqrt{\dfrac{GM}{r}} = \sqrt{\dfrac{6.67\times10^{-11}\times6.0\times10^{24}}{3.8\times10^{8}}} = 1.0\times10^{3}\ \text{m s}^{-1}$.

22 We must plot the function $E_P = -\dfrac{GM_{\text{earth}}m}{r} - \dfrac{GM_{\text{moon}}m}{d-r}$. Here m is the mass of the spacecraft and d the separation of the earth and the moon (centre-to-centre). Putting numbers in,

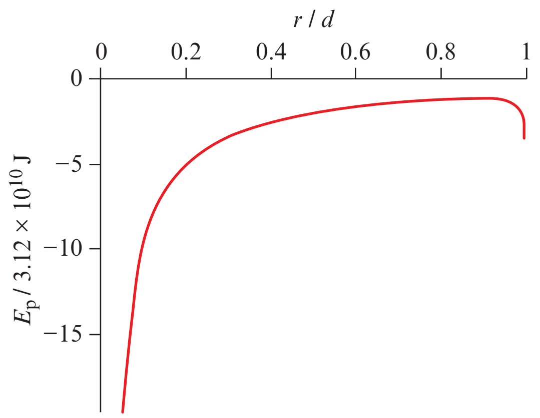
$$E_P = -\frac{6.67\times10^{-11}\times6.0\times10^{24}\times3.0\times10^{4}}{r} - \frac{6.67\times10^{-11}\times7.3\times10^{22}\times3.0\times10^{4}}{3.8\times10^{8}-r}$$

$$= \frac{-1.2\times10^{19}}{r} - \frac{1.5\times10^{17}}{3.8\times10^{8}-r}$$

$$= \frac{-1.2\times10^{19}/3.8\times10^{8}}{r/3.8\times10^{8}} - \frac{1.5\times10^{17}/3.8\times10^{8}}{1-r/3.8\times10^{8}}$$

$$= \frac{-3.2\times10^{10}}{x} - \frac{3.8\times10^{8}}{1-x}$$

where $x = \dfrac{r}{3.8\times10^{8}}$. In this way the function can be plotted on a calculator to give the graph:



23 **a** At $r = 0.75$, $g = \dfrac{GM_P}{(0.75d)^2} - \dfrac{GM_m}{(0.25d)^2} = 0$. Hence $\dfrac{M_P}{M_m} = \dfrac{(0.75d)^2}{(0.25d)^2} = 9$.

b The probe must have enough energy to get to the maximum of the graph. From then on the moon will pull it in. Then $W = \frac{1}{2}mv^2 = m\Delta V \Rightarrow v = \sqrt{2\Delta V} = \sqrt{2(-0.20\times10^{12} - (-6.45\times10^{12}))} = 3.5\times10^{6}\ \text{m s}^{-1}$.

24 The tangential component at A is in the direction of velocity, and so the planet increases its speed. At B it is opposite to the velocity and so the speed decreases.

From A to P the distance to the sun decreases so the potential energy decreases, hence the kinetic energy increases. The opposite is true for the motion from P to B.

25 The work done by an external agent in moving one of the masses from $r = a$ to $r = b$ at a small constant speed.

26 **a** The lines are not symmetrical and so the masses cannot be equal.

b Lines normal to the equipotentials.

c From a large distance the two masses appear as one, and so the equipotentials are spherical.

d The potential difference is the same so the work is the same.

27 The net force is the gravitational force, and this must point towards the centre of the earth. This happens only for orbit 2.

28 As shown in the text, the reaction force from the spacecraft floor is zero, giving the impression of weightlessness. More simply, both spacecraft and astronaut are in free fall with the same acceleration.

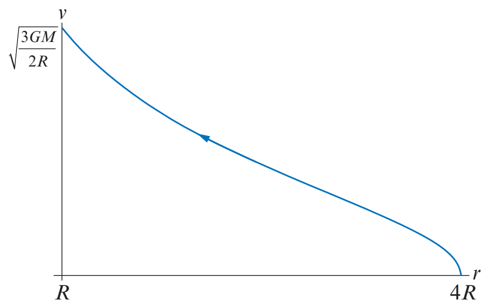
29 **a** The total energy of the rocket at the point where the fuel runs out is negative so the rocket cannot escape; it will fall back down.

b Apply energy conservation to determine that total energy at the point the fuel runs out is $E_T = \frac{1}{2}mv^2 - \frac{GMm}{2R} = \frac{1}{2}m\frac{GM}{2R} - \frac{GMm}{2R} = -\frac{GMm}{4R}$. At the highest point the kinetic energy is zero and so $-\frac{GMm}{4R} = -\frac{GMm}{r}$ leading to $r = 4R$.

c Apply energy conservation again between the points where the fuel runs out and the crash point to get $-\frac{GMm}{4R} = \frac{1}{2}mv^2 - \frac{GMm}{R}$ leading to $\frac{1}{2}v^2 = \frac{GM}{R} - \frac{GM}{4R} = \frac{3GM}{4R}$

$v = \sqrt{\frac{3GM}{2R}}$

d From energy conservation, when the rocket is a distance r from the centre of the planet $-\frac{GMm}{4R} = \frac{1}{2}mv^2 - \frac{GMm}{r}$. This simplifies to $v = \sqrt{\frac{2GM}{r} - \frac{GM}{2R}}$ (where $R \le r \le 4R$). We need to plot this function. It is best to write the equivalent form: $v = \sqrt{\frac{GM}{2R}}\sqrt{\frac{4R}{r} - 1}$. The graph is then:



30 We deduced many times that $v^2 = \frac{GM}{r}$ and so $E_T = \frac{1}{2}mv^2 - \frac{GMm}{r} = \frac{GMm}{2r} - \frac{GMm}{r} = -\frac{GMm}{2r} = -\frac{1}{2}mv^2$.

31 **a** The total energy is being reduced because thermal energy is being generated.

b Since $E_T = -\frac{GMm}{2r}$ and is decreasing, r has to get smaller.

c From $v = \sqrt{\frac{GM}{r}}$ and r decreasing, the speed is increasing.

d The potential energy is $E_p = -\frac{GMm}{r}$ and so decreases since r decreases.

OR

Kinetic energy increases and total energy decreases so potential energy decreases.

e The speed increases and the orbit radius decreases so the period decreases.

OR

Use Kepler's third law: with r decreasing, T decreases.

32 **a** B has the larger kinetic energy,

b A has the larger potential energy,

c A has the larger total energy.

33 **a** The total energy is negative so the satellite cannot escape.

b In orbit, $E_T = -\frac{GMm}{2r}$. Since we are told that $E_T = -\frac{GMm}{5R}$ and energy is conserved, $-\frac{GMm}{2r} = -\frac{GMm}{5R} \Rightarrow r = \frac{5R}{2}$.

34 Since $E_T = -\frac{GMm}{2r}$ and the orbit radius will increase, the total energy increases. The engines must do positive work.

35 **a** The potential energy is given by $E_P = -\frac{GMm}{r}$. This is least when the distance to the sun, r, is smallest (remember, E_P is negative).

b Therefore since the total energy is conserved, the kinetic energy and hence the speed are greatest at P.

c The torque on the planet is zero since the force is directed towards the sun which is the centre of rotation.

d $mv_P\text{SP} = mv_A\text{SA}$ and so $\frac{v_P}{v_A} = \frac{\text{SA}}{\text{SP}}$.

36 The area is approximately a sector and so has area $\Delta A = \frac{1}{2}r^2\Delta\theta$ where $\Delta\theta = \omega\Delta t$. Hence $\Delta A = \frac{1}{2}r^2\omega\Delta t$ and $\frac{\Delta A}{\Delta t} = \frac{1}{2}r^2\omega$. But $L = mvr = m(\omega r)r = m\omega r^2$ and hence $\frac{\Delta A}{\Delta t} = \frac{L}{2m}$. Since angular momentum is conserved, the rate of change of area is constant.

37 The escape speed is $v_{esc} = \sqrt{\frac{2GM}{R}}$. At the surface of the planet, $g = \frac{GM}{R^2} \Rightarrow GM = gR^2$. Substituting: $v_{esc} = \sqrt{\frac{2gR^2}{R}} = \sqrt{2gR}$. Also, since $V = -\frac{GM}{R}$, $v_{esc} = \sqrt{-2V}$. Finally, since $M = \rho\frac{4\pi R^3}{3}$, $v_{esc} = \sqrt{\frac{2G}{R}\rho\frac{4\pi R^3}{3}} = \sqrt{\frac{8\pi G\rho R^2}{3}} = R\sqrt{\frac{8\pi G\rho}{3}}$.

38 **a** See textbook.

b $T^2 = \frac{4\pi^2 r^3}{GM}$. Now $r \approx R$ and $\rho = \frac{M}{\frac{4\pi R^3}{3}} = \frac{3M}{4\pi R^3}$. Hence, $\frac{M}{R^3} = \frac{4\pi\rho}{3}$. Substituting, $T = \sqrt{\frac{4\pi^2}{G}\frac{3}{4\pi\rho}} = \sqrt{\frac{3\pi}{G\rho}}$.

c $\frac{T_{\text{planet}}}{T_{\text{earth}}} = \sqrt{\frac{\rho_{\text{earth}}}{\rho_{\text{planet}}}} \Rightarrow \frac{\rho_{\text{earth}}}{\rho_{\text{planet}}} = \left(\frac{169}{85}\right)^2 = 3.95 \approx 4$.

39 **a** We must use the formula $T^2 = \frac{4\pi^2 R^3}{GM}$ that we have derived many times already. Now $g = \frac{GM}{R^2} \Rightarrow GM = gR^2$. Substituting, $T^2 = \frac{4\pi^2 R^3}{gR^2} = \frac{4\pi^2 R}{g}$. Hence $T = 2\pi\sqrt{\frac{R}{g}}$.

b $T = 2\pi\sqrt{\frac{3.4 \times 10^6}{4.5}} = 5.46 \times 10^3 \text{ s} = 91 \text{ min}$.

c From $T^2 = \frac{4\pi^2 R^3}{GM}$ we deduce that $\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$ hence $\frac{91^2}{140^2} = \frac{(3.4 \times 10^6)^3}{R_2^3}$ and so $R_2 = 4.5 \times 10^6$ m.

The height is therefore $h = 4.5 \times 10^6 - 3.4 \times 10^6 = 1.1 \times 10^6$ m.

40 a $F = \frac{GM^2}{4R^2}$.

b $\frac{GM^2}{4R^2} = \frac{Mv^2}{R}$ and so $v^2 = \frac{GM}{4R}$. But $v^2 = \left(\frac{2\pi R}{T}\right)^2$ and so $\frac{GM}{4R} = \left(\frac{2\pi R}{T}\right)^2$. Hence $T^2 = \frac{16\pi^2 R^3}{GM}$.

c $T = \sqrt{\frac{16\pi^2(1.0 \times 10^9)^3}{6.67 \times 10^{-11} \times 1.5 \times 2.0 \times 10^{30}}} = 2.8 \times 10^4 \text{ s} = 7.8 \text{ h}$.

d $E_T = \frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 - \frac{GM^2}{2R}$. Since $v^2 = \frac{GM}{4R}$ we have that $E_T = \frac{1}{2}M\frac{GM}{4R} \times 2 - \frac{GM^2}{2R} = \frac{GM^2}{4R} - \frac{GM^2}{2R} = -\frac{GM^2}{4R}$.

e Since energy is being lost the total energy will decrease. This implies that the distance R will decrease. (From the period formula in **b** the period will decrease as well.)

f i The total energy is $E_T = -\frac{GM^2}{4R}$ and the period is $T^2 = \frac{16\pi^2 R^3}{GM}$. Combining the two gives $E_T = -\frac{GM^2}{4\left(\frac{GMT^2}{16\pi^2}\right)^{1/3}}$ OR $E_T = -cT^{-2/3}$ where c is a constant. Working as we do with propagation of uncertainties (or using calculus) we have that $\frac{\Delta E_T}{E_T} = \frac{2}{3}\frac{\Delta T}{T}$ OR $\frac{\frac{\Delta E_T}{\Delta t}}{E_T} = \frac{2}{3}\frac{\frac{\Delta T}{\Delta t}}{T}$.

ii $\frac{\frac{\Delta E_T}{E_T}}{\Delta t} = \frac{2}{3}\frac{\frac{\Delta T}{T}}{\Delta t} = \frac{2}{3} \times \frac{72 \times 10^{-6} \text{ s yr}^{-1}}{2.8 \times 10^4 \text{ s}} = 1.7 \times 10^{-9} \text{ yr}^{-1}$.

g The lifetime is therefore $\frac{1}{1.7 \times 10^{-9} \text{ yr}^{-1}} = 5.8 \times 10^8$ yr.

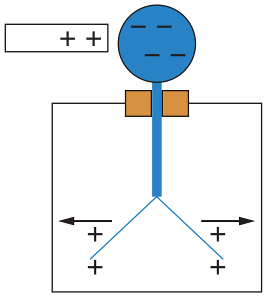
Chapter 18

Test your understanding

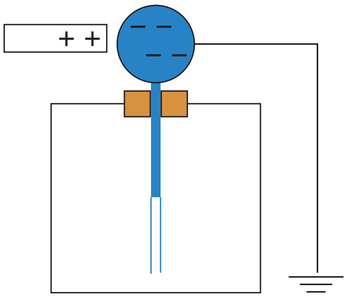
1 The net charge is +6.0 μC, and so each sphere will have 3.0 μC after separation.

2 The negative charge on the rod will push electrons away towards the leaves. The positive charge on the leaves will be reduced, and so the angle between the leaves will decrease.

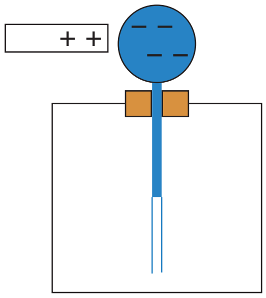
3 a



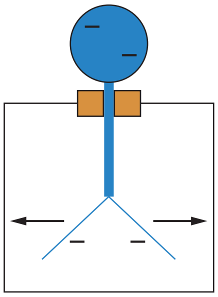
b



c



d



4 **a** $F = \frac{kq_1q_2}{r^2} = \frac{8.99 \times 10^9 \times 2.0 \times 10^{-6} \times 4.0 \times 10^{-6}}{(5.0 \times 10^{-2})^2} = 28.8 \approx 29$ N.

b **i** $F' = \frac{kq_1q_2}{(2r)^2} = \frac{F}{4}$;

ii $F' = \frac{k2q_1q_2}{(2r)^2} = \frac{F}{2}$;

iii $F' = \frac{k2q_1 \times 2q_2}{(2r)^2} = F$.

5 The middle charge is attracted to the left by the charge on the left with a force of $F_1 = \frac{kqQ_1}{r^2} = \frac{8.99 \times 10^9 \times 2.0 \times 10^{-6} \times 4.0 \times 10^{-6}}{(4.0 \times 10^{-2})^2} = 45$ N. It is attracted to the right by the charge on the right with a force of $F_2 = \frac{kqQ_2}{r^2} = \frac{8.99 \times 10^9 \times 2.0 \times 10^{-6} \times 3.0 \times 10^{-6}}{(2.0 \times 10^{-2})^2} = 135$ N. The net force is thus $135 - 45 = 90$ N directed towards the right.

6 Suppose we call the distance (in cm) from the left charge x. Then we need $\frac{8.99 \times 10^9 \times 2.0 \times 10^{-6} \times 4.0 \times 10^{-6}}{x^2} = \frac{8.99 \times 10^9 \times 2.0 \times 10^{-6} \times 3.0 \times 10^{-6}}{(6-x)^2} \Rightarrow \frac{4.0}{x^2} = \frac{3.0}{(6-x)^2}$

This means that

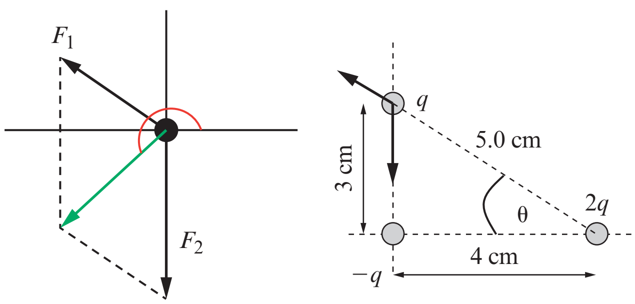
$4.0(6 - x)^2 = 3.0x^2$

$4(36 - 12x + x^2) = 3x^2$

$x^2 - 48x + 144 = 0$

The solution is $x = 3.22$ cm.

7 The forces are as shown. The distance between the charge q and the charge $2q$ is 5.0 cm.



The magnitudes are $F_1 = \frac{kq(2q)}{d^2} = \frac{8.99 \times 10^9 \times 3.0 \times 10^{-6} \times 6.0 \times 10^{-6}}{(5.0 \times 10^{-2})^2} = 64.7$ N and $F_2 = \frac{kqq}{d^2} = \frac{8.99 \times 10^9 \times (3.0 \times 10^{-6})^2}{(3.0 \times 10^{-2})^2} = 89.9$ N. We need to find the components of F_1: $F_{1x} = 64.7\cos\theta = 64.7 \times \frac{4}{5} = 51.76$ N and $F_{1y} = 64.7\sin\theta = 64.7 \times \frac{3}{5} = 38.82$ N. The components of the net force are $F_x = -51.76$ N and $F_y = 38.82 - 89.9 = -51.08$ N. The net force has magnitude $F = \sqrt{51.76^2 + 51.08^2} = 72.7 \approx 73$ N and direction $180° + \arctan\frac{51.08}{51.76} = 224.6° \approx 225°$.

8 $E = \frac{F}{q} = \frac{3.0 \times 10^{-5}}{5.0 \times 10^{-6}} = 6.0\ \text{N C}^{-1}$.

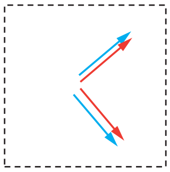
9 **a** In region I. It could be in a region where the fields from the two charges are opposite and that is regions I and III. It cannot be in III because the fields will never be equal in magnitude there since we are closer to the bigger charge.

b It has to be on the surface of the sphere with the largest charge. At the point on the surface of the blue sphere facing region II the field of the red charge gets added whereas at the point facing region III it is subtracted. Hence the maximum is in region II on the surface of the blue sphere.

10 **a** To the right.

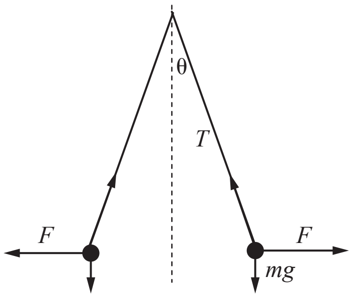
b The magnitude of the electric field at the centre from one charge alone is $E = \frac{kq}{r^2} = \frac{kq}{\frac{a^2}{2}} = \frac{2kq}{a^2}$.

The directions are:



So the net electric field is $E_{\text{net}} = 4 \times \frac{2kq}{a^2} \times \cos 45° = \frac{4kq\sqrt{2}}{a^2}$.

11 **a** In the following diagram, the angle θ of each string to the vertical is given by $\sin\theta = \frac{5}{85} \Rightarrow \theta = 3.37°$.



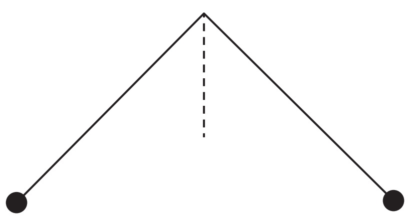
We have that

$T\cos\theta = mg$ and $T\sin\theta = F = \frac{kQ^2}{d^2}$ so that dividing side by side gives $\tan\theta = \frac{kQ^2}{mgd^2} \Rightarrow$

$$Q = \sqrt{\frac{mgd^2\tan\theta}{k}} = \sqrt{\frac{100 \times 10^{-6} \times 9.8 \times 0.1^2 \times \tan 3.37°}{8.99 \times 10^9}} = 8.0 \times 10^{-9}\text{ C}.$$

b This corresponds to $\frac{8.0 \times 10^{-9}}{1.6 \times 10^{-19}} = 5.0 \times 10^{10}$ electronic charges.

c The spheres will be further apart (symmetrically with respect to the vertical):



12 **a** Since the molar mass of water is 18 g per mole, a mass of 60 kg corresponds to $\frac{60 \times 10^3}{18} =$ 3333 moles, i.e. $3333 \times 6.02 \times 10^{23} = 2 \times 10^{27}$ molecules of water. A molecule of water contains 10 electrons (2 from hydrogen and 8 from oxygen) and so we have 2×10^{28} electrons in each person.

b The electric force is therefore $F_1 = \frac{kQ_1Q_2}{r^2} = \frac{9 \times 10^9 \times (2.0 \times 10^{28} \times 1.6 \times 10^{-19})^2}{(10)^2} = 9 \times 10^{26} \approx 10^{27}$ N, an enormous force.

c Assumptions include the use of Coulomb's law for objects that are not point charges, assuming the same distance between charges and so forth.

d We have neglected the existence of protons, which gives each person a zero electric charge and hence zero electric force.

13 The magnitude of each of the fields produced at P is $E = \frac{kQ}{r^2} = \frac{8.99 \times 10^9 \times 2.00 \times 10^{-6}}{\left(\sqrt{0.05^2 + 0.30^2}\right)^2} = 1.94 \times 10^5$ N C^{-1}.

The vertical components of the electric fields will cancel out leaving only the horizontal components.

The horizontal component is $E_x = 2E\cos\theta = 2 \times 1.94 \times 10^5 \times \frac{0.30}{\sqrt{0.05^2 + 0.30^2}} = 3.8 \times 10^5$ N C^{-1}.

(The factor of 2 is because we have two components to add.)

14 The two electric fields are $E_1 = E_2 = 1.94 \times 10^5$ N C^{-1}. Adding vectorially by taking components gives $E_x = 0$ and $E_y = 2 \times 1.94 \times 10^5 \times \sin\theta = 2 \times 1.94 \times 10^5 \times \frac{0.05}{\sqrt{0.05^2 + 0.30^2}} = 6.4 \times 10^4$ N C^{-1}.

15

$$W = B + D$$

$$mg = \rho_{air}Vg + 6\pi\eta rv$$

$$\rho\frac{4\pi}{3}r^3g = \rho_{air}\frac{4\pi}{3}r^3g + 6\pi\eta rv$$

$$\frac{4\pi}{3}r^3g(\rho - \rho_{air}) = 6\pi r\eta v$$

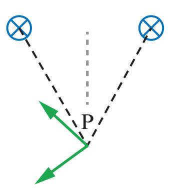
$$r^2 = \frac{3 \times 6\eta v}{4(\rho - \rho_{air})g}$$

$$r = \sqrt{\frac{9\eta v}{2(\rho - \rho_{air})g}}$$

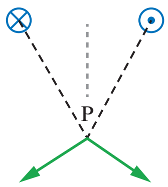
16 We look at the smallest charge and see if the other charges are integral multiples of it. In this case they are not. So we look at divisors of the smallest charge, for example 48×10^{-20}. All measured charges are multiples of this quantity. Of course any divisor of 48 would also do, but we have no evidence that this is the case so we would record 48×10^{-20} C as the fundamental charge.

17 At P it is zero. At Q it points towards the bottom of the page.

18



The diagram shows the magnetic fields from each wire. The net field is to the left.

19 

The diagram shows the magnetic fields from each wire. The net field is vertically down.

20 Into the page at P and Q and out of the page at R.

21 a B is into the page.

b F is into the page.

c B is out of the page.

d F is zero.

e F is zero.

22 The force is attractive; the ring behaves like a bar magnet with the north pole pointing left.

23 The force is attractive; the rings behave like a bar magnet with the north pole pointing left.

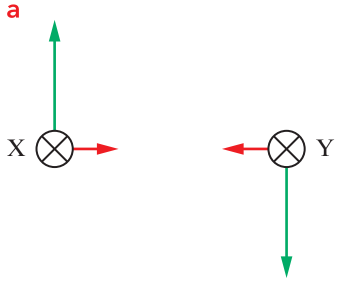
24 a The force is towards the top of the page.

b The force is zero because the magnetic field is zero.

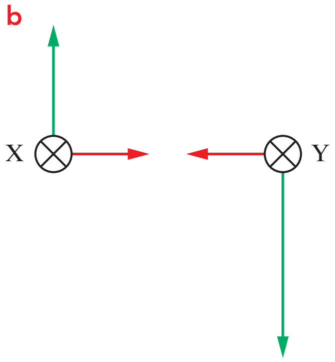
25 Into the page.

26 The ball will again be attracted to the left.

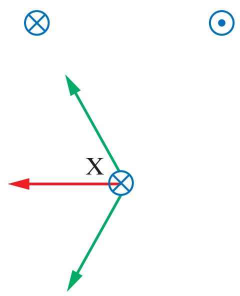
27 a



b



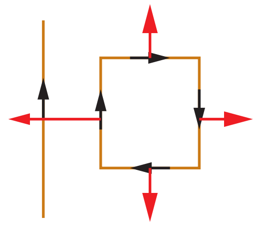
28



The diagram shows the forces from each wire. The net force is to the left.

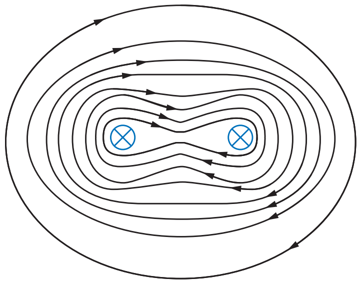
29 Out of the page; the magnetic field is pointing to the left.

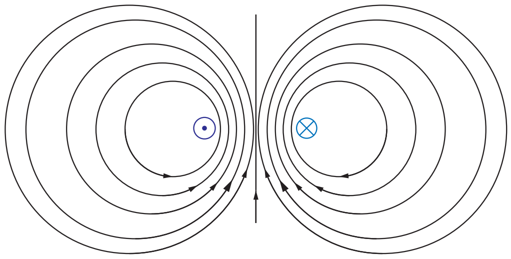
30



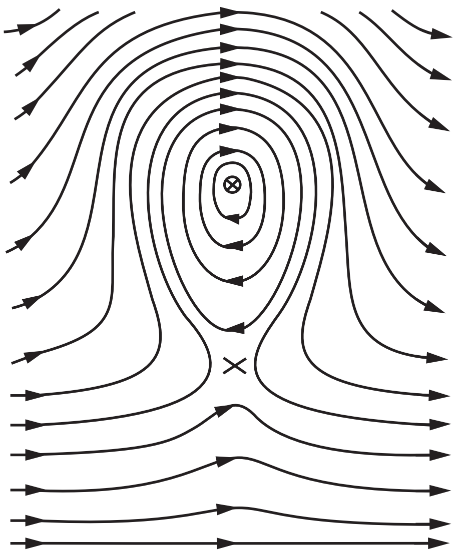
The net force is to the left.

31





32



33 The magnetic field is directed into the page. In **a** the right hand rule (for a negative charge) gives a force downwards away from the wire. In **b** it gives a force to the right.

34 **a** The field is to the right and so the force is into the page.

b The velocity is parallel to the field and the force is zero.

c The force is towards the magnet (up the page).

35 **a** There are equal and opposite forces at the poles of the magnet giving a net force of zero.

b The forces are opposite so they will rotate the magnet counterclockwise.

36 The force is $F = BIL\sin\theta = 5.00 \times 10^{-5} \times 3000 \times 30.0 \times \sin 30° = 2.25$ N.

37 **a** The combined magnetic field from the two wires at point R must point downwards so as to cancel the uniform field. Since R is closer to Q, the field of Q is larger than the field from P. Hence the current in Q must go out of the page.

b If the current increases, the net field from P and Q increases as well, so that the total field at R is no longer zero. If we move closer to Q the field from Q will be much larger than the field from P and so their combined field will be downwards and much larger than external field. Hence the point has to move to the left.

38 **a** $V = \frac{kq}{d/2} + \frac{kq}{d/2} = \frac{4kq}{d}$.

b $V = \frac{kq}{d/2} - \frac{kq}{d/2} = 0$.

39 The potential at P is $V = \frac{8.99 \times 10^9 \times 2.0 \times 10^{-6}}{0.4} - \frac{8.99 \times 10^9 \times 4.0 \times 10^{-6}}{0.6} = -1.5 \times 10^4$ V.

40 **a** $V = 4 \times \frac{kq}{r}$ where $r = 0.050\sqrt{2}$ m. Hence $V = 4 \times \frac{8.99 \times 10^9 \times 5.0 \times 10^{-6}}{0.050\sqrt{2}} = 2.5 \times 10^6$ V.

b $E = 0$

c The field is the gradient of the potential. The field can be zero when the potential is not.

41 a The work done is $W = q\Delta V = q\left(\frac{kQ}{r_2} - \frac{kQ}{r_2}\right) = 1.0 \times 10^{-3} \times \left(\frac{8.99 \times 10^9 \times 10}{2.0} - \frac{8.99 \times 10^9 \times 10}{10}\right) =$ 3.6×10^7 J.

b No, the work done by electric and gravitational forces is path independent.

42 The potential at a distance of 12 cm from the point charge is $V = \frac{kQ}{r} = \frac{8.99 \times 10^9 \times (-15 \times 10^{-9})}{0.12}$ $= -1.12 \times 10^3$ V. The work done is $W = e\Delta V = (-1.6 \times 10^{-19}) \times (-1.12 \times 10^3 - 0) = +1.8 \times 10^{-16}$ J.

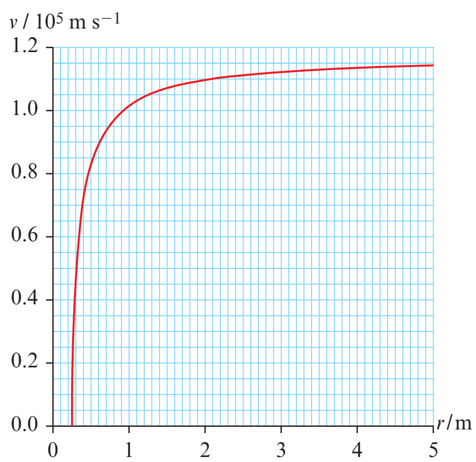
43 The work done on the electron by the *electric field* is $W = -q\Delta V = -(-1.6 \times 10^{-19}) \times (200 - 100) = +1.6 \times 10^{-17}$ J. This work equals the increase of the kinetic energy of the electron. Hence $\frac{1}{2}mv^2 = +1.6 \times 10^{-17}\ \text{J} \Rightarrow v = \sqrt{\frac{2 \times 1.6 \times 10^{-17}}{9.1 \times 10^{-31}}} = 5.9 \times 10^6\ \text{m s}^{-1}$

Alternatively, you can use conservation of energy, $E_A = E_B$ so that:

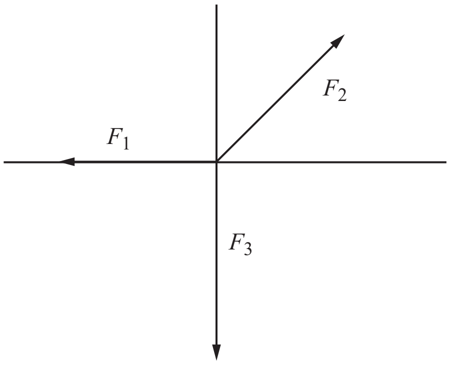
$$0 + \underbrace{(-1.6 \times 10^{-19}) \times 100}_{\text{electric potential energy}} = \underbrace{\frac{1}{2}mv^2}_{\text{kinetic energy}} + \underbrace{(-1.6 \times 10^{-19}) \times 200}_{\text{electric potential energy}}.$$

44 a The total energy of the proton when at the surface of the sphere is $E = \frac{kQe}{R} =$ $\frac{8.99 \times 10^9 \times 2.0 \times 10^{-9} \times 1.6 \times 10^{-19}}{0.25} = 1.151 \times 10^{-17}$ J. All of this energy will be converted to kinetic energy when far from the sphere: $\frac{1}{2}mv^2 = 1.151 \times 10^{-17} \Rightarrow v = \sqrt{\frac{2 \times 1.151 \times 10^{-17}}{1.67 \times 10^{-27}}} = 1.174 \times 10^5\ \text{m s}^{-1}$.

b At a distance r from the centre of the sphere we have $\frac{1}{2}mv^2 + \frac{kQe}{r} = 1.151 \times 10^{-17}$ J. Since $\frac{1}{2}m(1.174 \times 10^5)^2 = 1.151 \times 10^{-17}$ we may write $\frac{1}{2}mv^2 + \frac{kQe}{r} = \frac{1}{2}m(1.174 \times 10^5)^2 \Rightarrow$ $v = \sqrt{(1.174 \times 10^5)^2 - \frac{2kQe}{mr}}$, i.e. $v = 10^5\sqrt{1.378 - \frac{0.345}{r}}$. The graph is



45 a The forces are roughly as follows.



They have magnitudes:

$$F_1 = \frac{8.99 \times 10^9 \times 1 \times 10^{-6} \times 2 \times 10^{-6}}{0.05^2} = 7.19\ \text{N}$$

$$F_2 = \frac{8.99 \times 10^9 \times 4 \times 10^{-6} \times 2 \times 10^{-6}}{0.05^2 + 0.05^2} = 14.4\ \text{N}$$

$$F_3 = \frac{8.99 \times 10^9 \times 3 \times 10^{-6} \times 2 \times 10^{-6}}{0.05^2} = 21.6\ \text{N}$$

We must find the components of F_2:

$F_{2x} = F_2 \cos 45° = 10.2$ N and $F_{2y} = F_2 \sin 45° = 10.2$ N. So the net force has components:

$F_x = 10.2 - 7.2 = 3.0$ N and $F_y = 10.2 - 21.6 = 11.4$ N. The net force is then $F = \sqrt{(11.4)^2 + (3.0)^2} =$ 11.8 N. The direction of the net force is $\arctan\left(\frac{-11.4}{3.0}\right) = -75°$.

b The distance of the centre of the square from each of the vertices is $a = \sqrt{0.025^2 + 0.025^2} =$ 0.0354 cm.

So the potential at the centre is $V = \frac{kQ_1}{a} + \frac{kQ_2}{a} + \frac{kQ_3}{a} + \frac{kQ_4}{a} = \frac{8.99 \times 10^9}{0.0354} \times (-1 \times 10^{-6} + 2 \times 10^{-6} - 3 \times 10^{-6} + 4 \times 10^{-6})$

$V = 5.1 \times 10^5\,\text{V}$

c The work done is $W = q\Delta V = q(V - 0) = 1.0 \times 10^{-9} \times 5.1 \times 10^5 = 5.1 \times 10^{-4}$ J.

46 a The kinetic energy of the electron is $\frac{1}{2}mv^2 = \frac{1}{2} \times 9.1 \times 10^{-31} \times (1.5 \times 10^6)^2 = 1.02 \times 10^{-18}$ J. The potential energy of the electron at the positive plate is $8.0 \times (-1.6 \times 10^{-19}) = -1.28 \times 10^{-18}$ J and so the total energy is $E_T = (1.02 - 1.28) \times 10^{-18} = -0.26 \times 10^{-18}$ J. At the negative plate the potential energy would be zero. The kinetic energy would then be negative, which is impossible, so the electron will not make it to the negative plate.

b The electron stops at the point where its potential energy is -0.26×10^{-18} J. Hence the potential is $\frac{-0.26 \times 10^{-18}}{-1.6 \times 10^{-19}} = 1.625$ V. The potential inside the plates is given by $V = 8.0 - \frac{8.0}{0.12}d$, where d is the distance from the positive plate. So, $1.625 = 8.0 - \frac{8.0}{0.12}d \Rightarrow d = 9.6$ cm.

c The total energy at the positive plate has to be zero. So, $\frac{1}{2}mv^2 - 1.28 \times 10^{-18} = 0 \Rightarrow$

$v = \sqrt{\frac{2 \times 1.28 \times 10^{-18}}{9.1 \times 10^{-31}}} = 1.7 \times 10^6\ \text{m s}^{-1}$.

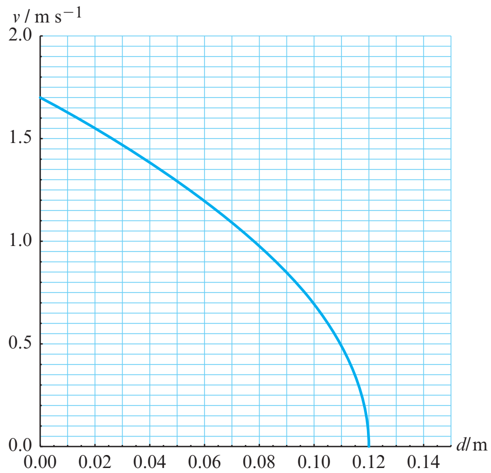
d $\frac{1}{2}mv^2 + (-e)V = 0$

$\frac{1}{2}mv^2 + (-e)\left(8.0 - \frac{8.0}{0.12}d\right) = 0$

$v = \sqrt{\frac{2e}{m}(8.0 - \frac{8.0}{0.12}d)} = 1.7 \times 10^6$

$\sqrt{1 - \frac{d}{0.12}}\ \mathrm{m\,s^{-1}}$

The graph is:



47 a Charge will move until both spheres are at the same potential. Then $\frac{kq_1}{r_1} = \frac{kq_2}{r_2}$. By conservation of charge, $q_1 + q_2 = Q$ where Q is the charge on the one sphere originally. Thus $\frac{q_1}{10} = \frac{q_2}{15} \Rightarrow 3q_1 = 2q_2$ and $q_1 + q_2 = 2.0$. Hence $q_1 = \frac{2}{5} \times 2.0 = 0.80\ \mu\mathrm{C}$ and $q_2 = \frac{3}{5} \times 2.0 = 1.2\ \mu\mathrm{C}$.

b $\sigma_1 = \frac{0.80 \times 10^{-6}}{4\pi \times 0.10^2} = 6.4 \times 10^{-6}\ \mathrm{C\,m^{-2}}$ and $\sigma_2 = \frac{1.2 \times 10^{-6}}{4\pi \times 0.15^2} = 4.2 \times 10^{-6}\ \mathrm{C\,m^{-2}}$.

c $E_1 = \frac{kq_1}{r_1^2} = 4\pi k\sigma_1 = 4\pi \times 8.99 \times 10^9 \times 6.4 \times 10^{-6} = 7.2 \times 10^5\ \mathrm{N\,C^{-1}}$ and

$E_2 = 4\pi k\sigma_2 = 4\pi \times 8.99 \times 10^9 \times 4.2 \times 10^{-6} = 4.8 \times 10^5\ \mathrm{N\,C^{-1}}$.

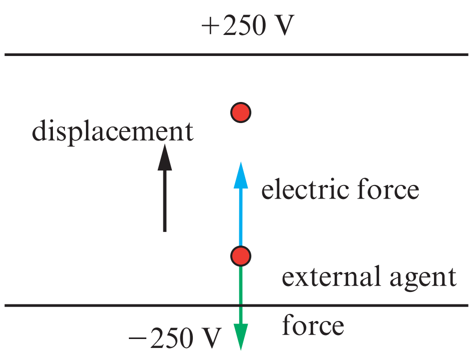
d The electric field is largest for the sphere with the larger charge density. The wire has to be long so that the charge of one sphere will not affect the charge distribution on the other so that both are uniformly charged.

48 Lines normal to the equipotentials.

49 a The potential (in V) a distance x from the bottom plate is given by $V = -250 + \frac{250 - (-250)}{0.15}x = (-250 + 3.33 \times 10^3 x)$ and so at $x = 3.00$ cm, $V = (-250 + 3.33 \times 10^3 \times 0.030) = -150$ V. Therefore the electric potential energy of the charge is $E_P = qV = (-2.00 \times 10^{-6}) \times (-150) = 0.300$ mJ.

b The potential at $x = 12.0$ cm is $V = (-250 + 3.33 \times 10^3 \times 0.120) = 150$ V and hence $E_P = qV = (-2.00 \times 10^{-6}) \times 150 = -0.300$ mJ.

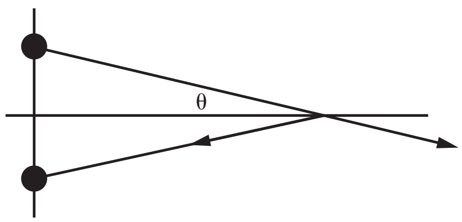
c The work done is $W = q\Delta V = \Delta E_P = -0.300 - 0.300 = -0.600$ mJ. (This is the work done by an external agent moving the charge at constant speed. It is negative because the external agent must exert a force (green arrow) opposite to the electric force (blue arrow). The work done by the electric force on the charge is + 0.600 mJ.)



50 a The kinetic energy of the electron $E_K = \frac{1}{2}mv^2 = \frac{1}{2} \times 9.1 \times 10^{-31} \times (1.59 \times 10^6)^2 = 1.15 \times 10^{-18}$ J gets converted to electric potential energy eV at the point where the electron stops. Hence the potential at P is $V = \frac{1.15 \times 10^{-18}}{-1.6 \times 10^{-19}} = -7.19$ V.

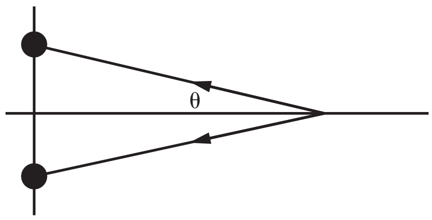
b $V = \frac{kq}{r} \Rightarrow q = \frac{Vr}{k} = \frac{(-7.19) \times 2.0 \times 10^{-10}}{8.99 \times 10^9} = -1.6 \times 10^{-19}$ C.

51 a The field due to each of the charges has the direction shown. It is clear that the net field will point in the negative y-direction.



The magnitude of the field due to one of the charges is $E = \frac{kQ}{r^2} = \frac{kQ}{a^2 + d^2}$. The y-component is $E_y = \frac{kQ}{a^2 + d^2} \sin\theta = \frac{kQ}{a^2 + d^2} \frac{a}{\sqrt{a^2 + d^2}} = \frac{kQa}{(a^2 + d^2)^{3/2}}$ and so the net field is $E_{net} = \frac{2kQa}{(a^2 + d^2)^{3/2}}$.

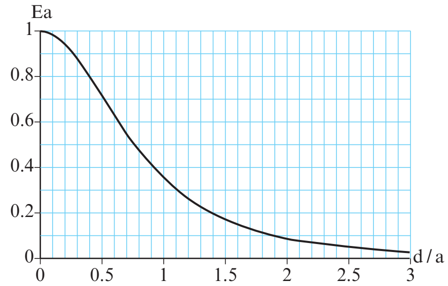
b For two negative charges:

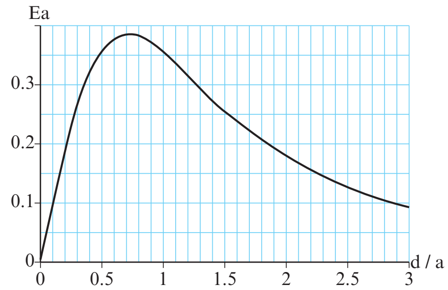


The net field is clearly directed to the left. It has magnitude $E_{net} = 2E_x = \frac{2kQ}{a^2 + d^2} \cos\theta = \frac{2kQ}{a^2 + d^2} \frac{d}{\sqrt{a^2 + d^2}}$ $= \frac{2kQd}{(a^2 + d^2)^{3/2}}$.

c We have $E_a = \frac{2kQa}{(a^2+d^2)^{3/2}} = \frac{2kQ}{a^2}\frac{1}{\left(1+\frac{d^2}{a^2}\right)^{3/2}}$ and $E_b = \frac{2kQd}{(a^2+d^2)^{3/2}} = \frac{2kQ}{a^3}\frac{d}{\left(1+\frac{d^2}{a^2}\right)^{3/2}} = \frac{2kQ}{a^2}\frac{d/a}{\left(1+\frac{d^2}{a^2}\right)^{3/2}}$

The plots are (the vertical axis is in units of $\frac{2kQ}{a^2}$):





52 The initial potential energy of the three protons is zero. When at the vertices of the triangle of side a the potential energy is $E_P = 3 \times \frac{k(e)(e)}{a} = \frac{3ke^2}{a}$ since there are three pairs of charges a distance a apart. This evaluates to $E_P = \frac{3 \times 8.99 \times 10^9 \times (1.6 \times 10^{-19})^2}{5.0 \times 10^{-15}} = 1.38 \times 10^{-13}$ J ≈ 0.86 MeV. This is the energy that must be supplied.

53 **a** Force towards the centre of the circle.

b We equate the electric force to the centripetal force to get $\frac{kq^2}{r^2} = m\frac{v^2}{r}$. Solving for the speed gives the answer.

c The total energy is kinetic plus electric potential energy: $E = \frac{1}{2}mv^2 - \frac{kq^2}{r}$. Using the previous result for speed: $v^2 = \frac{kq^2}{mr}$ gives $E = \frac{1}{2}m\frac{kq^2}{mr} - \frac{kq^2}{r} = \frac{1}{2}\frac{kq^2}{r} - \frac{kq^2}{r} = -\frac{1}{2}\frac{kq^2}{r}$.

d The change in energy is an increase of $\Delta E = -\frac{1}{2}\frac{kq^2}{2r} - \left(-\frac{1}{2}\frac{kq^2}{r}\right) = +\frac{kq^2}{4r}$, and this is the energy that must be supplied.

54 **a** As in the previous problem $v^2 = k\frac{e^2}{mr}$. Using also $v = \frac{2\pi r}{T}$ we get $\frac{4\pi^2 r^2}{T^2} = k\frac{e^2}{mr} \Rightarrow T^2 = \frac{4\pi^2 m}{ke^2}r^3$.

b $T = \sqrt{\frac{4\pi^2 \times 9.1 \times 10^{-31}}{8.99 \times 10^9 \times (1.6 \times 10^{-19})^2} \times (0.5 \times 10^{-10})^3} = 1.397 \times 10^{-16} \approx 1.4 \times 10^{-16}$ s.

c The energy is $E = -\frac{ke^2}{2r}$. In the first orbit this evaluates to $E_1 = -\frac{8.99 \times 10^9 \times (1.6 \times 10^{-19})^2}{2 \times 0.5 \times 10^{-10}} \approx$ -2.30×10^{-18} J. In the other orbit this becomes $E_2 = -\frac{8.99 \times 10^9 \times (1.6 \times 10^{-19})^2}{2 \times 2.0 \times 10^{-10}} \approx -5.75 \times 10^{-19}$ J.

The change is $+1.7 \times 10^{-18}$ J.

55 $\frac{1}{2}mv^2 = \frac{k(2e)(79e)}{d} \Rightarrow v = \sqrt{\frac{2k(2e)(79e)}{md}} = \sqrt{\frac{2 \times 8.99 \times 10^9 \times 2 \times 79 \times (1.6 \times 10^{-19})^2}{6.6 \times 10^{-27} \times 7.0 \times 10^{-15}}} = 4.0 \times 10^7\ \mathrm{m\,s^{-1}}$.

Chapter 19

Test your understanding

1 In a uniform electric field the force would have a constant direction. For circular motion, the force must be directed towards the centre of the circle and so keeps changing direction.

2 **a** The kinetic energy is $|q\Delta V|$ and so is the same.

b Since kinetic energy is the same: $\frac{1}{2}m_e v_e^2 = \frac{1}{2}m_p v_p^2 \Rightarrow \frac{v_e}{v_p} = \sqrt{\frac{m_p}{m_e}} \approx 43$.

c The acceleration is $a = \frac{F}{m} = \frac{qE}{m} = \frac{qV}{md}$. Then $v = at \Rightarrow t = \frac{v}{a} = \frac{vmd}{qV}$. Hence $\frac{t_e}{t_p} = \frac{v_e}{v_p} \times \frac{m_e}{m_p} = \sqrt{\frac{m_p}{m_e}}\frac{m_e}{m_p} = \sqrt{\frac{m_e}{m_p}} \approx \frac{1}{43}$.

3 **a** The kinetic energy is $q\Delta V$ and so that of the alpha particle is double that of the proton.

Therefore: $\frac{1}{2}m_\alpha v_\alpha^2 = 2 \times \frac{1}{2}m_p v_p^2 \Rightarrow \frac{v_p}{v_\alpha} = \sqrt{\frac{m_\alpha}{2m_p}} = \sqrt{2}$.

b The acceleration is $a = \frac{F}{m} = \frac{qE}{m} = \frac{qV}{md}$. Then $v = at \Rightarrow t = \frac{v}{a} = \frac{vmd}{qV}$. Hence $\frac{t_p}{t_\alpha} = \frac{v_p}{v_\alpha} \times \frac{m_p}{m_\alpha} \times \frac{2e}{e} = \sqrt{2} \times \frac{1}{4} \times 2 = \frac{\sqrt{2}}{2}$.

4 The proton will leave the plates after a time of $t = \frac{0.24}{1.8 \times 10^5} = 1.33 \times 10^{-6}$ s.

The vertical acceleration of the proton is $a = \frac{F}{m} = \frac{qE}{m} = \frac{qV}{md} = \frac{1.6 \times 10^{-19} \times 60}{1.67 \times 10^{-27} \times 0.08} = 7.186 \times 10^{10}\ \mathrm{m\,s^{-2}}$.

The vertical distance travelled is then $y = \frac{1}{2}at^2 = \frac{1}{2} \times 7.186 \times 10^{10} \times (1.33 \times 10^{-6})^2 = 6.36 \times 10^{-2}\ \mathrm{m} = 6.4$ cm.

5 **a** The vertical component of velocity is $u \sin 30° = \frac{u}{2}$. The acceleration of the proton in the vertical direction is $a = \frac{F}{m} = \frac{qE}{m} = \frac{qV}{md} = \frac{1.6 \times 10^{-19} \times 150}{1.67 \times 10^{-27} \times 0.04} = 3.593 \times 10^{11}\ \mathrm{m\,s^{-2}}$. The proton's vertical velocity must become zero before travelling 0.04 m in the vertical direction. So, $0 = \left(\frac{u}{2}\right)^2 - 2ad \Rightarrow u^2 = 8ad \Rightarrow u^2 = 4 \times 2 \times 3.593 \times 10^{11} \times 0.04$ and so $u = 2\sqrt{2 \times 3.593 \times 10^{11} \times 0.04} = 3.39 \times 10^5\ \mathrm{m\,s^{-1}}$.

b The proton reaches the top of its path in a time $\frac{0.04}{\frac{3.39 \times 10^5}{2}} = 2.36 \times 10^{-7}$ s. In this time the proton moves a horizontal distance $x = 3.39 \times 10^5 \times \cos 30° \times 2.36 \times 10^{-7} = 6.9$ cm. This is more than half the length of the plates so the proton will miss the lower plate.

c From $u = \sqrt{8ad} = \sqrt{\frac{8eV}{md}d} = \sqrt{\frac{8eV}{m}}$ and so changing d makes no difference to the maximum speed.

6 **a** Electric case: $0 = (u \sin\theta)^2 - 2ah \Rightarrow h = \frac{(u\sin\theta)^2}{2a}$. The acceleration is $a = \frac{F}{m} = \frac{qE}{m} = \frac{qV}{md}$ and so $h = \frac{(u\sin\theta)^2 md}{2qV}$. Thus doubling the mass doubles h. Gravitational case: $0 = (u\sin\theta)^2 - 2gH \Rightarrow H = \frac{(u\sin\theta)^2}{2g}$ so H is independent of mass.

b From $h = \frac{(u\sin\theta)^2 md}{2qV}$, doubling the charge halves h.

7 By moving along the magnetic field so that it experiences no magnetic force.

8 $qvB = \frac{mv^2}{R} \Rightarrow R = \frac{mv}{qB} = \frac{9.1 \times 10^{-31} \times 3.2 \times 10^7}{1.6 \times 10^{-19} \times 0.025} = 7.28$ mm

9 $qvB = \frac{mv^2}{R} \Rightarrow B = \frac{mv}{qR} = \frac{1.67 \times 10^{-27} \times 3.0 \times 10^7}{1.6 \times 10^{-19} \times 0.75} = 0.42$ T.

10 $qvB = \frac{mv^2}{R} \Rightarrow v = \frac{qBR}{m} = \frac{1.6 \times 10^{-19} \times 0.35 \times 0.12}{1.67 \times 10^{-27}} = 4.0 \times 10^6\ \text{m s}^{-1}$

11 a $evB = \frac{m_p v^2}{R} \Rightarrow R = \frac{m_p v}{eB}$. The time for one revolution is found from $T = \frac{2\pi R}{v} = \frac{2\pi}{v}\frac{m_p v}{eB} = \frac{2\pi m_p}{eB}$.

The number of revolutions per second is the frequency $\frac{1}{T} = \frac{eB}{2\pi m_p}$.

b It becomes half as large.

12 a $\frac{1}{2}mv^2 = eV \Rightarrow v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 250}{9.1 \times 10^{-31}}} = 9.376 \times 10^6 \approx 9.4 \times 10^6\ \text{m s}^{-1}$.

b $t = \frac{\pi R}{v} \Rightarrow R = \frac{vt}{\pi} = \frac{9.376 \times 10^6 \times 1.5 \times 10^{-9}}{\pi} = 4.477 \approx 4.5$ mm.

c $B = \frac{mv}{qR} = \frac{9.1 \times 10^{-31} \times 9.376 \times 10^6}{1.6 \times 10^{-19} \times 4.476 \times 10^{-3}} = 0.012$ T.

13 a $\frac{1}{2}mv^2 = eV \Rightarrow v^2 = \frac{2eV}{m}$. From $evB = \frac{mv^2}{R} \Rightarrow v = \frac{eBR}{m}$ and so $\frac{2eV}{m} = \frac{e^2B^2R^2}{m^2} \Rightarrow \frac{e}{m} = \frac{2V}{B^2R^2}$.

b $\frac{e}{m} = \frac{2 \times 490}{(1.2 \times 10^{-3})^2 \times 0.061^2} = 1.829 \times 10^{11}\ \text{C kg}^{-1}$.

c $m = \frac{1.6 \times 10^{-19}}{1.829 \times 10^{11}} = 8.7 \times 10^{-31}$ kg.

14 $R = \frac{mv}{qB} \Rightarrow m = \frac{qBR}{v}$ and so $\Delta m = \frac{qB}{v}\Delta R = \frac{1.6 \times 10^{-19} \times 0.21}{1.3 \times 10^6} \times 0.13 = 3.4 \times 10^{-27}$ kg.

15 a The electric force on the electron is directed towards the top of the page so the magnetic force must be in the opposite direction. This means the magnetic field is directed into the page.

$B = \frac{E}{v} = \frac{2.4 \times 10^3}{2.0 \times 10^5} = 1.2 \times 10^{-2}$ T

b For the electron we have that $qE = qvB$ and charge cancels out. So, for a proton with the same speed, the electric and magnetic forces are reversed in direction but they are still equal and opposite, so the proton is undeflected.

c The same applies to an alpha particle.

d The magnetic force is now larger than the electric force so the electron would be deflected towards the bottom of the page.

16 a The vertical component of velocity is parallel to the magnetic field. Hence there is no magnetic force and the particle moves in the vertical direction with constant speed. The horizontal velocity component is at right angles to the magnetic field so there is a force at right angles to both the velocity component and the magnetic field, forcing the particle in a circular path. The two motions combined result in a spiral.

b $ev\cos\theta B = \frac{m(v\cos\theta)^2}{R} \Rightarrow R = \frac{mv\cos\theta}{eB}$.

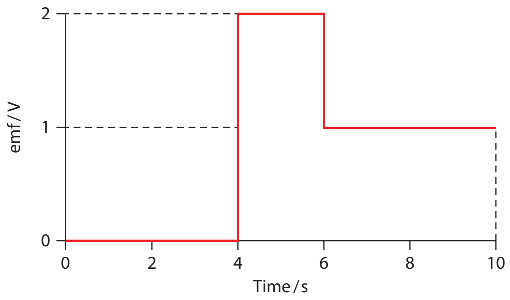
c The time of a revolution is $T = \frac{2\pi R}{v\cos\theta} = \frac{2\pi}{v\cos\theta}\frac{mv\cos\theta}{eB} = \frac{2\pi m}{eB}$. Hence $p = v\sin\theta\, T = \frac{2\pi m}{eB}v\sin\theta$.

Chapter 20

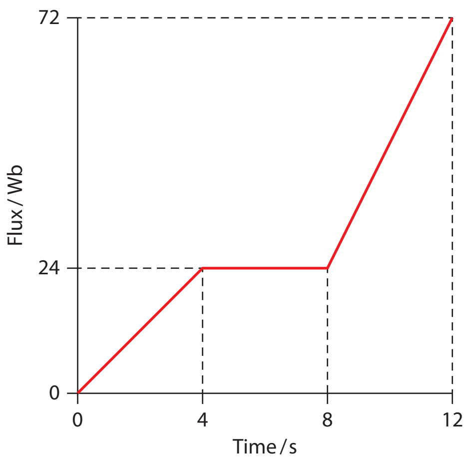
Test your understanding

1 The emf is constant at 2.0 V, the gradient of the graph.

2



3 a



b The same graph shifted up or down would give the same induced emf. It is the gradient that counts.

4 In the position shown, the flux is $5 \times 0.35 \times 0.20 = 0.35$ Wb.

After the rotation it will be −0.35 Wb. Hence $\bar{\varepsilon} = \frac{\Delta\Phi}{\Delta t} = \frac{0.70}{0.50} = 1.4$ V.

5 The induced emf is BvL so the loop entering first having a larger length L will have the larger induced emf.

6 Zero, because the magnetic field is tangent to the area of the cylinder.

7 The magnetic field in the small solenoid on the left is directed into the page. The flux is increasing and so the induced current will create a magnetic field coming out of the page. So the current must be counterclockwise.

The magnetic field in the small solenoid on the right is directed out of the page. The flux is increasing and so the induced current will create a magnetic field coming into the page. So the current must be clockwise.

8 **a** The flux in the ring is increasing because the magnetic field is greater near the pole so we have to oppose this increase by creating a magnetic field in the opposite direction, i.e. down. Hence the current has to be clockwise, as seen from above.

OR

The force on the ring must be upward. Hence the ring must behave like a bar magnet with its north pole down. Hence the current has to be clockwise as seen from above.

b The magnetic field at the loop is directed downwards and increasing so the induced current will produce a field upwards. Hence the current is clockwise as seen from above.

9 **a** The flux in the ring is increasing so we have to oppose this increase by creating a magnetic field in the opposite direction, i.e. up. Hence the current has to be counterclockwise as seen from above.

b The magnetic field is upward so opposing the increase means creating a field downward. Hence the current is clockwise as seen from above.

10 **a** The current is counterclockwise and so the ring behaves as a bar magnet with the north pole up. The magnets repel so the force on the ring is downward.

b The current is clockwise and so the ring behaves as a bar magnet with the north pole down. The magnets attract so the force on the ring is downward.

11 The velocity of an electron is towards the bottom of the page, and the field is into the page. So the magnetic force on the electron will be to left. Electrons will then accumulate on the left end leaving an excess positive charge on the right end.

12 **a** The flux is increasing and the field due to the wire is coming out of the page. So the loop will produce a field into the page, and hence the current is clockwise.

b The flux is decreasing and the field due to the wire is coming out of the page. So the loop will produce a field out of the page, and hence the current is counterclockwise.

13 The flux is increasing at the loop's position so to oppose this increase the loop will move away from the magnet i.e. to the left.

14 $\varepsilon = \frac{\Delta\Phi}{\Delta t} = N\frac{\Delta B}{\Delta t}A = 200 \times 0.45 \times \pi \times 0.01^2 = 2.8 \times 10^{-2}$ V.

15 The flux is increasing at the loop's position so to oppose this increase the rod will move so as to decrease the area of the loop, i.e. to the right.

OR

The field is increasing so the flux in the loop is increasing. A current will be induced to oppose the increase in flux. The current will produce a magnetic field directed out of the page. So the current will be counterclockwise. Hence the magnetic force on the rod will be directed to the right.

16 **a** Zero.

b $\overline{\varepsilon} = \frac{\Delta\Phi}{\Delta t} = N\frac{\Delta B}{\Delta t}A = 50 \times \frac{6.0}{8.0 \times 10^{-6}} \times 0.40 = 1.5 \times 10^7$ V.

17 **a** Zero; $\Phi = B\frac{A}{2} - B\frac{A}{2} = 0$.

b The flux is zero initially and stays zero so the flux does not change. Hence emf = 0.

c Suppose the fields are changing at a rate $r = \frac{\Delta B}{\Delta t}$. At $t = 0$, $\Phi = 0$. After time Δt the flux is $\Phi = (B_0 + r\Delta t)\frac{A}{2} - (B_0 - r\Delta t)\frac{A}{2} = rA\Delta t$ where B_0 is the field at $t = 0$. So the change in flux is $\Delta\Phi = rA\Delta t$ and there is an induced emf $\frac{\Delta\Phi}{\Delta t} = rA$.

18 **a** **i** Standard undamped harmonic oscillations graph.

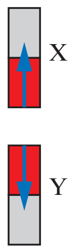
ii Damped harmonic oscillations with decreasing amplitude.

b When the switch is open there is no current induced and no magnetic force acting on the magnet. With the switch closed, the induced emf will force a current in the loop. This current will be opposing the changes in flux so there will be a magnetic force opposing the motion of the magnet.

19 There will be an induced emf in the loop to the right and so a current will be established and thermal energy generated. This energy will come from the kinetic energy of the loop. Hence the loop to the right will have a smaller average speed and will fall last.

20 **a** **i** The magnetic field at the lower loop is pointing downwards. The flux is increasing so the induced current in Y has to produce a magnetic field pointing upwards. So the current must be counterclockwise.

ii The loops repel so the force on Y is downwards. The loops behave as the bar magnets shown below.

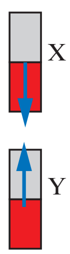


b **i** Yes.

ii No, because the flux is constant.

c **i** The flux will be decreasing so there will be an emf induced.

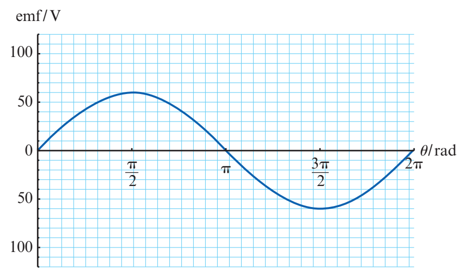
ii The magnetic field at the lower loop is pointing downwards. The flux is decreasing so the induced current in Y has to produce a magnetic field pointing downwards. So the current must be clockwise. The loops now attract, so the force on Y is upwards. The loops behave as the bar magnets below.



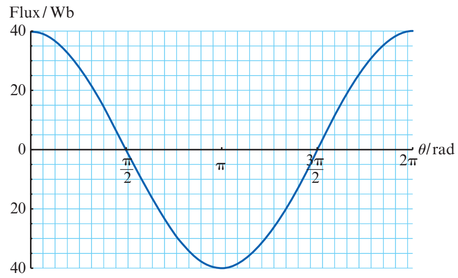
21 **a** $I = \frac{V}{R} = \frac{1.5}{3.0} = 0.50$ A.

b There will be an induced emf $BvL = 0.30 \times 5.0 \times 0.20 = 0.30$ V as the loop enters the magnetic field. This emf will oppose the change in flux so the net emf in the circuit is $1.5 - 0.30 = 1.20$ V. The current is then $I = \frac{V}{R} = \frac{1.2}{3.0} = 0.40$ A.

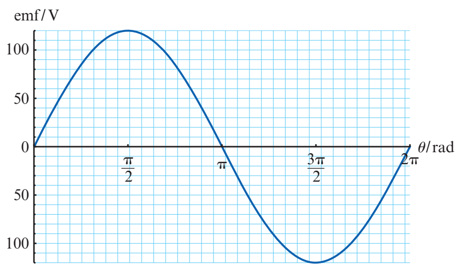
22 a



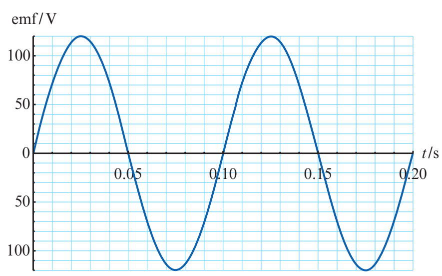
b



c

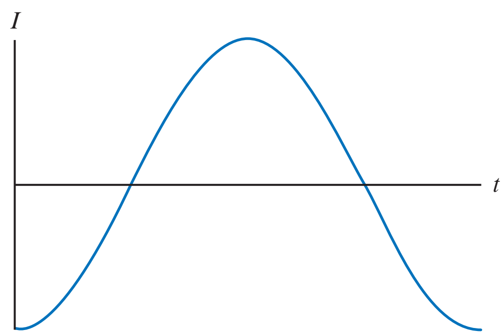


d



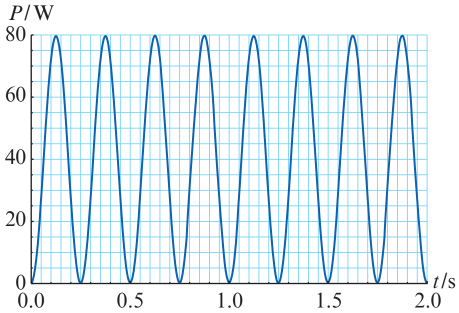
23 a The magnetic field is changing with time and goes through the secondary coil. Thus the flux in the secondary coil is changing and so there is an induced emf and therefore a current.

b The magnetic field and hence the flux are both sine functions. The induced emf, and hence the current, is the negative gradient of the sine graph and so a negative cosine.



24 a 1.0 s.

b



Chapter 21

Test your understanding

1 $\frac{10^{-10}}{10^{-15}} = 10^5$

2 The electric force.

3 a In order to avoid absorption of alpha particles as well as avoid multiple scatterings.

b In order to avoid collisions of alpha particles with air molecules, which would have deflected the alphas.

4 a Discrete energy means that the atom cannot have any continuous value of energy but rather one out of many separate, i.e. discrete, values.

b The existence of emission atomic spectra is the best evidence for the discreteness of energy in atoms: the emission lines have specific wavelengths implying specific energy differences between levels.

5 The bright lines are formed when an electron makes a transition from a high energy state H to a lower energy state L. The photon emitted will have a wavelength determined from $\frac{hc}{\lambda} = \Delta E_{LH} \Rightarrow \lambda = \frac{hc}{\Delta E_{LH}}$ where ΔE_{LH} is the difference in energy between states H and L. The dark lines are formed when a photon is absorbed by an electron in a low energy state L which then makes a transition to a high energy state H. For the absorption to be possible, the photon energy must equal the difference ΔE_{LH}. Hence this photon will have the same wavelength as the emission line wavelength.

6 $E = \frac{hc}{\lambda} = \frac{1.24 \times 10^{-6}\text{ eV m}}{480 \times 10^{-9}\text{ m}} = 2.58 \approx 2.6\text{ eV}$.

7 $\frac{hc}{\lambda} = 3.8\text{ eV} \Rightarrow \lambda = \frac{1.24 \times 10^{-6}}{3.8} = 3.26 \times 10^{-7} \approx 3.3 \times 10^{-7}\text{ m}$.

8 The energy difference is 2.55 eV. Hence, $\frac{hc}{\lambda} = E = 2.55\text{ eV} \Rightarrow \lambda = \frac{hc}{E} = \frac{1.24 \times 10^{-6}\text{ eV m}}{2.55\text{ eV}} =$ $4.86 \times 10^{-7} \approx 4.9 \times 10^{-7}\text{ m}$

9 The energy differences between levels get smaller as n increases. Therefore transitions down to $n = 2$ (the visible light transitions) have wavelengths that are close to each other.

10 a The ground state is the energy level with the least possible energy.

b The energy difference between the ground state and the first excited state is 10.2 eV. That between the ground state and the second excited state is 12.1 eV. The incoming photons do not have exactly these amounts of energy so the hydrogen atoms will not absorb any of these photons.

c With incoming electrons it is possible that some will give 10.2 eV of their energy to hydrogen atoms so that the atoms make a transition to the first excited state. The electrons that do give this energy will bounce off the atoms with a kinetic energy of about 0.2 eV.

11 a The discrete emission and absorption spectra.

b i The difference between $n = 1$ and $n = 2$ is 10.2 eV so no excitation takes place.

ii $n = 4$; the difference in energy is $13.6 - \frac{13.6}{16} = 12.75$ eV.

iii $n = 6$; the difference in energy is $13.6 - \frac{13.6}{36} = 13.22$ eV.

12 a The minimum energy needed to get an electron ejected from an atom.

b $E_3 = -\frac{13.6}{3^2} = -1.51$ eV so the energy required is 1.51 eV.

13 $\frac{1}{2}mv^2 = 13.6 \times 1.6 \times 10^{-19} \Rightarrow v = \sqrt{\frac{2 \times 13.6 \times 1.6 \times 10^{-19}}{9.1 \times 10^{-31}}} = 2.2 \times 10^6\ \text{m s}^{-1}$.

14 a $r_2 = a_0 2^2 = 0.529 \times 10^{-10} \times 4 = 2.116 \times 10^{-10} \approx 2.1 \times 10^{-10}$ m.

b $2\pi r_2 = n\lambda = 2\frac{h}{p} \Rightarrow p = \frac{h}{\pi r_2} = \frac{6.63 \times 10^{-34}}{\pi \times 2.116 \times 10^{-10}} = 9.9974 \times 10^{-25} \approx 1.0 \times 10^{-24}$ N s.

c $L = mvr = 2 \times \frac{h}{2\pi} = \frac{h}{\pi} = 2.1 \times 10^{-34}$ J s.

d $E_K = \frac{p^2}{2m} = \frac{(9.9974 \times 10^{-25})^2}{2 \times 9.1 \times 10^{-31}} = 5.49 \times 10^{-19} \approx 5.5 \times 10^{-19}$ J.

e $E_P = -2E_K = -2 \times 5.49 \times 10^{-19} = -1.098 \times 10^{-18} \approx -1.1 \times 10^{-18}$ J.

15 a $\frac{ke^2}{r^2} = \frac{mv^2}{r} \Rightarrow r = \frac{ke^2}{mv^2}$. From $mvr = n\frac{h}{2\pi}$ we get $r = n\frac{h}{2\pi mv}$. Equating the expressions for radius: $\frac{ke^2}{mv^2} = n\frac{h}{2\pi mv}$ and so $v = \frac{2\pi ke^2}{nh}$.

b $\frac{v_2}{v_1} = \frac{\frac{2\pi ke^2}{2h}}{\frac{2\pi ke^2}{h}} = \frac{1}{2}$

16 a From the previous problem we know that $v_n = \frac{2\pi ke^2}{nh}$ and also that $r_n = n^2 a_0$. Hence $T_n = \frac{2\pi r_n}{v_n} = \frac{2\pi a_0 n^2}{\frac{2\pi ke^2}{nh}} = n^3\frac{a_0 h}{ke^2} = n^3 T_1$, where $T_1 = \frac{a_0 h}{ke^2}$.

b The Bohr radius is (from the textbook) $a_0 = \frac{h^2}{4\pi^2 mke^2}$ and so $T_1 = \frac{a_0 h}{ke^2} = \frac{h^2}{4\pi^2 mke^2}\frac{h}{ke^2} = \frac{h^3}{4\pi^2 mk^2e^4}$.

c $T_1 = \frac{(6.63 \times 10^{-34})^3}{4\pi^2 \times 9.1 \times 10^{-31} \times (9.0 \times 10^9)^2 \times (1.6 \times 10^{-19})^4} = \frac{6.63^3}{4\pi^2 \times 9.1 \times 9.0^2 \times 1.6^4} \times 10^{-13} = 1.53 \times 10^{-16}$ s.

The frequency is $f_1 = \frac{1}{1.53 \times 10^{-16}} = 6.54 \times 10^{15}$ Hz.

d $T_n = n^3 T_1$ and $r_n = n^2 a_0$. Hence $T_n^2 = n^6 T_1^2 = \frac{T_1^2}{a_0^3} r_n^3$, i.e. $T_n^2 \propto r_n^3$.

17 a The energy difference in the transition is $\frac{13.6}{4} - \frac{13.6}{n^2}$. So the wavelength is $\lambda = \frac{hc}{\Delta E} = \frac{1.24 \times 10^{-6}\ \text{eV m}}{3.4\left(\frac{n^2-4}{n^2}\right)\ \text{eV}} = 3.65 \times 10^{-7} \times \frac{n^2}{n^2-4}$ m $= 365 \times \frac{n^2}{n^2-4}$ nm.

b $\lambda = 365 \times \frac{3^2}{3^2-4} = 657$ nm.

c From the previous problem, $T_2 = 2^3 T_1 = 8 \times 1.53 \times 10^{-16} = 1.22 \times 10^{-15}$ s, so $f_2 = \frac{1}{1.22 \times 10^{-15}} = 8.20 \times 10^{14}$ Hz. And $T_3 = 3^3 T_1 = 27 \times 1.53 \times 10^{-16} = 4.13 \times 10^{-15}$ s so $f_3 = \frac{1}{4.13 \times 10^{-15}} = 2.42 \times 10^{14}$ Hz. The frequency of the emitted photon is $f = \frac{c}{\lambda} = \frac{3.0 \times 10^8}{657 \times 10^{-9}} = 4.57 \times 10^{14}$ Hz, which is of the same order of magnitude as the frequencies in $n = 2$ and $n = 3$.

18 a $\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow v^2 = \frac{GM}{r}$. If $mvr = n\frac{h}{2\pi}$ we have, squaring, $m^2v^2r^2 = n^2\frac{h^2}{4\pi^2} \Rightarrow v^2 = n^2\frac{h^2}{4\pi^2 m^2 r^2}$.

Equating the two expressions for speed we get $\frac{GM}{r} = n^2\frac{h^2}{4\pi^2 m^2 r^2}$, i.e. $r = \frac{h^2}{4\pi^2 GMm^2}n^2$.

b $n = \sqrt{\frac{4\pi^2 GMm^2 r}{h^2}} = \sqrt{\frac{4\pi^2 \times 6.67 \times 10^{-11} \times 2.0 \times 10^{30} \times (6.0 \times 10^{24})^2 \times 1.5 \times 10^{11}}{(6.63 \times 10^{-34})^2}}$.

$n = \sqrt{\frac{4\pi^2 \times 6.67 \times 2.0 \times 6.0^2 \times 1.5 \times 10^{146}}{6.63^2}} \approx 2.5 \times 10^{74}$.

c It is meaningless. There is no meaningful change in an orbit with n and one with $n + 1$. The discreteness is unobservable.

19 $qvB = \frac{mv^2}{r} \Rightarrow v = \frac{qBr}{m}$. If $mvr = n\frac{h}{2\pi} \Rightarrow v = n\frac{h}{2\pi mr}$. Equating the two expressions for speed we get $\frac{qBr}{m} = n\frac{h}{2\pi mr} \Rightarrow r^2 = \frac{nh}{2\pi qB}$.

20 **a** The electron is acted upon by the electric force, and since the charge of the nucleus is $2e$:

$\frac{k(2e)\times e}{r^2} = \frac{mv^2}{r} \Rightarrow mv^2 = \frac{2ke^2}{r}$ and $E = \frac{1}{2}mv^2 - \frac{2ke^2}{r} = \frac{ke^2}{r} - \frac{2ke^2}{r} = -\frac{ke^2}{r}$

Squaring both sides of the Bohr condition equation:

$m^2v^2r^2 = n^2\frac{h^2}{4\pi^2}$

and so $mv^2 = \frac{n^2h^2}{4\pi^2mr^2}$

But earlier we found that $mv^2 = \frac{2ke^2}{r}$, so substituting for mv^2 we get:

$\frac{2ke^2}{r} = \frac{n^2h^2}{4\pi^2mr^2}$ hence $r = \frac{h^2}{8\pi^2ke^2m} \times n^2$. The Bohr radius $a_0 = \frac{h^2}{4\pi^2ke^2m}$. Hence for $n = 1$, $r = \frac{a_0}{2}$.

b $E_1 = -\frac{ke^2}{r} = -\frac{2ke^2}{a_0} = -4 \times 13.6 = -54.4$ eV.

21 Using formulae from the textbook, the difference in energy is $\Delta E = \frac{2\pi^2mk^2e^4}{h^2}\left(\frac{1}{n^2} - \frac{1}{(n+1)^2}\right) = \frac{2\pi^2mk^2e^4}{h^2}\frac{2n+1}{n^2(n+1)^2} \approx \frac{2\pi^2mk^2e^4}{h^2} \times \frac{2}{n^3}$ since n is very large. The frequency is then $f = \frac{\Delta E}{h} = \frac{4\pi^2mk^2e^4}{h^3} \times \frac{1}{n^3}$.

The frequency of revolution in the state n was calculated in an earlier problem and is $f_n = \frac{4\pi^2mk^2e^4}{h^3}\frac{1}{n^3}$, which is precisely what we found previously.

22 **a** $L = mvr$ and $v = \omega r$ so that $L = m\omega r^2$.

b $\frac{ke^2}{r^2} = \frac{mv^2}{r} = m\omega^2 r$ so $r^3 = \frac{ke^2}{m\omega^2}$. $L = m\omega r^2 = m\omega\left(\frac{ke^2}{m\omega^2}\right)^{2/3}$ hence $L^3 = m^3\omega^3\left(\frac{ke^2}{m\omega^2}\right)^2 = \frac{mk^2e^4}{\omega}$.

c From $L^3 = \frac{mk^2e^4}{\omega}$ we find $\omega = \frac{mk^2e^4}{L^3}$, and from $L = m\omega r^2$ we get $\omega = \frac{L}{mr^2}$. Thus, $\frac{mk^2e^4}{L^3} = \frac{L}{mr^2}$ and so $r^2 = \frac{L^4}{m^2k^2e^4}$, finally giving $r = \frac{L^2}{mke^2}$.

d The total energy is $E = -\frac{ke^2}{2r} = -\frac{ke^2}{2\frac{L^2}{mke^2}} = -\frac{mk^2e^4}{2L^2}$. Then $\frac{dE}{dL} = \frac{mk^2e^4}{L^3}$.

e But from (b) $L^3 = \frac{mk^2e^4}{\omega}$ and so $\frac{dE}{dL} = \frac{mk^2e^4}{\frac{mk^2e^4}{\omega}} = \omega$.

f $dE = hf = \frac{h\omega}{2\pi}$ so $\frac{h\omega}{2\pi} = \omega dL$ and then $dL = \frac{h}{2\pi}$.

Chapter 22

Test your understanding

1 $N \times \frac{hc}{\lambda} = P \Rightarrow N = \frac{P\lambda}{hc} = \frac{60 \times 560 \times 10^{-9}}{6.63 \times 10^{-34} \times 3.0 \times 10^{8}} \approx 1.7 \times 10^{20}\ \text{s}^{-1}$

2 Energy of a photon: $E = \frac{hc}{\lambda} = \frac{1.99 \times 10^{-25}}{620 \times 10^{-9}} = 3.21 \times 10^{-19}$ J.

Intensity is power per unit area so $I = NE$ where N is the number of photons per unit time per unit area. Thus

$N \times 3.21 \times 10^{-19} = 25 \Rightarrow N = 8 \times 10^{19}$.

3 The energy of one photon is $\frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{480 \times 10^{-9}} = 4.14 \times 10^{-19}$ J. The number of photons incident per second on an area A is $\frac{660 \times A}{4.14 \times 10^{-19}}$. The momentum change of one photon is $\frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{480 \times 10^{-9}} = 1.38 \times 10^{-27}$ N s. Hence the pressure is $\frac{\frac{660 \times A}{4.14 \times 10^{-19}}}{A} \times 1.38 \times 10^{-27}\ \text{N s} = 2.2 \times 10^{-6}$ Pa.

4 0.32 eV

5 1.8 eV + 2.2 eV = 4.0 eV

6 2.1 eV − 1.2 eV = 0.9 eV

7 Because the gradient is Planck's constant: $E_{max} = hf - \phi$.

8 **a** The stopping voltage is related to the energy of the emitted electrons. The energy of the emitted electrons depends only on the frequency of light so it does not change.

b The energy of the electrons will increase, and so the stopping voltage will increase.

9 **a** The energy of the emitted electrons depends only on the frequency of light. By the Doppler effect, the incident frequency is reduced so the energy of the electrons is also reduced.

b The number of emitted electrons depends on the number of incident photons. This number decreases so the number of electrons decreases.

10 From the Einstein formula $E_{max} = hf - \phi$. At the critical frequency, $E_{max} = 0$ and so $hf_c - \phi = 0 \Rightarrow$ $f_c = \frac{\phi}{h} = \frac{3.00 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} = 7.24 \times 10^{14}$ Hz.

11 **a** Evidence for photons comes from the photoelectric effect, Compton scattering and others.

b $\phi = hf_c$. Then $E_{max} = hf - hf_c = 6.63 \times 10^{-34} \times (3.87 - 2.25) \times 10^{14} = 1.074 \times 10^{-19}$ J.

The stopping voltage is $qV_s = E_{max} \Rightarrow V_s = \frac{E_{max}}{q} = \frac{1.074 \times 10^{-19}}{1.6 \times 10^{-19}} = 0.67$ V.

12 **a** Light consists of photons. When light is incident on the metal an electron from within the metal may absorb one photon and so its energy will increase by an amount equal to the photon energy. If this energy is sufficient to overcome the potential well the electron finds itself in, the electron may free itself from the metal.

b The number of electrons emitted per second is 10^{15} and so the charge that leaves the metal per second, i.e. the current, is $10^{15} \times 1.6 \times 10^{-19} = 1.6 \times 10^{-4}$ A.

c From $E_{max} = hf - \phi$ we get $\phi = hf - E_{max} = \frac{hc}{\lambda} - E_{max} = \frac{1.24 \times 10^{-6}}{5.4 \times 10^{-7}} - 2.1 = 0.196 \approx 0.20$ eV.

d The energy is independent of intensity and so we still have $E_{max} = 2.1$ eV.

e The current will double since current is proportional to intensity.

13 a
- The electrons are emitted without delay. In the photon theory of light the energy is supplied to an electron by a single photon in an instantaneous interaction.
- There is a critical frequency below which no electrons are emitted. The energy of the photon depends on frequency so if the photon energy is less than the work function, the electron cannot be emitted.
- The intensity of light has no effect on the energy of the emitted electrons. The intensity of light depends on the number of photons present, and so this will affect the number of electrons emitted, not their energy.

b i Stopping voltage is that voltage which makes the current in the photoelectric experiment zero.

ii $qV_S = \frac{hc}{\lambda} - \phi \Rightarrow \phi = \frac{hc}{\lambda} - qV_S = \frac{1.24 \times 10^{-6}}{2.08 \times 10^{-7}} - 1.40 = 4.56$ eV.

The longest wavelength corresponds to the smallest frequency, i.e. the critical frequency:

$\frac{hc}{\lambda_c} = \phi \Rightarrow \lambda_c = \frac{hc}{\phi} = \frac{1.24 \times 10^{-6}}{4.56} = 2.72 \times 10^{-7}$ m.

14 a i As long as the wavelength stays the same, the energy of the emitted electrons will stay the same.

ii Increasing the intensity of light increases the number of electrons emitted, i.e. the photocurrent.

b We have that $qV_S = \frac{hc}{2.3 \times 10^{-7}} - \phi$ and $q(2V_S) = \frac{hc}{1.8 \times 10^{-7}} - \phi$. Multiply the first equation by 2 to get $2qV_S = \frac{2hc}{2.3 \times 10^{-7}} - 2\phi$. Then $\frac{2hc}{2.3 \times 10^{-7}} - 2\phi = \frac{hc}{1.8 \times 10^{-7}} - \phi \Rightarrow \phi = \frac{2hc}{2.3 \times 10^{-7}} - \frac{hc}{1.8 \times 10^{-7}} = \frac{2 \times 1.24 \times 10^{-6}}{2.3 \times 10^{-7}} - \frac{1.24 \times 10^{-6}}{1.8 \times 10^{-7}} = 3.9$ eV

15 a The work function is $\phi = 3.0 \times 1.6 \times 10^{-19} = 4.8 \times 10^{-19}$ J. The energy incident on the given area is $5.0 \times 10^{-4} \times 1.0 \times 10^{-18} = 5.0 \times 10^{-22}$ J. To accumulate the energy equal to the work function we need a time of $5.0 \times 10^{-22} \times t = 4.8 \times 10^{-19} \Rightarrow t = 960$ s or 16 minutes.

b Since in photoelectric experiments there is no time delay in the emission of electrons, light cannot be treated as a wave as this calculation does.

c In the photon theory of light, the energy carried by a photon is given to the electron in one go, not gradually.

16 a i Extending the graph we find a horizontal intercept of about 5.0×10^{14} Hz.

ii The work function and the critical frequency are related through

$hf_c - \phi = 0 \Rightarrow \phi = hf_c = 6.63 \times 10^{-34} \times 5.0 \times 10^{14} = 3.315 \times 10^{-19}\ \text{J} = \frac{3.315 \times 10^{-19}}{1.6 \times 10^{-19}} = 2.1$ eV.

b Reading off the graph we find $2.0 \times 10^{-19}\ \text{J} = \frac{2.0 \times 10^{-19}}{1.6 \times 10^{-19}} = 1.25$ eV.

c It will be a line parallel to the original with a horizontal intercept at 6.0×10^{14} Hz.

17 a The intensity is $I = \frac{P}{A} = \frac{nhf}{A}$ where n is the number of photons incident per second. Then $I = \Phi hf$ where $\Phi = \frac{n}{A}$ is the number of photons per second per unit area.

b $I = \Phi hf = \Phi \frac{hc}{\lambda} = 3.8 \times 10^{18} \times \frac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{5.0 \times 10^{-7}} = 1.5\ \text{Wm}^{-2}$.

c $\Phi \frac{hc}{\lambda} = \Phi' \frac{hc}{\lambda'} \Rightarrow \Phi' = \Phi \frac{\lambda'}{\lambda} = 3.8 \times 10^{18} \times \frac{4}{5} = 3.0 \times 10^{18}\ \text{s}^{-1}\,\text{m}^{-2}$

d Since the intensity is the same, the flux for the shorter wavelength is less and hence fewer electrons will be emitted since fewer photons are incident.

e This assumes that the fraction of photons that eject electrons is the same for both wavelengths. This fraction is called the quantum efficiency, which in general does depend on wavelength in a complex way.

18 They would be too small and practically unobservable.

19 $\lambda' - \lambda = \frac{h}{mc}(1 - \cos 120°) = \frac{h}{mc}\left(1 - \left(-\frac{1}{2}\right)\right) = \frac{3h}{2mc} = \frac{3 \times 2.4 \times 10^{-12}}{2} = 3.6 \times 10^{-12}$ m.

20 $\lambda' - \lambda = \frac{h}{m_p c}(1 - \cos 120°) = \frac{3h}{2m_p c} = \frac{3 \times 6.63 \times 10^{-34}}{2 \times 1.67 \times 10^{-27} \times 3 \times 10^8} = 2.0 \times 10^{-15}$ m, so we have a much smaller shift than in the electron case.

21 The energy transferred is $\frac{hc}{\lambda} - \frac{hc}{\lambda'} = \frac{hc(\lambda' - \lambda)}{\lambda\lambda'}$. The fraction is then $\frac{\frac{hc(\lambda' - \lambda)}{\lambda\lambda'}}{\frac{hc}{\lambda}} = \frac{\lambda' - \lambda}{\lambda'}$.

22 $\lambda' - \lambda = \frac{h}{mc}(1 - \cos 0°) = 0$ so no energy is transferred.

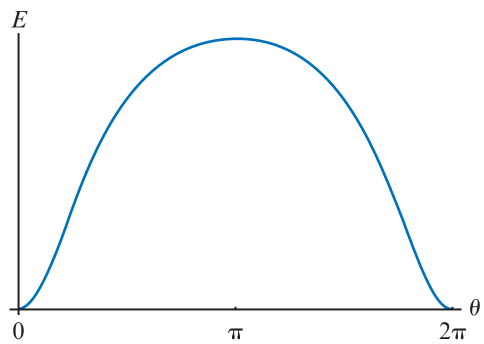
23 $\lambda' - \lambda = \frac{h}{mc}(1 - \cos 90°) = \frac{h}{mc} = 2.4 \times 10^{-12}$ m

So $\lambda' = 3.0 \times 10^{-12} + 2.4 \times 10^{-12} = 5.4 \times 10^{-12}$ m

The energy transferred is $\frac{hc}{\lambda} - \frac{hc}{\lambda'} = \frac{1.24 \times 10^{-6}}{3.0 \times 10^{-12}} - \frac{1.24 \times 10^{-6}}{5.4 \times 10^{-12}} = 0.18$ MeV.

24 a The energy transferred to the electron is $E = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = hc\frac{\lambda' - \lambda}{\lambda\lambda'} = hc\frac{\frac{h}{mc}(1 - \cos\theta)}{\lambda\left(\lambda + \frac{h}{mc}(1 - \cos\theta)\right)}$.

Plotting this as a function of θ shows a maximum at $\theta = \pi$. For example:



b With $\theta = \pi$, $E = hc\frac{\frac{2h}{mc}}{\lambda(\lambda + \frac{2h}{mc})}$ which becomes $\lambda^2 + \frac{2h}{mc}\lambda - \frac{2h^2}{mE} = 0$ OR

$\lambda^2 + 4.8 \times 10^{-12}\lambda - 2.0 \times 10^{-22} = 0$. The solution is $\lambda = 1.2 \times 10^{-11}$ m.

25 a $\frac{h}{\lambda} = p_x + \frac{h}{\lambda'}\cos\theta \Rightarrow p_x = \frac{h}{\lambda} - \frac{h}{\lambda'}\cos\theta$; $0 = \frac{h}{\lambda'}\sin\theta - p_y \Rightarrow p_y = \frac{h}{\lambda'}\sin\theta$

b $p^2 = p_x^2 + p_y^2 = \left(\frac{h}{\lambda} - \frac{h}{\lambda'}\cos\theta\right)^2 + \left(\frac{h}{\lambda'}\sin\theta\right)^2$

$$p^2 = \left(\frac{h}{\lambda}\right)^2 + \left(\frac{h}{\lambda'}\right)^2\cos^2\theta - 2\frac{h^2\cos\theta}{\lambda\lambda'} + \left(\frac{h}{\lambda'}\right)^2\sin^2\theta$$

$$= \left(\frac{h}{\lambda}\right)^2 + \left(\frac{h}{\lambda'}\right)^2(\cos^2\theta + \sin^2\theta) - 2\frac{h^2\cos\theta}{\lambda\lambda'}$$

$$= \left(\frac{h}{\lambda}\right)^2 + \left(\frac{h}{\lambda'}\right)^2 - 2\frac{h^2\cos\theta}{\lambda\lambda'}$$

c Energy of electron at rest is mc^2. Energy conservation gives $\frac{hc}{\lambda} + mc^2 = \frac{hc}{\lambda'} + \sqrt{p^2c^2 + (mc^2)^2}$.

d $\sqrt{p^2c^2 + (mc^2)^2} = \frac{hc}{\lambda} - \frac{hc}{\lambda'} + mc^2$

$$p^2c^2 + (mc^2)^2 = \left(\frac{hc}{\lambda}\right)^2 + \left(\frac{hc}{\lambda'}\right)^2 + (mc^2)^2 - 2\frac{hc}{\lambda} \times \frac{hc}{\lambda'} + 2\frac{hc}{\lambda}mc^2 - 2\frac{hc}{\lambda'}mc^2$$

$$p^2 = \left(\frac{h}{\lambda}\right)^2 + \left(\frac{h}{\lambda'}\right)^2 - 2\frac{h}{\lambda} \times \frac{h}{\lambda'} + 2\frac{h}{\lambda}mc - 2\frac{h}{\lambda'}mc$$

Use

$$p^2 = \left(\frac{h}{\lambda'}\right)^2 + \left(\frac{h}{\lambda}\right)^2 - 2\frac{h^2\cos\theta}{\lambda\lambda'}$$

$$\left(\frac{h}{\lambda'}\right)^2 + \left(\frac{h}{\lambda}\right)^2 - 2\frac{h^2\cos\theta}{\lambda\lambda'} = \left(\frac{h}{\lambda}\right)^2 + \left(\frac{h}{\lambda'}\right)^2 - 2\frac{h}{\lambda} \times \frac{h}{\lambda'} + 2\frac{h}{\lambda} \times mc - 2\frac{h}{\lambda'} \times mc$$

$$\frac{h\cos\theta}{\lambda\lambda'} = \frac{h}{\lambda\lambda'} - \frac{mc}{\lambda} + \frac{mc}{\lambda'}$$

$$\frac{h(1-\cos\theta)}{\lambda\lambda'} = \frac{mc}{\lambda} - \frac{mc}{\lambda'}$$

$$\frac{h(1-\cos\theta)}{\lambda\lambda'} = mc\frac{(\lambda' - \lambda)}{\lambda\lambda'}$$

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)$$

26 $\frac{\lambda_p}{\lambda_\alpha} = \frac{\frac{h}{p}}{\frac{h}{p}} = 1.$

27 $\frac{\lambda_p}{\lambda_\alpha} = \frac{\frac{h}{m_p v}}{\frac{h}{m_\alpha v}} = \frac{m_\alpha}{m_p} \approx 4.$

28 $E_K = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mE_K}$. Hence $\frac{\lambda_p}{\lambda_\alpha} = \frac{\frac{h}{\sqrt{2Em_p}}}{\frac{h}{\sqrt{2Em_\alpha}}} = \sqrt{\frac{m_\alpha}{m_p}} \approx 2.$

29 a $\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{0.250 \times 10} \approx 3 \times 10^{-34}$ m.

b Diffraction effects for the brick would be observed if it went through holes of opening 10^{-34} m which is an absurdity. It makes no sense to treat the brick as a wave.

30 a The Davisson–Germer experiment—see text.

b $p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{680 \times 10^{-9}} = 9.75 \times 10^{-28}$ N s. Hence $v = \frac{p}{m} = \frac{9.75 \times 10^{-28}}{9.1 \times 10^{-31}} = 1.1 \times 10^3$ m s^{-1}.

31 a The work done in accelerating the electron will go into kinetic energy and so $E_K = eV$. Then $\frac{p^2}{2m} = eV \Rightarrow p = \sqrt{2meV}$. Then $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$.

b $\frac{\lambda_p}{\lambda_\alpha} = \frac{\sqrt{2m_\alpha q_\alpha V}}{\sqrt{2m_p q_p V}} = \sqrt{\frac{m_\alpha}{m_p} \times \frac{q_\alpha}{q_p}} = \sqrt{4 \times 2} = \sqrt{8}.$

c $\lambda = \frac{h}{\sqrt{2meV}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 520}} = 5.4 \times 10^{-11}$ m.

32 From $E_K = \frac{p^2}{2m_p} \Rightarrow p = \sqrt{2m_p E_K}$ and so $\lambda = \frac{h}{\sqrt{2m_p E_K}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 1.67 \times 10^{-27} \times 200 \times 10^6 \times 1.6 \times 10^{-19}}} =$ 2.03×10^{-15} m.

33 $2\pi r = 2\lambda$. The $n = 2$ state has $r = 4a_0$ and so $\lambda = 4\pi a_0$ (i.e. $\lambda = 6.6 \times 10^{-10}$ m).

34 $n = 2$: $2\pi r_2 = 2\lambda_2 \Rightarrow \lambda_2 = \pi r_2 = 4\pi a_0$

$n = 3$: $2\pi r_3 = 3\lambda_3 \Rightarrow \lambda_3 = \frac{2\pi r_3}{3} = \frac{2\pi 9a_0}{3} = 6\pi a_0$ So we get $\frac{\lambda_2}{\lambda_3} = \frac{4\pi a_0}{6\pi a_0} = \frac{2}{3}$.

35 a The electrons will diffract through the aperture and spread.

b The momentum is: $E_K = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mE_K} = \sqrt{2 \times 9.1 \times 10^{-31} \times 5.0 \times 1.6 \times 10^{-19}} = 1.21 \times 10^{-24}$ N s

and so the de Broglie wavelength is $\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}}{1.21 \times 10^{-24}} = 5.49 \times 10^{-10}$ m. Hence $\theta = 2 \times \frac{\lambda}{b} =$

$2 \times \frac{5.49 \times 10^{-10}}{8 \times 10^{-10}} \approx 1.4$ rad.

36 $\theta = \frac{\lambda}{\Delta y} = \frac{h}{\Delta y p}$ but also $\theta \approx \frac{\Delta p_y}{p}$. Hence $\frac{h}{\Delta y p} = \frac{\Delta p_y}{p} \Rightarrow \Delta y \times \Delta p_y \approx h$.

37 $\sqrt{\frac{hG}{2\pi c^5}}$: $\sqrt{\frac{\text{J s N m}^2\,\text{kg}^{-2}}{\text{m}^5\,\text{s}^{-5}}} = \sqrt{\frac{\text{kg m s}^{-2}\,\text{m s kg m s}^{-2}\,\text{m}^2\,\text{kg}^{-2}}{\text{m}^5\,\text{s}^{-5}}} = \sqrt{\text{s}^2} = \text{s}$.

$\sqrt{\frac{hG}{2\pi c^5}} = \sqrt{\frac{6.63 \times 10^{-34} \times 6.67 \times 10^{-11}}{2\pi \times (3.0 \times 10^8)^5}} \approx 5 \times 10^{-44}$ s.

$\sqrt{\frac{hG}{2\pi c^3}}$: $\sqrt{\frac{\text{J s N m}^2\,\text{kg}^{-2}}{\text{m}^3\,\text{s}^{-3}}} = \sqrt{\frac{\text{kg m s}^{-2}\,\text{s kg m}^2\,\text{s}^{-2}\,\text{m}^2\,\text{kg}^{-2}}{\text{m}^3\,\text{s}^{-3}}} = \sqrt{\text{m}^2} = \text{m}$.

$\sqrt{\frac{hG}{2\pi c^3}} = \sqrt{\frac{6.63 \times 10^{-34} \times 6.67 \times 10^{-11}}{2\pi \times (3.0 \times 10^8)^3}} \approx 2 \times 10^{-35}$ m.

Chapter 23

Test your understanding

1 $+2e$

2 82 protons and 127 neutrons.

3 a $65 \times (3.0 \times 10^8)^2 = 5.8 \times 10^{18}$ J.

b $65 \times 931.5 = 6.1 \times 10^4$ MeV.

4 a Isotopes are nuclei of the same element (hence they have the same proton (atomic) number) that differ in the number of neutrons, i.e. they have different nucleon (mass) numbers.

b They have different mass and different radius and different neutron number.

5 The existence of neutrons.

6 Because they have the same number of electrons.

7 Because there is also the strong nuclear force that acts between the nucleons, and this force is attractive.

8 a Strong nuclear force.

b Electric force.

9 Because the masses involved are negligibly small.

10 $\mu = 3m_p + 4m_n - M = 3 \times 1.007276 + 4 \times 1.008665 - 7.0144 = 4.21 \times 10^{-2}$ u.

BE $= 4.21 \times 10^{-2} \times 931.5 = 39.2$ MeV.

11 $\mu = 7 \times 1.007825 + 5 \times 1.008665 - 12.0186 = 0.07950$ u.

BE $= 0.07950 \times 931.5 = 74.1$ MeV.

12 BE $= 8.7903 \times 56 = 492$ MeV.

13 The binding energy is $8.6933 \times 91 = 791$ MeV; $\mu = \frac{BE}{c^2} = \frac{791\text{ MeV}}{c^2} = 791\text{ MeVc}^{-2}$.

Remember: 1 u = 931.5 MeVc^{-2}. So 1 MeVc^{-2} = $\frac{1}{931.5}$ u. Hence $\mu = 791$ MeVc^{-2} = $791 \times \left(\frac{1}{931.5}\text{u}\right) = 0.849$ u.

14 $\mu = 2m_p + m_n - M \Rightarrow M = 2m_p + m_n - \mu$. So, $M = 2m_p + m_n - \mu = 2 \times 938 + 940 - 7.7180 =$ 2808 MeVc^{-2} OR $M = \frac{2808}{931.5} = 3.015$ u.

15 $\mu = 54m_p + 77m_n - M \Rightarrow M = 54m_p + 77m_n - \mu$.

$\mu = \frac{BE}{c^2} = \frac{8.4237 \times 131\text{ MeV}}{c^2} = 1103.5\text{ MeVc}^{-2}$. So, $M = 54 \times 938 + 77 \times 940 - 1103.5 =$ 1.219×10^5 MeVc^{-2} OR $M = \frac{1.219 \times 10^5}{931.5} = 130.86$ u.

16 $\mu = 28 \times 1.007825 + 34 \times 1.008665 - 61.9283 = 0.58541$ u; BE $= 0.58541 \times 931.5 = 545.3$ MeV.

BE per nucleon is then $\frac{545.3}{62} = 8.795$ MeV.

17 For C-12: $\mu = 6 \times 1.007825 + 6 \times 1.008665 - 12.00000 = 0.09894$ u; BE = $0.09894 \times 931.5 = 92.16$ MeV.

For C-13: $\mu = 6 \times 1.007825 + 7 \times 1.008665 - 13.0033 = 0.10431$ u; BE = $0.10431 \times 931.5 = 97.16$ MeV.

The BE per nucleon is then $\frac{92.16}{12} = 7.68$ MeV for C-12 and $\frac{97.16}{13} = 7.47$ MeV for C-13. Hence C-12 is more tightly bound.

18 **a** $Z = 2$, $A = 4$ so ${}^4_2\text{He}$

b $Z = 0$, $A = 1$ so ${}^1_0\text{n}$

c $Z = 1$, $A = 1$ so ${}^1_1\text{p}$

19 The mass difference is $\Delta m = 238.0003 - 1.007276 - 237.0012 = -0.00818$ u. So the energy that would have to be *supplied* to remove the proton is $0.0018 \times 931.5 = 7.62$ MeV, very close to the binding energy per nucleon.

20 The mass difference is $\Delta m = 254.0873 - 117.9190 - 131.9086 - 4 \times 1.008665 = 0.22504$ u. So the energy released is $0.22504 \times 931.5 = 209.6$ MeV.

21 $\Delta m = 1.008665 + 13.9992 - 14.0000 - 1.007276 = 5.89 \times 10^{-4}$ u. So the energy released is $5.89 \times 10^{-4} \times 931.5 = 0.549$ MeV.

22 $\Delta m = 2.0141 + 6.0151 - 2 \times 4.0026 = 2.40 \times 10^{-2}$ u. So $Q = 2.40 \times 10^{-2} \times 931.5 = 22.4$ MeV.

23 The emission of particles and energy from unstable nuclei.

24 Random: we cannot predict which nucleus will decay and when.

Spontaneous: we cannot influence the decay process in any way.

25 ${}^3_1\text{H} \rightarrow {}^3_2\text{He} + {}^{0}_{-1}\text{e} + \bar{\nu}$

26 ${}^{23}_{12}\text{Mg} \rightarrow {}^{23}_{11}\text{Na} + {}^{0}_{+1}\bar{\text{e}} + \nu$

27 ${}^{210}_{83}\text{Bi} \rightarrow {}^{210}_{84}\text{Po} + {}^{0}_{-1}\text{e} + \bar{\nu}$

${}^{210}_{84}\text{Po} \rightarrow {}^{210}_{84}\text{Po} + \gamma$

So $Z = 84$, $A = 210$.

28 ${}^{239}_{94}\text{Pu} \rightarrow {}^{235}_{92}\text{U} + {}^4_2\alpha$; uranium.

29 The resulting nucleus must have proton number Z. So we need one alpha decay and two beta minus decays:

${}^{A}_{Z}\text{X} \rightarrow {}^{A-4}_{Z-2}\text{Y} + {}^4_2\text{He}$

${}^{A-4}_{Z-2}\text{Y} \rightarrow {}^{A-4}_{Z-1}\text{Z} + {}^{0}_{-1}\text{e} + \bar{\nu}$

${}^{A-4}_{Z-1}\text{Y} \rightarrow {}^{A-4}_{Z}\text{X} + {}^{0}_{-1}\text{e} + \bar{\nu}$

30 Beta plus decay.

31 Beta minus decay.

32 ${}^8_4\text{Be} \rightarrow {}^4_2\text{He} + {}^4_2\text{He}$; $\Delta m = 8.0053 - 2 \times 4.0026 = 1.00 \times 10^{-4}$ u, so the energy released is 0.0932 MeV.

33 ${}^{A}_{Z}\text{X} \rightarrow {}^{A-4}_{Z-2}\text{Y} + {}^{4}_{2}\text{He}$

${}^{A-4}_{Z-2}\text{Y} \rightarrow {}^{A-4}_{Z-3}\text{Z} + {}^{0}_{1}\bar{\text{e}} + \nu$

so alpha decay followed by beta plus decay.

34 ${}^{A}_{Z}\text{X} \rightarrow {}^{A-4}_{Z-2}\text{Y} + {}^{4}_{2}\text{He}$

${}^{A-4}_{Z-2}\text{Y} \rightarrow {}^{A-8}_{Z-4}\text{Z} + {}^{4}_{2}\text{He}$

${}^{A-8}_{Z-4}\text{Z} \rightarrow {}^{A-8}_{Z-3}\text{W} + {}^{0}_{-1}\text{e} + \bar{\nu}$

so 2 alpha decays followed by beta minus decay.

35 ${}^{A}_{Z}\text{X} \rightarrow {}^{A-4}_{Z-2}\text{Y} + {}^{4}_{2}\text{He}$

${}^{A-4}_{Z-2}\text{Y} \rightarrow {}^{A-4}_{Z-3}\text{Z} + {}^{0}_{1}\bar{\text{e}} + \nu$

${}^{A-4}_{Z-3}\text{Z} \rightarrow {}^{A-4}_{Z-4}\text{W} + {}^{0}_{1}\bar{\text{e}} + \nu$

so an alpha decay followed by 2 beta plus decays.

36 Activity is the rate of decay, i.e. the number of decays per unit time.

37 60 Bq

38 18 min is 6 half-lives and so the sample will be reduced by $2^6 = 64$ times, i.e. to 0.50 mg.

39

Time	Number of X nuclei	Number of Y nuclei	Ratio of Y to X
0	N	0	0
T	$N/2$	$N/2$	1
$2T$	$N/4$	$3N/4$	3
$3T$	$N/8$	$7N/8$	7

Ratio is 7.

40 From the previous table 2 half-lives went by and so the half-life is 6.0 min.

41 The probability of decay within one half-life is always 0.5.

42 $\frac{10\,000}{625} = 16 = 2^4$ so 4 half-lives went by. The half-life is then 9.0 days.

43 The actual count rate from the sample is $310 - 20 = 290$. After a half-life it will halve to 145 and so the count rate will be $145 + 20 = 165$ counts per minute.

44 The actual count rate from the sample is $565 - 25 = 540$. $540 \rightarrow 270 \rightarrow 135$ in 2 half-lives. The count rate would then be $135 + 25 = 160$ which is what we want. So, the half-life is 6.0 days.

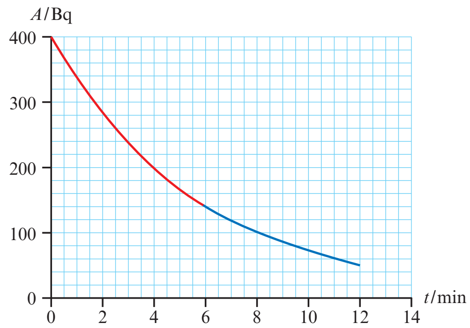
45 The equation is $C = \frac{k}{(d + d_0)^2}$, i.e. $d + d_0 = \sqrt{\frac{k}{C}}$. So a graph of d versus $\frac{1}{\sqrt{C}}$ would give a straight line with slope $\sqrt{k}$ and intercept $-d_0$.

46 The ratio is 1:7 after 3 half-lives. So the age is $3 \times 1.25 \times 10^9 = 3.75 \times 10^9$ years.

47 **a** Activity is the number of decays per unit time, the rate of decay.

b 4.0 min

c



d We make the following table of numbers of X and Y nuclei:

Time/min	Number of X nuclei	Number of Y nuclei	Ratio of Y to X
0	N	0	0
4	$N/2$	$N/2$	1
8	$N/4$	$3N/4$	3
12	$N/8$	$7N/8$	7

So the required time is 12 min.

48 Above a certain nuclear size any one nucleon has the same number of nearest neighbours. Because the strong nuclear force has a short range it is only the closest neighbours that exert the strong force on the nucleon. If this nucleon is a proton, it repels all other protons in the nucleus because the electric force has infinite range. So adding more protons tends to destabilise the nucleus. To counter this we need lots of neutrons both because they make the distance between protons larger (and so reduce the electric tendency for a breakup) and because they themselves contribute to binding through the nuclear force.

49 $E = \frac{k(2e)(Ze)}{d} \Rightarrow d = \frac{2kZe^2}{E} = \frac{2 \times 8.99 \times 10^9 \times 26 \times (1.6 \times 10^{-19})^2}{4.10 \times 10^6 \times 1.6 \times 10^{-19}} = 1.8 \times 10^{-14}$ m.

50 It will not change because both nuclei have the same electric charge.

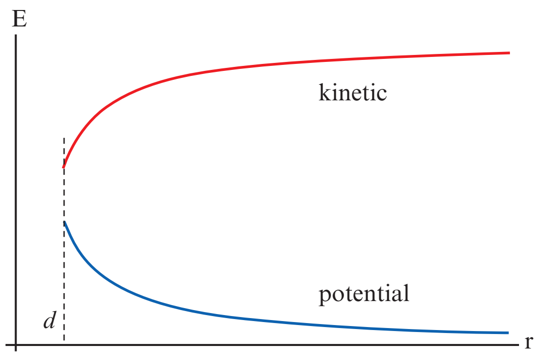
51 **a** For the alpha particle, $R = 1.2 \times 10^{-15} \times (4)^{\frac{1}{3}} = 1.9 \times 10^{-15}$ m. For platinum $R = 1.2 \times 10^{-15} \times 195^{\frac{1}{3}} = 6.96 \times 10^{-15}$ m.

b $E = \frac{k(2e)(78e)}{d} = \frac{8.99 \times 10^9 \times 2 \times 78 \times (1.6 \times 10^{-19})^2}{(6.96 + 1.9) \times 10^{-15}} = 4.05 \times 10^{-12}\ \text{J} = 25$ MeV.

52 $E = \frac{k(2e)(79e)}{d} = \frac{8.99 \times 10^9 \times 2 \times 79 \times (1.6 \times 10^{-19})^2}{8.5 \times 10^{-15}} = 4.28 \times 10^{-12}$ J

$\frac{1}{2}mv^2 = E \Rightarrow v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2 \times 4.28 \times 10^{-12}}{6.64 \times 10^{-27}}} = 3.6 \times 10^7\ \text{m s}^{-1}$.

53 **a** and **b** For the data given the two curves do not cross.



c From angular momentum conservation we have that $mvb = mud$. So to find b we need to first find u, the speed at the point of closest approach. Using conservation of energy:

$\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + \frac{kZe^2}{d}$ and so $u = \sqrt{v^2 - \frac{2kZe^2}{md}} = \sqrt{(3.0 \times 10^7)^2 - \frac{2 \times 8.99 \times 10^9 \times 28 \times (1.6 \times 10^{-19})^2}{1.67 \times 10^{-27} \times 2.0 \times 10^{-14}}} = 2.27 \times 10^7\ \mathrm{m\,s^{-1}}$.

Hence $b = \frac{u}{v}d = \frac{2.27 \times 10^7}{3.0 \times 10^7} \times 2.0 \times 10^{-14} = 1.5 \times 10^{-14}$ m.

54 The radius of a nucleus of mass number A is $R = 1.2 \times 10^{-15} \times A^{\frac{1}{3}}$ m and its mass is $M = Am_n$ (here m_n is the mass of a nucleon). The density is therefore

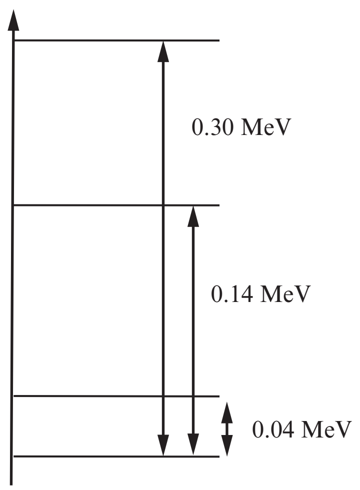
$$\rho = \frac{M}{\frac{4}{3}\pi R^3} = \frac{A \times m_n}{\frac{4}{3}\pi(1.2 \times 10^{-15} \times A^{1/3})^3} = \frac{m_n}{\frac{4}{3}\pi(1.2 \times 10^{-15})^3} = \frac{1.67 \times 10^{-27}}{\frac{4}{3}\pi(1.2 \times 10^{-15})^3} \approx 10^{17}\ \mathrm{kg\,m^{-3}}.$$

Notice how A cancelled out so density is independent of A.

55 **a** As the energy increases the alpha particle can approach closer and closer to the nucleus. Eventually it will be within the range of the strong nuclear force and some alphas will be absorbed by the nucleus and will not scatter.

b Since the nuclear charge of aluminum is smaller than that of gold, the alphas will get closer to aluminum and so will experience the nuclear force first. Hence deviations will first be seen for aluminum.

56



57 **a** $E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = \frac{1.24 \times 10^{-12}\ \mathrm{MeV\,m}}{0.051\ \mathrm{MeV}} = 2.4 \times 10^{-11}$ m.

b Gamma ray.

58 I: beta plus decay

II: gamma decay

III: beta minus decay

IV: beta plus decay

59 **a** The discrete energies of alpha particles and gamma rays.

b ${}^{226}_{88}\mathrm{Ra} \rightarrow {}^{226}_{88}\mathrm{Ra} + \gamma$

c $E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{E} = \frac{1.24 \times 10^{-12}\ \mathrm{MeV\,m}}{0.0678\ \mathrm{MeV}} = 1.8 \times 10^{-11}$ m.

60 **a** We know that $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{3.0} = 0.231\ \text{s}^{-1}$.

b We start with $\frac{1}{100} \times 6.02 \times 10^{23} = 6.02 \times 10^{21}$ nuclei and so:

i $6.02 \times 10^{21} \times e^{-0.231 \times 1} = 4.8 \times 10^{21}$;

ii $6.02 \times 10^{21} \times e^{-0.231 \times 2} = 3.8 \times 10^{21}$;

iii $6.02 \times 10^{21} \times e^{-0.231 \times 3} = 3.0 \times 10^{21}$.

61 **a** The probability of decay within a half-life is always $\frac{1}{2}$.

b The probability that the nucleus will not decay after the passage of three half-lives is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$.

Hence the probability that the nucleus will decay some time within three half-lives is $1 - \frac{1}{8} = \frac{7}{8}$.

c The probability of decay in any one half-life interval is 0.5.

62 **a** We have that $A = \lambda N_0 e^{-\lambda t}$ so that the initial activity is λN_0. A mass of 1.0 g of polonium corresponds to $\frac{1.0}{210} = 4.762 \times 10^{-3}$ moles and hence $N_0 = 4.762 \times 10^{-3} \times 6.02 \times 10^{23} = 2.867 \times 10^{21}$ nuclei.
Since $\lambda = \frac{\ln 2}{138} = 5.023 \times 10^{-3}\ \text{day}^{-1}$ we find an initial activity of $5.023 \times 10^{-3} \times 2.867 \times 10^{21} = 1.44 \times 10^{19}$ Bq.

b The number of lead nuclei produced in 31 days is $N_0 - N_0 e^{-\lambda t} = 2.867 \times 10^{21} \times (1 - e^{-5.023 \times 10^{-3} \times 31}) = 4.13 \times 10^{20}$.

So the mass is $4.13 \times 10^{20} \times \frac{206}{6.02 \times 10^{23}} = 0.141$ g.

63 The decay constant is $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{12} = 0.0578\ \text{d}^{-1}$ and so $A = \lambda N_0 e^{-\lambda t} = 3.5 \times e^{-0.0578 \times 20} = 1.1$ MBq.

64 The decay constant is $\lambda = \frac{\ln 2}{6.0} = 0.1155\ \text{day}^{-1}$. Then $0.50 = A_0 e^{-\lambda t} = A_0 e^{-0.1155 \times 1.0}$ so that $A_0 = \frac{0.50}{e^{-0.1155 \times 1.0}} = 0.56$ MBq

65 After time t the number of uranium atoms remaining in the rocks is $N = N_0 e^{-\lambda t}$ and so the number that decayed (and hence eventually became lead) is $N_0 - N_0 e^{-\lambda t} = N_0(1 - e^{-\lambda t})$. Hence we have that $\frac{N_0(1 - e^{-\lambda t})}{N_0 e^{-\lambda t}} = 0.80$. This means that $1 - e^{-\lambda t} = 0.80\, e^{-\lambda t}$. Solving, we get $\lambda t = 0.5878 \Rightarrow t = \frac{0.5878}{\frac{\ln 2}{4.5 \times 10^9}} = 3.8 \times 10^9$ years.

66 The number of moles is $\frac{150}{12} = 12.5$ and the number of nuclei $12.5 \times 6.02 \times 10^{23} = 7.525 \times 10^{24}$.
Of these $7.525 \times 10^{24} \times 1.3 \times 10^{-12} = 9.7825 \times 10^{12}$ are C-14 nuclei. The decay constant is $\lambda = \frac{\ln 2}{5730 \times 365 \times 24 \times 60 \times 60} = 3.83587 \times 10^{-12}\ \text{s}^{-1}$. The initial activity was $\lambda N_0 = 3.83587 \times 10^{-12} \times 9.7825 \times 10^{12} = 37.5$ Bq. Hence from $11 = 37.5 e^{-\frac{\ln 2}{5730}t}$ OR $\ln\left(\frac{11}{37.5}\right) = -\frac{\ln 2}{5730}t \Rightarrow t = 10\,140$ years.

Note: In calculating the number of moles we assumed all the carbon was C-12.
Doing so introduces a negligibly small error.

67 The activity is given by $A = \lambda N = \lambda N_0 e^{-\lambda t}$ where $\lambda = \frac{\ln 2}{T_{1/2}}$ is the decay constant.

a $\frac{A_A}{A_B} = \frac{\lambda_A N_{0A}}{\lambda_B N_{0B}} = \frac{3}{4} \times 1 = 0.75.$

b $\frac{A_A}{A_B} = \frac{\lambda_A N_{0A} e^{-\frac{\ln 2}{4} \times 4}}{\lambda_B N_{0B} e^{-\frac{\ln 2}{3} \times 4}} = \frac{3}{4} \times \frac{e^{-\frac{\ln 2}{4} \times 4}}{e^{-\frac{\ln 2}{3} \times 4}} = 0.95.$

c $\frac{A_A}{A_B} = \frac{\lambda_A N_{0A} e^{-\frac{\ln 2}{4} \times 12}}{\lambda_B N_{0B} e^{-\frac{\ln 2}{3} \times 12}} = \frac{3}{4} \times \frac{e^{-\frac{\ln 2}{4} \times 12}}{e^{-\frac{\ln 2}{3} \times 12}} = 1.5.$

68 a $N = N_0 e^{-\lambda t} = N_0 e^{-1.2 \times 1} = 0.30119$, so the number of nuclei that decayed is $0.69881 N_0$, a fraction of 0.6988.

b $\lambda = \frac{1.2}{60} = 0.020 \text{ min}^{-1}$. The probability of decay within 60 minutes is then $1 - (1 - 0.020)^{60} = 1 - (0.98)^{60} \approx 0.70245$.

c The two answers are very close. We can make them even closer if we make the decay constant 'smaller' by expressing it in inverse seconds: $\lambda = \frac{1.2}{60 \times 60} = 3.333 \times 10^{-4} \text{ s}^{-1}$. Repeating the calculation above, the probability of decay within one hour is $1 - (1 - 3.333 \times 10^{-4})^{3600} \approx 0.6989$.

69 a If the mass (in grams) is m and the molar mass is μ, the number of moles of the radioactive isotope is $\frac{m}{\mu}$. The initial number of nuclei is then $N = \frac{m}{\mu} N_A$ since one mole contains Avogadro's number of molecules.

b The activity is $A = \lambda N = \lambda N_0 e^{-\lambda t} = \lambda \frac{m}{\mu} N_A e^{-\lambda t}$ and the initial activity is thus $A_0 = \lambda \frac{m}{\mu} N_A$. Measuring the initial activity then allows determination of the decay constant and hence the half-life from $\lambda = \frac{\ln 2}{T_{1/2}}$.

70 $A = \lambda N = \frac{\ln 2}{5.0 \times 60} \times 2.0 \times 10^{20} = 4.6 \times 10^{17}$ Bq.

Chapter 24

Test your understanding

1 **a** The mass difference is $235.0439 + 1.008665 - (97.9128 + 134.9165 + 3 \times 1.008665) = 0.1973$ u. The energy released is $Q = 0.1973 \times 931.5 = 184$ MeV OR $Q = 98 \times 8.5814 + 135 \times 8.3465 - 235 \times 7.5909 = 184$ MeV.

b In fission, a large nucleus splits into two lighter nuclei. The binding energy per nucleon of the products is *higher* than that of the splitting nucleus. The energy released is the difference in the binding energy of the products minus that of the splitting nucleus.

2 **a** ${}^{236}_{92}\text{U} \rightarrow {}^{117}_{46}\text{Pd} + {}^{117}_{46}\text{Pd} + 2{}^{1}_{0}\text{n}$.

b Two neutrons are produced as well as photons.

c The mass difference is $236.0456 - (2 \times 116.9178 + 2 \times 1.008665) = 0.1927$ u. The energy is $Q = 0.1927 \times 931.5 = 180$ MeV. (Since equal numbers of electron masses have to be subtracted from the atomic masses on each side of the reaction equation, we are allowed to use atomic masses here.)

3 $Q = 234 \times 7.60 + 4 \times 7.07 - 238 \times 7.57 = 5.02$ MeV.

4 $x = 35$ and $y = 88$

$Q = 145 \times 8.26 + 88 \times 8.56 - 235 \times 7.59 = 167$ MeV

5 See textbook.

6 See textbook.

7 **a** Kinetic energy of the neutrons produced.

b The thermal energy extracted from the moderator is used to form steam at high pressure that turns a generator.

8 The energy needed is $60 \times 12 \times 60 \times 60 = 2.59 \times 10^6$ J. In one reaction the energy released is $170 \times 10^6 \times 1.6 \times 10^{-19} = 2.72 \times 10^{-11}$ J. So we need $\frac{2.59 \times 10^6}{2.72 \times 10^{-11}} = 9.52 \times 10^{16}$ reactions. In each reaction the mass of uranium used is $235\text{ u} = 235 \times 1.66 \times 10^{-27} = 3.9 \times 10^{-25}$ kg. Hence the total mass needed is $9.52 \times 10^{16} \times 3.9 \times 10^{-25} = 3.7 \times 10^{-8}$ kg.

9 The radioactive products of a nuclear fission reaction.

10 Higher level waste is mainly the leftover uranium fuel in a reactor's fuel rods and the lighter elements produced in the fission of the uranium fuel, such as strontium-90 and cesium-137. These elements are highly radioactive. The radiation produced when they decay is very penetrating and delivers fatal doses even during short exposures, decades after they have been removed from the reactor.

Low level waste includes material that has become radioactive due to neutron absorption or that has been contaminated with radioactive material. It includes protective clothing, plastic shoe covers, cleaning materials such as mops and filters, tools and other equipment. These are not very radioactive, and after storage for some time, so that their radioactivity drops to below background, they can be disposed of as ordinary trash. Also included in low level waste are products generated in the mining of uranium and in the process of extracting the uranium from the ore.

11 Leakage of radioactive waste material into groundwater, lakes and rivers may enter the food chain, contaminating a large number of people. The energy produced when they decay produces heat, and so their storage must ensure cooling to avoid fires. The absorption of neutrons by uranium-238 eventually produces plutonium-239 and other heavy elements. These produce less heat and penetrating radiation, but they have very long half-lives which means they pose a danger even after thousands of years. Higher level waste products are kept in steel containers surrounded by thick reinforced concrete which are filled with water. The water serves both as a coolant as well as a radiation shield. After many years in these containers, the waste can be moved into dry casks; these are steel vessels surrounded by concrete. These storage facilities are temporary until satisfactory permanent storage is found. Many countries are considering burying the material in containers in special facilities deep into the earth.

Chapter 25

Test your understanding

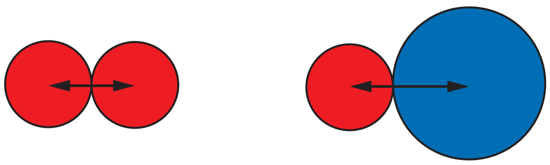
1 High temperatures so that nuclei move fast enough to overcome the Coulomb barrier (their electric repulsion).

High density to increase the probability for collisions, resulting in fusion.

In future commercial applications, the time for which the material can be kept hot enough.

2 The mass difference is 1.007825 + 2.014102 − 3.016029 = 5.898×10^{-3} u. This corresponds to an energy $5.898 \times 10^{-3} \times 931.5 = 5.5$ MeV.

3 The nucleus of tritium is larger and so the fusing nuclei do not have to come as close to each other as in deuterium–deuterium fusion.



4 A main issue is the lack of a reliable design that will be able to withstand the large temperatures and pressures of the fusing materials without frequent breakdowns and maintenance.

5 Like nuclear fission, nuclear fusion has the potential to produce large amounts of energy from abundant fuel (hydrogen) without the emission of greenhouse gases. Unlike fission, fusion will not produce radioactive waste in the same degree. Some of the products of fusion reactions are radioactive but generally with short half-lives and more easily containable.

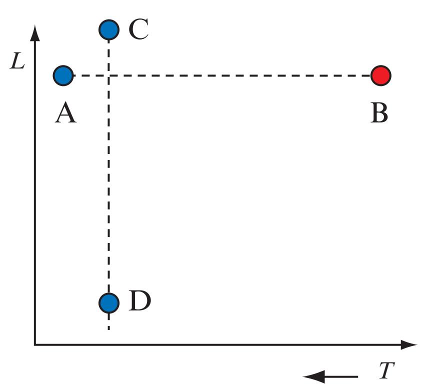
6 **a** An HR diagram is a plot of the luminosity of a star versus its surface temperature. Temperature is plotted increasing to the left on the horizontal axis.

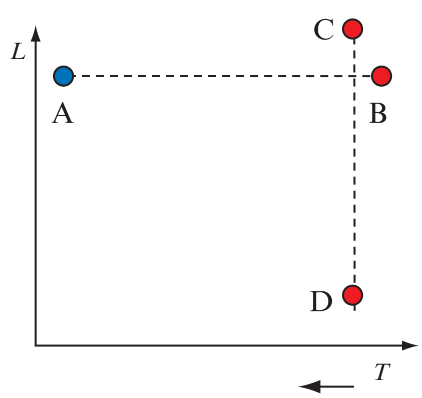
b Such a plot reveals that stars are grouped into large classes: the main sequence, which occupies a diagonal strip from top left to bottom right; the white dwarfs in the lower left corner and the red giants and supergiants in the top right corner.

7 **a** $\sigma 4\pi R_A^2 \times (3.2 \times 10^4)^4 = \sigma 4\pi R_B^2 \times (2.9 \times 10^3)^4$ hence $\frac{R_A}{R_B} = \sqrt{\frac{(2.9 \times 10^3)^4}{(3.2 \times 10^4)^4}} = 8.2 \times 10^{-3}$.

b $\frac{L_C}{L_D} = \frac{5.0 \times 10^{28}}{2.4 \times 10^{22}} = \frac{\sigma 4\pi R_C^2\, T^4}{\sigma 4\pi R_D^2 T^4} = \frac{R_C^2}{R_D^2}$ hence $\frac{R_C}{R_D} = \sqrt{\frac{5.0 \times 10^{28}}{2.4 \times 10^{22}}} = 1.4 \times 10^3$.

c A and B must be on the same horizontal line and C and D on the same vertical line. Two possibilities are shown.





8 The luminosity is approximately $10^{-4}\ L_{\odot}$. So approximately, $10^{-4}\ L_{\odot} = \sigma 4\pi R^2 \times 3000^4$, i.e. $10^{-4}\ \sigma 4\pi R_{\odot}^2 \times 6000^4 = \sigma 4\pi R^2 \times 3000^4$ OR $R = 4 \times 10^{-2}\ R_{\odot}$.

9 The distance is $\frac{1}{0.0125} = 80$ pc. This is $80 \times 3.086 \times 10^{16} = 2.47 \times 10^{18}$ m.

10 The distance is $\frac{1}{0.022} = 45.45$ pc. This is $45.45 \times 3.086 \times 10^{16} = 1.40 \times 10^{18}$ m. The luminosity is

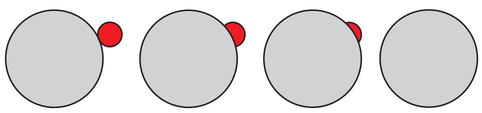
$b = \frac{L}{4\pi d^2} \Rightarrow L = 4\pi d^2 b = 4\pi \times (1.40 \times 10^{18})^2 \times 5.0 \times 10^{-10} = 1.23 \times 10^{28}$ W.

$L = \sigma 4\pi R^2 T^4 \Rightarrow R = \sqrt{\frac{L}{\sigma 4\pi T^4}} = \sqrt{\frac{1.23 \times 10^{28}}{5.67 \times 10^{-8} \times 4\pi \times 4500^4}} = 6.5 \times 10^9$ m.

11 a $\omega = \frac{2\pi}{T} = \frac{2\pi}{27 \times 24 \times 3600} = 2.69 \times 10^{-6}$ rad s^{-1}.

b $\Delta\theta = \omega t = 2.69 \times 10^{-6} \times 17 \times 10^{-3} = 4.57 \times 10^{-8}$ rad.

The figure shows how the moon hides the star.



c The distance is $\frac{1}{0.167} = 5.99$ pc OR $5.99 \times 3.086 \times 10^{16} = 1.85 \times 10^{17}$ m. The diameter of the star is then $4.57 \times 10^{-8} \times 1.85 \times 10^{17} = 8.45 \times 10^9$ m. The radius is then 4.2×10^9 m.

12 A red giant forms out of a main sequence star when a certain percentage of the hydrogen of the star is used up in nuclear fusion reactions. The core of the star collapses, and this releases gravitational potential energy that warms the core to sufficiently high temperatures for fusion of helium in the core to begin. The suddenly released energy forces the outer layers of the star to expand rapidly and to cool down. The star thus becomes a bigger but cooler star—a red giant.

13 a A planetary nebula refers to the explosion of a red giant star that ejects most of the mass of the star into space.

b Not all planetary nebulae (of the 3000 or so that are known to exist) appear as rings the way the famous helix and ring nebulae appear. The ring-like appearance is because the gas surrounding the centre is very thin. A line of sight through the outer edges of the nebula goes through much more gas than a line of sight through the centre. Hence the centre looks transparent while the edges do not.

c An equal and opposite force is exerted on the core, heating it up as it compresses it.

14 For the first reaction we must use nuclear masses:

$(1.007825 - m_e) + (1.007825 - m_e) - (2.014102 - m_e + m_e) = 4.50 \times 10^{-4}$ u so $Q = 4.50 \times 10^{-4} \times 931.5 = 0.42$ MeV (used $m_e = 0.000549$ u).

Here we can use atomic masses:

$1.007825 + 2.014102 - 3.016029 = 5.898 \times 10^{-3}$ u so $Q = 5.898 \times 10^{-3} \times 931.5 = 5.49$ MeV.

$2 \times 3.016029 - 4.002603 - 2 \times 1.007825 = 1.3805 \times 10^{-2}$ u so $Q = 1.3805 \times 10^{-2} \times 931.5 = 12.86$ MeV.

We showed in the textbook that the reactions are equivalent to:

$2 \times {}^1_1\text{H} + 2 \times {}^1_1\text{H} \rightarrow 2 \times {}^2_1\text{H} + 2 \times {}^0_1\text{e}^+ + 2 \times {}^0_0\nu$

$2 \times {}^1_1\text{H} + 2 \times {}^2_1\text{H} \rightarrow 2 \times {}^3_2\text{He} + 2 \times {}^0_0\gamma$

${}^3_2\text{He} + {}^3_2\text{He} \rightarrow {}^4_2\text{He} + 2 \times {}^1_1\text{H}$

$4 \times {}^1_1\text{H} \rightarrow {}^4_2\text{He} + 2 \times {}^0_1\text{e}^+ + 2 \times {}^0_0\nu + 2 \times {}^0_0\gamma$

Hence the total energy released is $Q = 2 \times 0.42 + 2 \times 5.49 + 12.86 = 24.68$ MeV. Adding 2×1.02 MeV gives 26.72 MeV. We would get the same answer if we did the calculation just for the combined reaction $4 \times {}^1_1\text{H} \rightarrow {}^4_2\text{He} + 2 \times {}^0_1\text{e}^+ + 2 \times {}^0_0\nu + 2 \times {}^0_0\gamma$.

15 a A 2 solar mass star would evolve to become a red giant. As the star expands in size into the red giant stage, nuclear reactions inside the core of the star are able to produce heavier elements than helium because the temperature of the core is sufficiently high. The red giant star will explode as a planetary nebula, ejecting most of the mass of the star into space and leaving behind a dense core. The core is no longer capable of nuclear reactions, and the star continues to cool down. The core is stable under further collapse because of electron degeneracy pressure. The core has a mass that is less than the Chandrasekhar limit and so ends up as a white dwarf.

b A 20 solar mass star would evolve to become a red supergiant. The red supergiant star will explode as a supernova ejecting most of the mass of the star into space and leaving behind a dense core. The core is no longer capable of nuclear reactions and the star continues to cool down. The core is stable under further collapse because of neutron degeneracy pressure. The core must have a mass that is less than Oppenheimer–Volkoff limit and so ends up as a neutron star.

16 a A white dwarf forms as the core left behind after planetary nebula explosion of a red giant star.

b White dwarfs are very small (earth size in radius) and very dense.

17 A main sequence star provides energy by nuclear fusion; no fusion takes place in a white dwarf.

With the exception of a few of the smallest main sequence stars (the red dwarfs), main sequence stars are larger in radius than a white dwarf.

The density of main sequence stars is much less than that of white dwarfs.

Main sequence stars are in equilibrium under the action of gravitational and radiation pressure whereas in white dwarfs the equilibrium is between a gravitational and a quantum mechanical pressure.

18 $\rho = \dfrac{M}{V} = \dfrac{1.0 \times 10^{30}}{\frac{4\pi}{3} \times (6.4 \times 10^6)^3} = 9.1 \times 10^8 \text{ kg m}^{-3}$

19 a An almost vertical region of the HR diagram above the main sequence with stars of variable luminosity.

b A supernova is the more violent explosion of a red supergiant star that also throws most of the mass of the star into space. A supernova is so bright that it can even be seen during the day if not too far away.

20 Nuclear fusion reactions make the star expand to a large size.

The outer layers cool down as they expand and so the peak wavelength shifts towards the red.

21 A main sequence star provides energy by nuclear fusion; no fusion takes place in a neutron star.

Even the smallest main sequence star is larger in radius than a neutron star.

The density of main sequence stars is much less than that of neutron stars.

Main sequence stars are in equilibrium under the action of gravitational and radiation pressure whereas in neutron stars the equilibrium is between a gravitational and a quantum mechanical pressure.

22 a A neutron star forms as the core left behind after a supernova in a red supergiant star.

b Neutron stars are very small (tens of kilometres in radius) and very dense.

23 a The beryllium produced in the first stage is used up in the second stage so the net result is three helium nuclei, making one nucleus of carbon.

b Use $3 \times {}^{4}_{2}\text{He} \rightarrow {}^{12}_{6}\text{C} + 2 \times \gamma$. Then the mass difference is $3 \times 4.0026 - 12.0000 = 7.8 \times 10^{-3}$ u and so the energy released is $7.8 \times 10^{-3} \times 931.5 = 7.2657$ MeV.

c One fusion reaction uses three helium nuclei, i.e. a mass of $3 \times 6.64 \times 10^{-27} = 1.99 \times 10^{-26}$ kg. The energy released per reaction is $7.27 \times 10^{6} \times 1.6 \times 10^{-19} = 1.16 \times 10^{-12}$ J. So we need $\frac{1.6 \times 10^{34}}{1.16 \times 10^{-12}} = 1.38 \times 10^{46}$ reactions per second. So the mass fused per second is $1.38 \times 10^{46} \times 1.99 \times 10^{-26} = 2.75 \times 10^{20}\ \text{kg s}^{-1}$. The time to fuse the mass of the star is then $\frac{3.0 \times 10^{32}}{2.75 \times 10^{20}} = 1.1 \times 10^{12}$ s OR 3.5×10^{4} years.

24 a The nuclei involved in intermediate stages have large electric charges and the Coulomb barrier is high. This requires higher kinetic energies and so higher temperatures. These temperatures are found in massive stars.

b The heavy nuclei in intermediate stages are all used up again in a later stage.